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DETERMINING OUT-OF-CONTROL VARIABLE(S) IN A MULTIVARIATE QUALITY CONTROL CHART

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ABSTRACT

The Mason Young and Tracy (MYT) decomposition is considered to be the most promising techniques for identifying variables that significantly cause abnormality in any monitoring process. This technique has been applied to two and three process variables. In this work, we extend the technique to four process variables which can be used for identifying out-of-control variables when signal occurs in a control chart. Twenty four (24) decompositions are presented of which one of the decompositions is used to identify out-of-control variables. Shift in the mean as well as in the variance-covariance structure is detected using the MYT decomposition model.

Key words: Hotelling's T^2 control chart, MSQC library, Multivariate Statistical Process control, MYT decomposition.

INTRODUCTION

Process monitoring in which several variables are of interest is called Multivariate Statistical Process Control (MSPC). Multivariate control charts are widely used in practice to monitor the simultaneous performance of several related quality characteristics. The origin of multivariate quality control charts can be attributed to Hotelling (1947). A multivariate control scheme has a better sensitivity than the one constructed based on univariate control charts in monitoring multivariate quality process (Lu *et al.*, 1998). Ulen and Demir (2010), mentioned that multivariate control charts have a number of advantages when compared to the univariate control methods, this is because it monitors not only a single variable effects but also relationship among the variables by taking cognizance of all examined groups.

The need to execute MSPC in production process for quality improvements increases daily. Statistical methods play a very important role in quality improvement in manufacturing industries (Woodall, 2000). Despite the need to implement MSPC, there has been a serious challenge in the aspect of identifying variable(s) responsible for an out-of-control condition. The Hotelling's T^2 statistic is considered to be the most important techniques used for monitoring the mean vector in a multivariate quality control (Lowry and Montgomery, 1995). In this work, the Hotelling's T^2 statistic is used to identify out-of-control variables by decomposing the T^2 statistic using four quality characteristics.

Interpretation of Out-of-Control Variables

When T^2 indicates a change in the mean vector, corrective action is necessary. However, a T^2 value does not provide direct information about which variable causes the overall out-of-control condition. This information is relevant to quality engineers/analysts because they need to know which variable requires adjustment after the process has been declared out-of-control. Identifying the potential cause which involves a single variable.

Many approaches have been proposed for identifying which of the *p* variables is responsible for an out-of-control signal. Alt (1985), was the pioneer that recommended using a set of Bonferroni limits on each *p* individual variables as the method of choice for interpreting an out-of-control signal on a multivariate control chart. Doganaksoy*et al.*, (1991) later extended the idea of Bonferroni-type control limits by combining several procedures that resulted in a priority ranking of the variables. Hayter and Tsui (1994) buttressed the idea of Bonferroni-type control limits by providing a procedure for exact simultaneous control confidence intervals for each of the variable means, using simulation. Jackson (1985) and Pignatiello and Runger (1990) recommended the use of principal components to help in the interpretation of an out-of-control signal. They pointed out that by using the individual variables and the principal components with the univaiate charts, the information about the correlation effect of the variables is retained.

Verron*et al.*, (2010) developed a new method of detection and isolation using Bayesian network. They combined two approaches which are the causal decomposition of the T^2 statistic and the detection of fault with Bayesian network. According to them, the method permits the isolation of variables implicated in the fault.

Aparisi and Sanz (2010) developed a software interpretation based on the use of neural networks for MEWMA chart. They observed that users of MEWMA control chart have an easy tool that helps to take decision when the MEWMA control chart detects an out-of-control state. The potential drawback to this approach is that a very large training (in-control) data set is required to create the chart. The sample size required also required also increases as the number of variables being monitored increases. The use of T^2 decomposition proposed by Mason *et al.*, (1995) is considered as the most valuable technique (see Agog *et al*., 2014). The main idea of this method is to decompose the T^2 statistic into independent parts, each of which reflects the contribution of an individual variable. However, the drawback to the method is that the decomposition of the T^2 statistic into *p* independent T^2 components is not unique as *p!*for different non-independent partitions are possible. Mason *et al.*, (1997), provides an appropriate computing scheme that can greatly reduce the computational effort. This method was designed to deal with individual observations, but it can easily be generalized to handle rational subgroups. Ulen and Demir (2010), applied the T^2 decomposition using three variables obtained from a pharmaceutical industry to determine variables that contributed to out-of-control signal.

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METHODS

The MYT Decomposition technique of the Hotelling's T^2 statistic into orthorgonal components is used for identifying variables that significantly vary. The Hotelling'sT^2 statistic is a common tool used in multivariate process control chart. This method has two distinct phases of control charting, Phase I and Phase II. In Phase I, charts are used for retrospectively testing whether the process was in control when the first samples were being drawn. The second Phase or Phase II is used for monitoring future production. This phase is aimed at detecting whether subsequent production is capable of causing any of the observation vectors from the historical dataset to be out-of- control. The two Phases of the control charts were employed for monitoring out-of control condition.

