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RANDOM NOISE CHARACTERIZATION ON THE CARRYING CAPACITIES OF A COMPLEX SPECIES INTERACTION IN A HARSH OGONI ECOSYSTEM

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ABSTRACT
The process of the survival of species dependent on a limited resource in a polluted environment which isnot a new idea can be described by the technique of a mathematical modelling. We have utilised the technique of a numerical simulation to study the impact of environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment such as the Ogoni devastated ecosystem. The computational method of tackling this scientific problem is based on the fact that such a harsh ecosystem is characteristically stochastic and can be said to be heterogeneous as in most ecosystem scenarios. Arelatively lower random noise intensity is associated with a lesser proportion of the carrying capacity that is destroyed in contrast to a severe random noise intensity that produces a bigger impact on the carrying capacity. Since environmental pollution and other inherent toxicant-type inhibiting extrinsic factors can affect the Ogoni ecosystem simultaneously, we propose some sort of a sustainable mitigation strategy that is capable of providing a long term solution to the impact of crude oil pollution on the Ogoni ecosystem.

INTRODUCTION
From the current UNEP report on the Ogoni ecosystem [3], we can concur that its ecosystem should be characterised as being harsh in the sense that with the high level of crude oil pollution, this incidence makes accessibility to vital scientific data difficult to collect and interpret. Therefore, our present method of numerical simulation will be used to mimic the most near-evidence of the fierceness of the crude oil impact on the carrying capacities of species interaction over a longer period of time in the unit of days. One of the most likely simulation analysis methods is that of the random noise inclusion on the species interaction. The present method is expected to inform the appropriate policy implementation over the Ogoni devastated ecosystem which will likely take several decades before full recovery will be realised.The fact is that severe uncertainties which characterize most polluted ecosystems in the developing countries necessitate the effect of a few environmental factors that are capable in contributing to some sort of random noise in such ecosystems of the present Ogoni ecosystem is no exception [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. While all these cited authors have made substantial contributions to solve a few environmental problems using the tool of a mathematical modelling, the application

of a computational approach to measure the impact of a high random noise on the carrying capacities of interacting populations remains to be an un-resolved open environmental problem. One of the methods of studying the impact of the random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment is called numerical simulation using a Matlab programming scheme. Since the ecosystem is characteristically stochastic on one hand and the precise values of model parameters are estimated based on an approximated model formulation, it will be an interesting challenge to decide on how to measure the impact of environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment such as the Ogoni polluted ecosystem which has turned out to be a typical polluted environment in the world. One of the typical examples of induced random noise on the carrying capacities can be attributed to the oil spillage. The impact of this extrinsic factor on the carrying capacities which may take a longer period of time and huge funding for experimental scientists to quantify will be cost-effective and beneficial when a computational approach is successfully implemented. It is this approach we propose to utilise in a bid to mitigate this challenging environmental problem on the basis of a sound mathematical reasoning.

Mathematical Formulation

Following Dubey et al. [1], this model formulation under some simplifying assumptions is

$$\frac{dN}{dt} = \mathrm{r(B)N} - \frac{r_0 N^2}{K(B,T)}, \quad (1)$$

$$\frac{dB}{dt} = \mathrm{r_B(U,N)B} - \frac{r_{B0} B^2}{K_B(T)}, \quad (2)$$

$$\frac{dT}{dt} = \mathrm{Q(t)} - \delta_0 \mathrm{T} - \alpha \mathrm{BT} + \theta_1 \delta_1 \mathrm{U} + \pi\upsilon \mathrm{BU} \quad (3)$$

$$\frac{dU}{dt} = \beta \mathrm{B} + \theta_0 \delta_0 \mathrm{T} - \delta_1 \mathrm{U} + \alpha \mathrm{BT} - \upsilon \mathrm{BU} \quad (4)$$

where N(t)stands for the density of the biological species, B(t) stands forthe density of the resource biomass, T (t) stands forthe concentration of the pollutantpresent in the environment and U(t) stands forthe concentration of the pollutant taken up by thepopulation.

Here, the initial conditions are N (0) > 0, B (0) > 0, T(0) > 0, U(0) > 0 when the independent variable time t is equal to zero. For the purpose of this simulation study, the two carrying capacities are defined by $K(B,T) = K_0 + K_1B - K_2T$ and $K_B(T) = K_{B0} - K_{B1}T$. Following Dubey et al. [1], we have considered the following precise parameter values: $K_0 = 60$, $K_1 = 0.02$, $K_2 = 0.03$, B = 1.46, T = 9, $K_{B1} = 0.05$. We have assumed the precise value of the model parameter K_{B0} to be 50. The induced- random noise carrying capacity can be studied on the model parameter values K_1 and K_2.

