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NUMERICAL SIMULATION OF SPECIES DEPENDENT INTERACTION IN A POLLUTED ENVIRONMENT: ISSUES IN RANDOM NOISE, CONTROL AND POLICY

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ABSTRACT

In this study, we have utilized a sound numerical simulation technique to derive the conditions under which a legally-binding control related policy is necessary in order to mitigate the endemic Niger Delta polluted environmental issue. The implication of this present analysis if implemented will have several benefits for the Nigerian sustainable development formulation and is capable to revive the national economy.

INTRODUCTION

The fact is that severe uncertainties which characterize most polluted ecosystems in the developing countries necessitate the effect of a few environmental factors that are capable in contributing to some sort of random noise in such ecosystems. The present Ogoni ecosystem is no exception [1-10]. While all these cited authors have made substantial contributions to solve a few environmental problems using the tool of mathematical modelling, the application of a computational approach to measure the impact of a high random noise on the carrying capacities of interacting populations remains to be an un-resolved open environmental problem. The notion of a carrying capacity in the context of an ecological system is a popular biological parameter that is rarely quantified in the event of an environmental perturbation such as the induced-random noise of ecosystem characterization. Since the carrying capacity specifies the maximum size of a population that can sustain the growth of that population, it is a good scientific practice to quantify the impact of random on the carrying capacity. In this preliminary study, we have proposed the utilization of a low random noise intensity of 2.4 and a severe random noise intensity of 60 and used these noise values to study their impacts on the two carrying capacities. These random noise intensities were generated using the Normal probability distribution.

MATHEMATICAL FORMULATION

Following Dubey et al. [1], the model formulation under some simplifying assumptions is

$$\frac{dN}{dt} = r(B)N - \frac{r_0 N^2}{K(B,T)} \quad (1)$$

$$\frac{dB}{dt} = r_B(U,N)B - \frac{r_{B0} B^2}{K_B(T)} \quad (2)$$

$$\frac{dT}{dt} = \mathrm{Q(t)} - \delta_0 \mathrm{T} - \alpha \mathrm{BT} + \theta_1 \delta_1 \mathrm{U} + \pi v \mathrm{BU} \quad (3)$$

$$\frac{dU}{dt} = \beta \mathrm{B} + \theta_0 \delta_0 \mathrm{T} - \delta_1 \mathrm{U} + \alpha \mathrm{BT} - v \mathrm{BU} \quad (4)$$

Where N(t) stands for the density of the biological species, B(t) stands for the density of the resource biomass, T(t) stands for the concentration of the pollutant present in the environment and U(t) stands for the concentration of the pollutant taken up by the population.

Here, the initial conditions are $N(0) > 0$, $B(0) > 0$, $U(0) > 0$ when the independent variable time t is equal to zero. For the purpose of this simulation study, the two carrying capacities are defined by $K(B,T) = K_0 + K_1B - K_2T$ and $K_B(T) = K_{B0} - K_{B1}T$. Following Dubey and Hussain [1], we have considered the following precise parameter values: $K_0 = 60$, $K_1 = 0.02$, $K_2 = 0.03$, $B = 1.46$, $T = 9$, $K_{B1} = 0.05$. We have assumed the precise value of the model parameter K_{B0} to be 50.

METHOD OF SOLUTION

Following Dubey and Hussain [1], the function K(B,T) specifies the maximum density of the species population that the environment can support in the presence of the resource biomass and the environmental pollutant. Following these authors, this function is said to increase as the density of the resource biomass increases while it is said to decrease as the environmental concentration of the pollutant increases. In contrast, the function $K_B(T)$ specifies the maximum density of the resource biomass that the environment can support in the presence of the pollutant and it is said to decrease as the environmental concentration of the pollutant increase. In the event of an extrinsic fact such as the extreme climate change, it is highly probable that the model parameters which can be impacted are the K_0 and K_{B0} having the precise values of 60 and 50 respectively. In this study, the random noise intensity of 2.4 on these two parameters is calculated. The impact of the repeated random noise intensity on the two carrying capacities was determined over ten (10) repeated simulations. Then, the proportion (%) of the carrying capacity that is destroyed by the environmental noise was calculated using a realistic ecological-mathematical expression.

RESULTS AND DISCUSSION

The selected method of this present analysis has produced the following results as presented in the following list of Tables.

Table 1: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 2.4: Simulation 1

Example	K_{1new}	K_{1old}	Pd_1	K_{2new}	K_{2old}	Pd_2
1	57.8039	59.7592	3.27	47.3761	49.5500	4.40
2	59.4544	59.7592	0.51	47.3579	49.5500	4.42
3	58.2415	59.7592	2.54	49.3159	49.5500	0.47
4	59.0908	59.7592	1.12	48.2375	49.5500	2.65
5	57.4612	59.7592	3.85	47.2343	49.5500	4.67
6	59.3809	59.7592	0.63	47.2206	49.5500	4.70
7	57.4620	59.7592	3.84	48.3851	49.5500	2.35
8	57.8385	59.7592	3.21	49.2095	49.5500	0.70
9	58.7470	59.7592	1.69	47.3522	49.5500	4.44
10	57.8579	59.7592	3.18	47.2472	49.5500	4.65