Individual Observations

Most chemical and process industries have a subgroup size of*n*=1. This is because there are many quality characteristics that must be monitored. Suppose that *m* samples each of size *n=1* are to be monitored, and that *p* is the number of quality characteristics observed in each sample. Let $\bar{X}$ be the sample mean vector and **S** be the covariance matrix for the individual observations. The T^2 statistic for a *p* dimensional observation vector $X' = (x_1, x_2, ..., x_p)$ can be represented as

$$T^2 = (X - \bar{X})' S^{-1} (X - \bar{X}) \qquad \text{(i)}$$

The vector $(X - \bar{X})$ is partitioned as

$$(X - \bar{X}) = [(X^{(p-1)} - \bar{X}^{(p-1)}, x_p - \bar{x}_p)] \qquad \text{(ii)}$$

where

$X^{(p-1)} = (x_1, x_2, ..., x_p)$ represents the $(p-1)$ dimensional variables vector excluding the *pth* variable x_p.

$\bar{x}_p$ represents the mean of the *pth* variable $\bar{X}^{(p-1)}$ represents the corresponding $(p-1)$ elements of the mean vector.

Also, the variance-covariance matrix S can be partition as

$$S = \begin{pmatrix} S_{XX} & S_{xX} \\ S'_{xX} & S_p^2 \end{pmatrix}, \qquad \text{(iii)}$$

where

$S_{XX} = (p-1) \times (p-1)$ is the covariance matrix for the first $(p-1)$ variables.

S_p^2 is the variance of x_p

S_{xX} is the $(p-1)$ dimensional vector containing the covariances between x_p and the remaining $(p-1)$ variables.

Data Description

The data for the analysis were obtained from an Indomie Company in Northern Nigeria for percentage free fatty acid (% FFA) recorded for four (4) different machines in the production at the same time interval of thirty days. Let x_1, x_2, x_3 and x_4 denote the four machines. One of the 24 decompositions for the observation vector $(x_1, x_2, .x_3.., x_4)$ is given as

$$T^2_{(x_1,x_2,x_3,x_4)} = T_1^2 + T_{3.1}^2 + T_{2.1,3}^2 + T_{4.1,2,3}^2 \qquad \text{(iv)}$$

The computation begins by first determining the value of the conditional term $T^2_{4.1,2,3}$ from the above equation.

$$T^2_{4.1,2,3} = T^2_{(x_1,x_2,x_3,x_4)} - T^2_{(x_1,x_2,x_3)} \qquad \text{(v)}$$

The variance-covariance matrix and the mean vector for the observation vector $(x_1, x_2, .x_3.., x_4)$ is given as;

$$S_{44} = \begin{pmatrix} S_1^2 & S_{12} & S_{13} & S_{14} \\ S_{21} & S_2^2 & S_{23} & S_{24} \\ S_{31} & S_{32} & S_3^2 & S_{34} \\ S_{41} & S_{42} & S_{43} & S_4^2 \end{pmatrix} \text{ and}$$

$$\bar{X}^{(4)} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{pmatrix} \qquad \text{(vi)}$$

Thus the computation of $T^2_{(x_1,x_2,x_3,x_4)}$ is as follows;

$$T^2_{(x_1,x_2,x_3,x_4)} = (X^{(4)} - \bar{X}^{(4)})' S_{44} (X^{(4)} - \bar{X}^{(4)}) \quad \text{(vii)}$$

To obtain $T^2_{(x_1,x_2,x_3)}$, the original estimates of the mean vector and covariance structure is partitioned to obtain the mean vector and covariance matrix of the sub vector $X^{(3)} = (x_{1,} x_{2,} x_{3,})$. The corresponding partitioning is given as;

$$S_{33} = \begin{pmatrix} S_1^2 & S_{12} & S_{13} \\ S_{21} & S_2^2 & S_{23} \\ S_{31} & S_{32} & S_3^2 \end{pmatrix} \text{ and } \bar{X}^{(3)} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} \quad \text{(viii)}$$

Thus the computation $T^2_{(x_1,x_2,x_3)}$ is as follows;

$$T^2_{(x_1,x_2,x_3)} = (X^{(3)} - \bar{X}^{(3)})' S_{33} (X^{(3)} - \bar{X}^{(3)}) \text{ (ix)}$$

Also the decomposition of $T^2_{(x_1,x_2,x_3)}$ is given by

$$T^2_{(x_1,x_2,x_3)} = T_1^2 + T_{3.1}^2 + T_{2.1,3}^2 \qquad \text{(x)}$$

The above equation can be obtained by first computing the conditional term $T^2_{2.1,3}$ as follows

$$T^2_{2.1,3} = T^2_{(x_1,x_2,x_3)} - T^2_{(x_1,x_3)} \text{ (xi)}$$

To obtain the term $T^2_{(x_1,x_3)}$, the original estimates of the mean vector and covariance structure is partitioned to obtain the mean vector and covariance matrix of the sub vector $X^{(2)} = (x_1, x_3)$. The corresponding partition is given as

$$S_{22} = \begin{pmatrix} S_1^2 & S_{13} \\ S_{31} & S_3^2 \end{pmatrix} \text{ and } \bar{X}^{(2)} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_3 \end{pmatrix} \qquad \text{(xii)}$$