Method of Solution

The posed problem demands the application of a simple calculation scheme without a resort to the technique of a solution trajectory over a longer period of time which does not yield the expected results in

Scientia Africana, Vol. 13 (No.2), December 2014. Pp328-335
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this context. By the application of the two carrying capacities formulae as specified in the last section of this paper, the detailed method of analysis is defined next.

According to Dubey and Hussain [1], the function K(B, T)specifies the maximum density of the species population that the environment can support in the presence of the resource biomass and the environmental pollutant. Following these authors, this function is said to increase as the density of the resource biomass increases while it is said to decrease as the environmental concentration of the pollutant increases. In contrast, the function K_B(T)specifies the maximum density of the resource biomass that the environment can support in the presence of the pollutant and it is said to decrease as the environmental concentration of the pollutant increases. In the event of an extrinsic factor such as the extreme climate change, it is highly probable that the model parameters which can be impacted are the K_0 and K_{B0} having the precise values of 60 and 50 respectively. In this study, the random noise intensities of 14.4, 36 and 57.6 on these two parameters were numerically simulated. The impact of the random noise intensity on the two carrying capacities was determined over ten (10) repeated simulations. The proportion of the carrying capacity that is destroyed by the environmental noise was calculated using a realistic ecological-mathematical expression.

Results and Discussion

The proposed method of analysis has produced the following results as presented in the following list of Tables.

Table 1: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 14.4: Simulation 1

Example	K_{1new}	K_{1old}	Pd_1	K_{2new}	K_{2old}	Pd_2
1	56.0902	59.7592	**6.14**	46.3238	49.5500	6.5
2	50.1424	59.7592	16.09	37.3908	49.5500	**24.5**
3	54.7989	59.7592	8.30	38.3105	49.5500	22.7
4	50.0344	59.7592	16.27	49.4533	49.5500	0.2
5	51.0879	59.7592	14.51	43.9805	49.5500	11.2
6	46.5689	59.7592	**22.07**	49.5334	49.5500	**0.03**
7	53.0999	59.7592	11.14	43.4394	49.5500	12.3
8	53.1220	59.7592	11.11	38.4597	49.5500	22.4
9	55.1156	59.7592	7.77	38.2498	49.5500	22.8
10	52.9717	59.7592	11.36	49.0350	49.5500	1.04

In this sample of ten repeated simulations, the maximum impact on the carrying capacity of the biological species N (t) is 22.07 percent whereas the maximum impact on the carrying capacity of the resource biomass B (t) is 24.5 percent. The interval of impact for N (t) is [6.14, 22.07] whereas the interval of impact for B (t) is [0.03, 24.5]. The range for the data sets Pd_1 is 15.93 percent whereas the range for the data sets Pd_2 is 24.47 percent.

Table 2: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 14.4: Simulation 2

Example	K_{1new}	K_{1old}	Pd_1	K_{2new}	K_{2old}	Pd_2
1	57.2266	59.7592	4.24	39.1567	49.5500	20.98
2	52.9410	59.7592	11.41	47.3508	49.5500	**4.44**
3	54.8470	59.7592	8.22	40.8036	49.5500	17.65
4	56.9981	59.7592	4.62	38.9167	49.5500	21.46
5	56.2622	59.7592	5.85	36.3391	49.5500	**26.66**
6	55.8847	59.7592	6.48	38.5268	49.5500	22.25
7	57.0425	59.7592	4.55	45.4100	49.5500	8.36
8	58.4472	59.7592	**2.20**	41.2526	49.5500	16.75
9	49.9188	59.7592	**16.47**	41.6791	49.5500	15.89
10	53.6287	59.7592	10.26	40.2700	49.5500	18.73

In the second scenario, the maximum impact on the carrying capacity of the biological species N (t) is 16.47 percent whereas the maximum impact on the carrying capacity of the resource biomass B (t) is 26.66 percent. The interval of impact for N (t) is [2.20, 16.47] whereas the interval of impact for B (t) is [4.44, 26.66]. The range for the data sets Pd_1 is 14.27 percent whereas the range for the data sets Pd_2 is 22.22 percent.