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Unlike previous analysis, we have found that in a sample of ten repeated simulations, the maximum impact on the carrying capacity of the biological species N(t) is 3.85 percent whereas the maximum impact on the carrying capacity of the resource biomass B(t) is 4.70 percent. The interval impact for N(t) is [0.51, 3.85] whereas the interval impact for B(t) is [0.47, 4.70]. It is a universal knowledge that the standard deviation is the most popularly used measure of variation. In this context, the standard deviation is simply a measure of the variation from the mean. That is, the size of the standard deviation provides vital information about how spread out the data are from the mean. With respect to this simulation data, the standard deviations for $Pd_1$and Pd_2 data sets are 1.2937 and 1.6832 whereas the means for Pd_1 and Pd_2 data sets are 2.384 and 3.345.

Table 2: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 2.4: Simulation 2

Example	K_{1new}	K_{1old}	Pd_1	K_{2new}	K_{2old}	Pd_2
1	58.1854	59.7592	2.63	49.4643	49.5500	0.17
2	57.7213	59.7592	3.41	47.3084	49.5500	4.52
3	58.1302	59.7592	2.73	47.7314	49.5500	3.67
4	57.9757	59.7592	2.99	48.6087	49.5500	1.90
5	58.1861	59.7592	2.63	49.1392	49.5500	0.83
6	58.0647	59.7592	2.84	49.4736	49.5500	0.15
7	59.0946	59.7592	1.11	49.4392	49.5500	0.22
8	59.5261	59.7592	0.39	47.5737	49.5500	3.99
9	58.0916	59.7592	2.79	48.7890	49.5500	1.54
10	57.4787	59.7592	3.82	49.4673	49.5500	0.17

The maximum impact on the carrying capacity of the biological species N(t) is 3.82 percent while the maximum impact on the carrying capacity of the resource biomass B(t) is 4.52 percent. The interval impact for N(t) is [0.39, 3.82] whereas the interval impact for B(t) is [0.15, 4.52]. Similarly, the standard deviations for Pd_1 and Pd_2 data sets are 1.0258 and 1.7376 whereas the means for Pd_1 and Pd_2 data sets are 2.5340 and 1.7160.

From these 20 repeated simulations when the random noise value is 2.4, we have observed that the impacts on the carrying capacities can be considered to be less significant. This common observation is repeatedly observed after several numerical simulations. Therefore, the twenty numerical results obtained are adequate to reinforce the same incidence. On the other hand, how do impact measures behave when the noise intensity is fierce?

Table 3: A typical impact of the environmental ransom noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 60: Simulation 3

Example	K_{1new}	K_{1old}	Pd_1	K_{2new}	K_{2old}	Pd_2
1	38.6596	59.7592	35.31	0.2997	49.5500	99.40
2	24.6434	59.7592	58.76	16.5666	49.5500	66.57
3	4.7276	59.7592	92.10	32.3997	49.5500	34.61
4	14.3272	59.7592	76.03	4.3263	49.5500	91.27
5	36.9324	59.7592	38.20	15.4807	49.5500	68.76
6	55.2079	59.7592	7.62	46.3130	49.5500	6.53
7	27.9113	59.7592	53.29	2.8000	49.5500	94.35
8	3.7186	59.7592	93.78	41.7556	49.5500	15.73
9	25.6298	59.7592	57.11	21.3866	49.5500	56.84
10	59.0451	59.7592	1.195	29.3226	49.5500	40.82

From this random noise impact analysis, the minimum impact when the random noise intensity is 60 is about 2 times bigger than the minimum impact on N (t) as reported in Table 1 while the minimum impact when the random noise intensity is 60 is about 3 times bigger than the minimum impact on N (t) as reported in Table 2. When the random noise intensity is 60, the maximum impact on N (t) is about 24 times bigger than the maximum impact on N (t) as reported in Table 1 whereas when the random noise intensity is 60, the maximum impact on N (t) is about 25 times bigger than the maximum impact on N (t) as reported in Table 2.

In the same manner, the minimum impact when the random noise intensity is 60 is about 14 times bigger than the minimum impact on B (t) as reported in Table 1 while the minimum impact when the random noise intensity is 60 is about 44 times bigger than the minimum impact on B (t) as reported in Table 2. Similarly, when the random noise intensity is 60, the maximum impact on B (t) is about 21 times bigger than the maximum impact on B (t) is about 22 times bigger than the maximum impact on B (t) as reported in Table 2.

It is clear from this detailed numerical simulation that a severe environmental random noise is associated with a bigger impact on the carrying capacities of the density of the biological species and the density of the resource biomass. In contrast, it is unanimously established that a lower noise impact leads to a relatively weaker impact on the carrying capacities of the density of the biological species and the density of the resource biomass. On the basis of this analysis, we propose that a legally-binding control on the factors that promote severe environmental degradation should be implemented to mitigate the present challenging devastating Niger Delta environmental-ecosystem problem.

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