Hence, the computation of the term $T^2_{(x_1,x_3)}$ is as follows;

$$T^2_{(x_1,x_3)} = (X^{(2)} - \bar{X}^{(2)})' S_{22}^{-1} (X^{(2)} - \bar{X}^{(2)}) \text{ (xiii)}$$

Also, the decomposition for $T^2_{(x_1,x_3)}$ is given by

$$T^2_{(x_1,x_3)} = T_1^2 + T_{3.1}^2 \quad \text{(xiv)}$$

The term $T^2_{3.1}$ is obtained by computing the T^2 value of the sub vector $\bar{X}^{(1)} = (x_1)$. Hence, the unconditional term T_1^2 is computed by

$$T_1^2 = \frac{(x_1 - \bar{x}_1)^2}{s_1^2} \text{ (xv)}$$

Thus, $T^2_{3.1}$ is computed as

$$T^2_{3.1} = T^2_{(x_1,x_3)} - T^2_{(x_1)} = T^2_{(x_1,x_3)} - T_1^2 \quad \text{(xvi)}$$

The phase II control limits for individual observations are

$$UCL = \frac{p(m+1)(m-1)}{m^2 - mp} F_{\alpha,p,m-p} \text{ (xvii)}$$

$$LCL = 0.$$

MYT Decomposition Model

The observation vectors $(x_1, x_2, ..., x_p)$ are disintegrated into *p!*which generates the same overall T^2 statistic. For *p*=4, there are 4!=24 possible decompositions of the Hotelling's T^2 having 96 terms. Agog *et al*.,(2014) presented stepwise procedure for T^2 decomposition using four variables. The result is as follows;

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$$T^2 = T_1^2 + T_{2.1}^2 + T_{3.1,2}^2 + T_{4.1,2,3}^2$$

$$T^2 = T_1^2 + T_{3.1}^2 + T_{4.1,3}^2 + T_{2.1,3,4}^2$$

$$T^2 = T_1^2 + T_{4.1}^2 + T_{2.1,4} + T_{3.1,2,4}^2$$

$$T^2 = T_1^2 + T_{2.1}^2 + T_{4.1,2} + T_{3.1,2,4}^2$$

$$T^2 = T_1^2 + T_{3.1}^2 + T_{2.1,3}^2 + T_{4.1,2,3}^2$$

$$T^2 = T_1^2 + T_{4.1}^2 + T_{3.1,4}^2 + T_{2.1,3,4}^2$$

$$T^2 = T_2^2 + T_{1.2}^2 + T_{3.1,2}^2 + T_{4.1,2,3}^2$$

$$T^2 = T_2^2 + T_{3.2}^2 + T_{4.2,3}^2 + T_{1.2,3,4}^2$$

$$T^2 = T_2^2 + T_{4.2}^2 + T_{1.2,3}^2 + T_{3.1,2,4}^2$$

$$T^2 = T_2^2 + T_{1.2}^2 + T_{4.1,2}^2 + T_{3.1,2,4}^2$$

$$T^2 = T_2^2 + T_{3.2}^2 + T_{1.2,3}^2 + T_{4.1,2,3}^2$$

$$T^2 = T_2^2 + T_{4.2}^2 + T_{3.2,4}^2 + T_{1.2,3,4}^2$$

$$T^2 = T_3^2 + T_{1.3}^2 + T_{2.1,3}^2 + T_{4.1,2,3}^2$$

$$T^2 = T_3^2 + T_{2.3}^2 + T_{4.2,3}^2 + T_{1.2,3,4}^2$$

$$T^2 = T_3^2 + T_{4.3}^2 + T_{1.3,4}^2 + T_{2.1,3,4}^2$$

$$T^2 = T_3^2 + T_{1.3}^2 + T_{4.1,3}^2 + T_{2.1,3,4}^2$$

$$T^2 = T_3^2 + T_{2.3}^2 + T_{1.2,3}^2 + T_{4.1,2,3}^2$$

$$T^2 = T_3^2 + T_{4.3}^2 + T_{2.3,4}^2 + T_{1.2,3,4}^2$$

$$T^2 = T_4^2 + T_{1.4}^2 + T_{2.1.4}^2 + T_{3.1,2,4}^2$$

$$T^2 = T_4^2 + T_{2.4}^2 + T_{3.2,4}^2 + T_{1.2,3,4}^2$$

$$T^2 = T_4^2 + T_{3.4}^2 + T_{1.3,4}^2 + T_{2.1,3,4}^2$$

$$T^2 = T_4^2 + T_{1.4}^2 + T_{3.1,4}^2 + T_{2.1,3,4}^2$$

$$T^2 = T_4^2 + T_{2.4}^2 + T_{1.2,4}^2 + T_{3.1,2,4}^2$$

$$T^2 = T_4^2 + T_{3.4}^2 + T_{2.3,4}^2 + T_{1.2,3,4}^2$$

The first term after the equality sign in each of the decompositions above is called the unconditional term while the other terms three terms in each expression of the decompositions are called the conditional terms. This procedure requires the examination of $p \times 2^{(p-1)}$ pieces of terms within all possible decompositions. The conditional terms to be monitored are $p \times (2^{(p-1)} - 1)$ term.