Table 3: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 14.4: Simulation 3

Example	K_{1new}	K_{1old}	Pd_1	K_{2new}	K_{2old}	Pd_2
1	51.3748	59.7592	14.03	41.7634	49.5500	15.72
2	47.2320	59.7592	20.96	45.7372	49.5500	7.70
3	55.1789	59.7592	7.67	47.8333	49.5500	3.47
4	46.2257	59.7592	**22.65**	40.2541	49.5500	18.76
5	52.8549	59.7592	11.55	40.3438	49.5500	18.58
6	51.9153	59.7592	13.13	40.2287	49.5500	18.81
7	51.9272	59.7592	13.11	39.1669	49.5500	20.96
8	52.2353	59.7592	12.59	35.2407	49.5500	**28.88**
9	56.6103	59.7592	5.27	48.0265	49.5500	3.08
10	58.1796	59.7592	**2.64**	48.6343	49.5500	**1.85**

In the third scenario, the maximum impact on the carrying capacity of the biological species N (t) is 22.65 percent whereas the maximum impact on the carrying capacity of the resource biomass B (t) is 28.88 percent. The interval of impact for N (t) is [2.64, 22.65] whereas the interval of impact for B (t) is [1.85, 28.88]. The range for the data sets Pd_1 is 20.01 percent whereas the range for the data sets Pd_2 is 27.03 percent.

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Table 4: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 14.4: Simulation 4

Example	K_{1new}	K_{1old}	Pd_1	K_{2new}	K_{2old}	Pd_2
1	53.9332	59.7592	9.75	43.0934	49.5500	13.03
2	54.4914	59.7592	8.82	38.5555	49.5500	22.19
3	50.7175	59.7592	15.13	38.4335	49.5500	22.44
4	46.3261	59.7592	**22.48**	35.5425	49.5500	**28.27**
5	56.9940	59.7592	**4.63**	47.5502	49.5500	4.04
6	49.7330	59.7592	16.78	48.1990	49.5500	**2.73**
7	52.1934	59.7592	12.66	41.9130	49.5500	15.41
8	47.3588	59.7592	20.75	42.5681	49.5500	14.09
9	54.0934	59.7592	9.48	39.8814	49.5500	19.51
10	49.0851	59.7592	17.86	42.0612	49.5500	15.11

In the fourth scenario, the maximum impact on the carrying capacity of the biological species N (t) is 22.48 percent whereas the maximum impact on the carrying capacity of the resource biomass B (t) is 28.27 percent. The interval of impact for N (t) is [4.63, 22.48] whereas the interval of impact for B (t) is [2.73, 28.27]. The range for the data sets Pd_1 is 17.85 percent whereas the range for the data sets Pd_2 is 25.54 percent.

Table 5: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 14.4: Simulation 5

Example	K_{1new}	K_{1old}	Pd_1	K_{2new}	K_{2old}	Pd_2
1	54.7521	59.7592	8.38	47.3900	49.5500	4.36
2	51.3195	59.7592	14.12	45.7751	49.5500	7.62
3	59.1191	59.7592	**1.07**	38.6790	49.5500	**21.94**
4	56.2631	59.7592	5.85	43.1794	49.5500	12.86
5	49.8549	59.7592	16.57	44.3771	49.5500	10.44
6	49.1559	59.7592	**17.74**	43.8662	49.5500	11.47
7	49.9180	59.7592	16.47	39.4117	49.5500	20.46
8	53.3900	59.7592	10.66	49.2681	49.5500	**0.57**
9	54.9948	59.7592	7.97	43.4399	49.5500	12.33
10	55.8673	59.7592	6.51	46.7124	49.5500	5.73

In the fifth scenario, the maximum impact on the carrying capacity of the biological species N (t) is 17.74 percent whereas the maximum impact on the carrying capacity of the resource biomass B (t) is 21.94 percent. The interval of impact for N (t) is [1.07, 17.74] whereas the interval of impact for B (t) is [0.57, 21.94]. The range for the data sets Pd_1 is 16.67 percent whereas the range for the data sets Pd_2 is 21.37 percent.

In the worst case scenario, the Ogoni ecosystem can suddenly become harsh instead of being benign as would have been the reality in other ecosystems with less severe environmental devastation. Here, we considered when the random noise intensity takes the value of 36. The impact of this high random noise value on the carrying capacities of species interaction within the Ogoni ecosystem reported as follows:

Table 6: Measuring the typical impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 36: Simulation 6

Example	K_{1new}	K_{1old}	Pd_1	K_{2new}	K_{2old}	Pd_2
1	41.2400	59.7592	30.99	17.7159	49.5500	**64.25**
2	38.5903	59.7592	35.42	43.9789	49.5500	11.24
3	52.5641	59.7592	12.04	34.8996	49.5500	29.57
4	32.8058	59.7592	45.10	19.8290	49.5500	59.98
5	31.3205	59.7592	**47.59**	38.0831	49.5500	23.14
6	40.5329	59.7592	32.17	46.3118	49.5500	6.54
7	55.7378	59.7592	6.73	44.6435	49.5500	9.90
8	35.3277	59.7592	40.88	31.7236	49.5500	35.98
9	52.9296	59.7592	11.43	31.7298	49.5500	35.96
10	54.4453	59.7592	**8.89**	47.5709	49.5500	**3.99**