Presentation of Hotelling's Control Charts

Before multivariate quality control chart can be implemented, the quality characteristics must be related. Thus, the first step in this analysis is the calculation of the correlation matrix between the four machines. The correlation matrix R below indicates that there is a strong positive relationship between the four machines and thus the multivariate technique is required.

$$R = \begin{pmatrix} 1 & 0.9181 & 0.6510 & 0.5228 \\ 0.9181 & 1 & 0.6405 & 0.5104 \\ 0.6510 & 0.6405 & 1 & 0.3926 \\ 0.5228 & 0.5104 & 0.3926 & 1 \end{pmatrix}$$

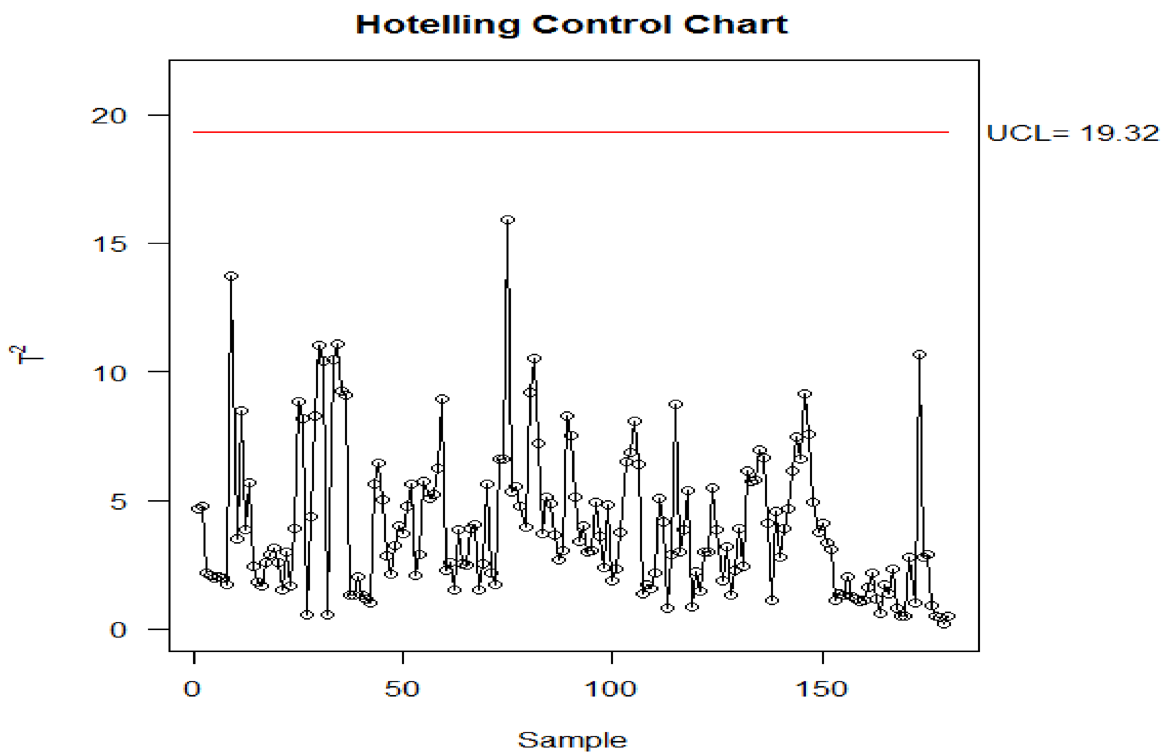

Figure 1: Phase I Hotelling's control chart for 180 observations

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Figure 1 indicates that the fatty acid reading in the four machines are in-control for the sampled observations. In this stage, the control limit is calculated based on *F* distribution with $\alpha = 0.01$. Since the process is in-control in the Phase I analysis, Phase II is performed to confirm the result obtained in Phase I. In Phase II, the Hotelling's T^2 control chart is used to determine whether the process remains in-control from 181 point and beyond.

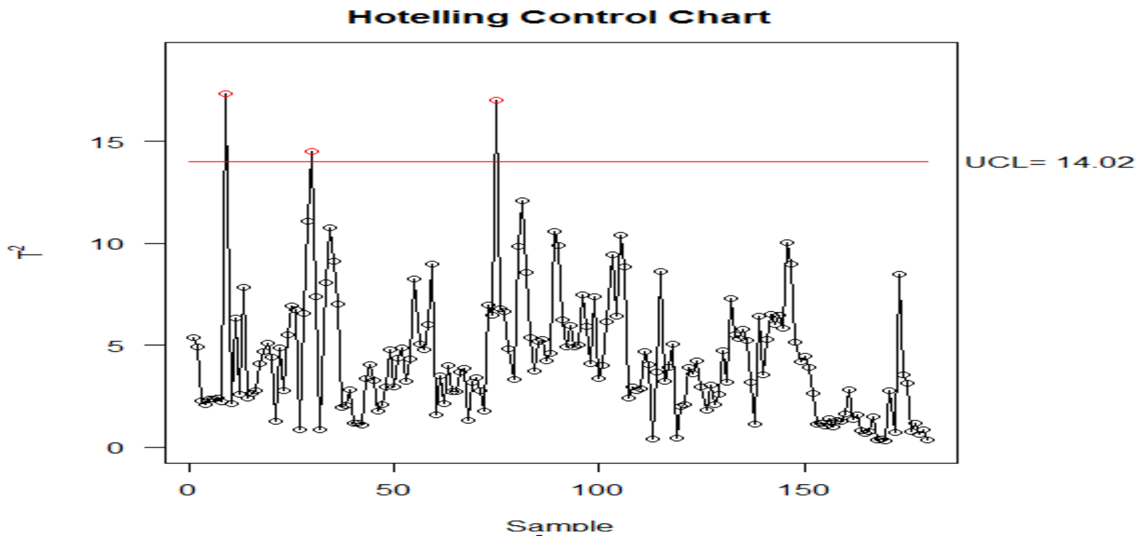

Figure 2: Phase II Hotelling's T^2 control chart for 180 observations

In figure 2, it is discovered that three points are above the upper control limit thereby making the process out-of-control. In this stage, the control limit is calculated based on *F* distribution with $\alpha = 0.01$. The samples that fall outside the upper control limit; sample 9 $(T^2 = 17.3379)$, sample 30 $(T^2 = 14.5099)$ and sample 75 $(T^2 = 17.0154)$. Thus, the machine responsible for the out-of-control is to be sourced out.

The variance-covariance matrix S, is

$$S = \begin{pmatrix} 0.00060 & 0.00057 & 0.00046 & 0.00049 \\ 0.00057 & 0.00065 & 0.00047 & 0.00049 \\ 0.00046 & 0.00047 & 0.00082 & 0.00042 \\ 0.00049 & 0.00049 & 0.00042 & 0.00140 \end{pmatrix}$$

and the process mean vector is

$$\bar{X} = \begin{pmatrix} 0.1629 \\ 0.1565 \\ 0.1416 \\ 0.1578 \end{pmatrix}$$

For sample 9, the observation point is

$$X_9 = \begin{pmatrix} 0.146 \\ 0.104 \\ 0.126 \\ 0.140 \end{pmatrix},$$

where the Hotelling's T^2 statistic for sample 9 is $T^2 = (X - \bar{X})' S^{-1} (X - \bar{X}) = 17.3379$

whichcan be obtained by summing the following decomposition terms;

$$T_1^2 + T_{3.1}^2 + T_{2.1,3}^2 + T_{4.1,2,3}^2 = 0.3267 + 0.0228 + 16.9883 + 0.0001 = 17.3379$$

For sample 30, the observation point is

$$X_{30} = \begin{pmatrix} 0.147 \\ 0.110 \\ 0.140 \\ 0.130 \end{pmatrix},$$

where the Hotelling'sT^{2}statistic for sample 30 is $T^2 = (X - \bar{X})' S^{-1} (X - \bar{X}) = 14.5099$ which can be obtained by summing the following decomposition terms;

$$T_1^2 + T_{3.1}^2 + T_{2.1,3}^2 + T_{4.1,2,3}^2 = 0.2817 + 0.2125 + 13.8553 + 0.1604 = 14.5099$$

For sample 75, the observation point is

$$X_{75} = \begin{pmatrix} 0.182 \\ 0.163 \\ 0.124 \\ 0.279 \end{pmatrix}$$

Where the Hotelling's T^2 statistic for sample 75 is $T^2 = (X - \bar{X})' S^{-1} (X - \bar{X}) = 17.0154$ which can be obtained by summing the following decomposition terms;

$$T_1^2 + T_{3.1}^2 + T_{2.1,3}^2 + T_{4.1,2,3}^2 = 0.8067 + 2.3114 + 2.2861 + 11.6112 = 17.0154$$

Table 1 shows the unique terms in the decomposition of four process variables. The unique terms are often used for identifying the sources of variation. There are 32 unique terms in the decomposition of the four machines. The T^2 value of the 32 unique terms are presented as follows;

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Table 1 Unique Terms of the T^2 Decomposition for Phase II Chart