In the sixth scenario, the maximum impact on the carrying capacity of the biological species N (t) is 47.59 percent whereas the maximum impact on the carrying capacity of the resource biomass B (t) is 64.25 percent. The interval of impact for N (t) is [8.89, 47.59] whereas the interval of impact for B (t) is [3.99, 64.25]. The range for the data sets Pd_1 is 38.7 percent whereas the range for the data sets Pd_2 is 60.26 percent.

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Table 7: Measuring the typical impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 57.6: Simulation 7

Example	K_{1new}	K_{1old}	Pd_1	K_{2new}	K_{2old}	Pd_2
1	27.9472	59.7592	53.23	13.2687	49.5500	73.22
2	57.9165	59.7592	**3.08**	14.1425	49.5500	71.46
3	38.8843	59.7592	34.93	46.6969	49.5500	**5.76**
4	31.5600	59.7592	47.19	38.4614	49.5500	22.38
5	52.6696	59.7592	11.86	37.7135	49.5500	23.89
6	51.3199	59.7592	14.12	38.6594	49.5500	21.98
7	57.3024	59.7592	4.11	12.9626	49.5500	**73.84**
8	43.5237	59.7592	27.17	18.5268	49.5500	62.61
9	19.7178	59.7592	**67.01**	20.8009	49.5500	58.02
10	28.8971	59.7592	51.64	23.9074	49.5500	51.75

In the seventh scenario, the maximum impact on the carrying capacity of the biological species N (t) is 67.01 percent whereas the maximum impact on the carrying capacity of the resource biomass B (t) is 73.84 percent. The interval of impact for N (t) is [3.08, 67.01] whereas the interval of impact for B (t) is [5.76, 73.84]. The range for the data sets Pd_1 is 63.93 percent whereas the range for the data sets Pd_2 is 68.08 percent.

The key contribution of this study is the fact that smaller environmental random noise intensity is associated with a smaller impact on the biological species and the resource biomass whereas relatively bigger environmental random noise intensity is associated with a bigger impact on the biological species and the resource biomass simultaneously. In the absence of an immediate sustainable mitigation strategy, we propose that a workable environmentally remediation process with some sort of lower noise intensity may provide an insight to alleviate the present Ogoni ecosystem oil polluted endemic problem. This cutting-edge research report is expected to inform an important aspect of a sustainable development initiative devoid of any political interest that complements the recommendation of the UNEP suggestions.

REFERENCES

B. Dubey and J. Hussain (2006), Modelling the survival of species dependent on a resource in a polluted environment, Nonlinear Anal.: Real World Appl. 7, pp. 187-210.

E.N. Ekaka-a, N.M. Nafo and A.I. Agwu (2012), Examining the extent of the sensitivity of a mathematical model parameter of survival of species dependent on a resource in a polluted environment, International Journal of Information Science and Computer

Mathematics, Volume 6, Numbers 1-2, pp.1-8.

UNEP Ogoni Special Final Draft: Stakeholder Democracy Network.

B. Dubey, B. Das (1999), Models for the survival of species dependent on resource in industrial environments, J. Math. Anal. Appl. 231, pp. 374–396.

B. Dubey, J. Hussain (2003), Nonlinear models for the survival of two competing species dependent on resource in industrial environments, Nonlinear Anal.: Real World Appl. 4, pp. 21–44.

B. Dubey (1997), Modelling the effect of toxicant on forestry resources, Ind. J. Pure Appl. Math. 28, pp. 1–12.

J.B. Shukla, B. Dubey (1997), Modelling the depletion and conservation of forestry resources: effects of population and pollution, J. Math. Biol. 36, pp. 71–94.

J.B. Shukla, B. Dubey (1996), H.I. Freedman, Effect of changing habitat on survival of species, Ecol. Model. 87 (1–3), pp. 205–216.

J.B. Shukla, H.I. Freedman, V.N. Pal, O.P. Misra, M. Agarwal, A. Shukla (1989), Degradation and subsequent regeneration of forestry resource: a mathematical model, Ecol. Model. 44, pp. 219–229.

D.M. Thomas, T.W. Snell, M. Jaffar (1996), A control problem in a polluted environment, Math. Biosci. 133, pp. 139–163.