Components of decomposition	T^2 values of Observation 9	T^2 values of observation 30	T^2 values of observation 75	Critical Value
T_1^2	0.3267	0.2817	0.8067	6.8163
T_2^2	4.8246	3.8462	0.0138	6.8163
T_3^2	0.2390	0.0000	0.3122	6.8163
T_4^2	0.2857	0.6429	**10.1150***	6.8163
$T_{1.2}^2$	**12.3066***	9.5009	3.6218	9.5587
$T_{1.3}^2$	0.1105	0.4942	2.8059	9.5587
$T_{1.4}^2$	0.1144	0.0145	0.9011	9.5587
$T_{2.1}^2$	**16.8045***	**13.0647***	2.9531	9.5587
$T_{2.3}^2$	6.0473	6.5684	0.3892	9.5587
$T_{2.4}^2$	5.0178	3.2607	3.1219	9.5587
$T_{3.1}^2$	0.0228	0.2125	2.3114	9.5587
$T_{3.2}^2$	1.4617	2.7222	0.6876	9.5587
$T_{3.4}^2$	0.0922	0.1167	3.8514	9.5587
$T_{4.1}^2$	0.0734	0.3757	**10.2094***	9.5587
$T_{4.2}^2$	0.4789	0.0574	**13.0989***	9.5587
$T_{4.3}^2$	0.1389	0.7596	**13.6542***	9.5587
$T_{1.2,3}^2$	11.0515	7.7811	4.7028	11.8802
$T_{1.2,4}^2$	11.8288	9.5619	1.6269	11.8802
$T_{1.3,4}^2$	0.0401	0.1497	0.0468	11.8802
$T_{2.1,3}^2$	**16.9883***	**13.8553***	2.2861	11.8802
$T_{2.1,4}^2$	**16.7322***	**12.8081***	3.5988	11.8802
$T_{2.3,4}^2$	6.2467	5.8162	0.6485	11.8802
$T_{3.1,2}^2$	0.2066	1.0031	1.6444	11.8802
$T_{3.1,4}^2$	0.0179	0.2519	2.9971	11.8802
$T_{3.2,4}^2$	1.3211	2.6722	1.3780	11.8802
$T_{4.1,2}^2$	0.0011	0.1191	11.1040	11.8802
$T_{4.1,3}^2$	0.0685	0.4151	10.8951	11.8802
$T_{4.2,3}^2$	0.3383	0.0074	**13.9135***	11.8802
$T_{1.2,3,4}^2$	10.7133	7.9341	2.4005	14.0213
$T_{2.1,3,4}^2$	**16.9199***	13.6006	3.0022	14.0213
$T_{3.1,2,4}^2$	0.2056	1.0444	2.1516	14.0213
$T_{4.1,2,3}^2$	0.0001	0.1604	11.6112	14.0213

*denotes significant value at 1%blevel of significance.

In sample 9, it is discovered that none of the unconditional terms has significantly contributed to the out-of-control signal. However, the variance-covariance structure between machine 1 and machine 2 has significantly contributed to the signal in sample 9.
$T^2 - T_{1.2}^2 = 17.3379 - 12.3066 = 5.0313$, where 5.0313 < 14.0213 (UCL)

also,

$T^2 - T_{2.1}^2 = 17.3379 - 16.8045 = 0.5334$, where 0.5334 < 14.0213 (UCL)
The result shows that the sub vector is in-control after removing the effect of machine 1 and 2. This indicates that the variance-covariance structure of the two machines Deviates significantly from the historical dataset. The same attention is given to sample 30.

Sample 75 indicates that the value of the unconditional T^2 component associated with quality machine 4 $(T_4^2 = 10.1150)$ significantly contributes to the out-of-control signal. Thus, the operating personnel should focus on machine 4 for this out-of-control point. Hence, the fatty acid reading of machine 4 should be removed from the observation vector in sample 75 as follows;

$T^2 - T_4^2 = 17.0154 - 10.1150 = 6.9004$, where 6.9004 < 14.0213 (UCL)
By removing the reading of machine 4 from sample 75, the sub vector is in-control.

Percentage Free Fatty Acid (%FFA) reading for daily dough moulding from four machines

Sample	Time	X_1	X_2	X_3	X_4
1	8:00am	0.125	0.116	0.141	0.144
2	10:00am	0.124	0.120	0.144	0.138
3	12:00am	0.128	0.123	0.109	0.120
4	2:00pm	0.129	0.125	0.109	0.122
5	4:00pm	0.130	0.123	0.111	0.120
6	6:00pm	0.130	0.123	0.110	0.120
7	8:00am	0.133	0.125	0.106	0.122
8	10:00am	0.135	0.127	0.108	0.124
9	12:00am	0.146	0.104	0.126	0.140
10	2:00pm	0.131	0.139	0.128	0.125
11	4:00pm	0.105	0.118	0.112	0.119
12	6:00pm	0.124	0.129	0.103	0.140
13	8:00am	0.132	0.107	0.104	0.128
14	10:00am	0.127	0.122	0.106	0.140
15	12:00am	0.144	0.132	0.106	0.138
16	2:00pm	0.146	0.132	0.111	0.136
17	4:00pm	0.146	0.128	0.109	0.134
18	6:00pm	0.152	0.132	0.111	0.136
19	8:00am	0.161	0.140	0.116	0.137
20	10:00am	0.158	0.138	0.118	0.137
21	12:00am	0.161	0.160	0.121	0.138
22	2:00pm	0.073	0.153	0.123	0.151
23	4:00pm	0.146	0.132	0.111	0.136
24	6:00pm	0.144	0.123	0.112	0.164
25	8:00am	0.101	0.112	0.109	0.13

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26	10:00am	0.102	0.110	0.117	0.120
27	12:00am	0.150	0.145	0.142	0.140
28	2:00pm	0.148	0.124	0.142	0.146
29	4:00pm	0.149	0.117	0.140	0.164
30	6:00pm	0.147	0.110	0.140	0.130
31	8:00am	0.155	0.174	0.102	0.132
32	10:00am	0.150	0.145	0.142	0.140
33	12:00am	0.166	0.184	0.147	0.099
34	2:00pm	0.179	0.180	0.093	0.130
35	4:00pm	0.174	0.173	0.092	0.131
36	6:00pm	0.165	0.178	0.102	0.136
37	8:00am	0.165	0.159	0.160	0.134
38	10:00am	0.173	0.166	0.167	0.146
39	12:00am	0.181	0.174	0.178	0.155
40	2:00pm	0.180	0.179	0.171	0.179
41	4:00pm	0.185	0.183	0.166	0.176
42	6:00pm	0.183	0.180	0.166	0.179
43	8:00am	0.165	0.180	0.171	0.179
44	10:00am	0.161	0.177	0.168	0.190
45	12:00am	0.164	0.175	0.169	0.199
46	2:00pm	0.178	0.184	0.172	0.192
47	4:00pm	0.175	0.173	0.177	0.190
48	6:00pm	0.176	0.176	0.181	0.203
49	8:00am	0.163	0.153	0.170	0.205
50	10:00am	0.160	0.163	0.163	0.205
51	12:00am	0.159	0.160	0.175	0.202
52	2:00pm	0.156	0.160	0.175	0.202
53	4:00pm	0.177	0.163	0.163	0.199
54	6:00pm	0.175	0.159	0.168	0.199
55	8:00am	0.180	0.153	0.169	0.191
56	10:00am	0.213	0.212	0.171	0.191
57	12:00am	0.210	0.211	0.170	0.218
58	2:00pm	0.209	0.207	0.162	0.237
59	4:00pm	0.193	0.188	0.165	0.270
60	6:00pm	0.185	0.188	0.170	0.196
61	8:00am	0.202	0.192	0.170	0.195
62	10:00am	0.190	0.182	0.171	0.191
63	12:00am	0.187	0.183	0.185	0.22
64	2:00pm	0.189	0.185	0.180	0.207
65	4:00pm	0.194	0.189	0.178	0.207
66	6:00pm	0.205	0.204	0.175	0.215
67	8:00am	0.206	0.205	0.164	0.203
68	10:00am	0.179	0.177	0.160	0.201
69	12:00am	0.196	0.187	0.166	0.211
70	2:00pm	0.177	0.191	0.165	0.206
71	4:00pm	0.197	0.189	0.160	0.196
72	6:00pm	0.188	0.184	0.159	0.202
73	8:00am	0.190	0.181	0.121	0.218
74	10:00am	0.172	0.166	0.126	0.237
75	12:00am	0.182	0.163	0.124	0.279

76	2:00pm	0.193	0.175	0.128	0.196
77	4:00pm	0.181	0.165	0.113	0.195
78	6:00pm	0.189	0.184	0.123	0.191
79	8:00am	0.177	0.177	0.137	0.22
80	10:00am	0.196	0.185	0.111	0.207
81	12:00am	0.197	0.177	0.11	0.207
82	2:00pm	0.195	0.177	0.125	0.215
83	4:00pm	0.198	0.181	0.182	0.203
84	6:00pm	0.191	0.200	0.188	0.201
85	8:00am	0.176	0.171	0.190	0.211
86	10:00am	0.178	0.159	0.164	0.206
87	12:00am	0.176	0.158	0.161	0.196
88	2:00pm	0.178	0.160	0.163	0.202
89	4:00pm	0.209	0.193	0.194	0.136
90	6:00pm	0.207	0.190	0.184	0.134
91	8:00am	0.197	0.190	0.185	0.136
92	10:00am	0.183	0.171	0.178	0.137
93	12:00am	0.176	0.159	0.177	0.137
94	2:00pm	0.178	0.161	0.168	0.138
95	4:00pm	0.179	0.161	0.173	0.151
96	6:00pm	0.161	0.137	0.161	0.135
97	8:00am	0.188	0.166	0.170	0.164
98	10:00am	0.178	0.163	0.151	0.130
99	12:00am	0.179	0.158	0.161	0.120
100	2:00pm	0.175	0.162	0.165	0.140
101	4:00pm	0.178	0.163	0.170	0.146
102	6:00pm	0.184	0.161	0.169	0.164
103	8:00am	0.191	0.166	0.166	0.130
104	10:00am	0.152	0.151	0.081	0.132
105	12:00am	0.153	0.134	0.101	0.076
106	2:00pm	0.154	0.131	0.103	0.099
107	4:00pm	0.164	0.153	0.126	0.13
108	6:00pm	0.163	0.150	0.121	0.131
109	8:00am	0.176	0.165	0.139	0.136
110	10:00am	0.155	0.148	0.163	0.134
111	12:00am	0.147	0.149	0.175	0.146
112	2:00pm	0.162	0.162	0.184	0.155
113	4:00pm	0.173	0.173	0.156	0.179
114	6:00pm	0.173	0.165	0.184	0.176
115	8:00am	0.156	0.149	0.077	0.179
116	10:00am	0.137	0.129	0.121	0.179
117	12:00am	0.137	0.131	0.126	0.19
118	2:00pm	0.134	0.131	0.124	0.199
119	4:00pm	0.145	0.144	0.128	0.155
120	6:00pm	0.148	0.144	0.113	0.179
121	8:00am	0.152	0.141	0.123	0.176
122	10:00am	0.143	0.130	0.137	0.179
123	12:00am	0.144	0.132	0.111	0.179
124	2:00pm	0.133	0.137	0.110	0.190
125	4:00pm	0.146	0.148	0.125	0.199

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126	6:00pm	0.142	0.139	0.145	0.162
127	8:00am	0.144	0.142	0.157	0.173
128	10:00am	0.189	0.181	0.159	0.165
129	12:00am	0.183	0.179	0.137	0.149
130	2:00pm	0.188	0.182	0.144	0.129
131	4:00pm	0.182	0.176	0.149	0.131
132	6:00pm	0.125	0.113	0.149	0.131
133	8:00am	0.112	0.110	0.123	0.144
134	10:00am	0.193	0.198	0.165	0.133
135	12:00am	0.206	0.215	0.178	0.167
136	2:00pm	0.204	0.214	0.170	0.175
137	4:00pm	0.192	0.198	0.187	0.181
138	6:00pm	0.142	0.137	0.113	0.147
139	8:00am	0.189	0.171	0.191	0.187
140	10:00am	0.152	0.146	0.146	0.102
141	12:00am	0.177	0.167	0.171	0.118
142	2:00pm	0.189	0.171	0.191	0.192
143	4:00pm	0.209	0.211	0.194	0.166
144	6:00pm	0.184	0.191	0.206	0.206
145	8:00am	0.191	0.198	0.16	0.128
146	10:00am	0.159	0.156	0.156	0.064
147	12:00am	0.154	0.146	0.136	0.063
148	2:00pm	0.149	0.148	0.141	0.083
149	4:00pm	0.128	0.123	0.114	0.087
150	6:00pm	0.137	0.134	0.135	0.087
151	8:00am	0.142	0.137	0.135	0.092
152	10:00am	0.137	0.135	0.094	0.145
153	12:00am	0.142	0.137	0.113	0.147
154	2:00pm	0.140	0.137	0.111	0.145
155	4:00pm	0.142	0.140	0.112	0.138
156	6:00pm	0.138	0.138	0.107	0.146
157	8:00am	0.140	0.138	0.114	0.136
158	10:00am	0.146	0.141	0.112	0.134
159	12:00am	0.145	0.139	0.113	0.136
160	2:00pm	0.157	0.149	0.117	0.137
161	4:00pm	0.159	0.145	0.117	0.137
162	6:00pm	0.137	0.139	0.108	0.138
163	8:00am	0.144	0.135	0.114	0.151
164	10:00am	0.149	0.144	0.122	0.135
165	12:00am	0.149	0.153	0.123	0.164
166	2:00pm	0.149	0.151	0.125	0.130
167	4:00pm	0.148	0.152	0.121	0.120
168	6:00pm	0.150	0.150	0.126	0.140
169	8:00am	0.151	0.148	0.124	0.146
170	10:00am	0.155	0.152	0.128	0.164
171	12:00am	0.122	0.118	0.113	0.130
172	2:00pm	0.142	0.141	0.123	0.132
173	4:00pm	0.124	0.140	0.137	0.076
174	6:00pm	0.143	0.135	0.111	0.099
175	8:00am	0.122	0.115	0.110	0.130
176	10:00am	0.143	0.141	0.125	0.131
177	12:00am	0.151	0.142	0.127	0.136
178	2:00pm	0.152	0.148	0.131	0.134
179	4:00pm	0.157	0.148	0.133	0.146
180	6:00pm	0.155	0.152	0.128	0.165

When monitoring multivariate quality control, it is important to construct the second phase of the Hotelling's T^2 control chart. The construction of Phase I alone is not efficient for monitoring the presence of signal in the process control chart. From the result it was discovered that Phase I retrospective analysis indicates that the process is in-control while in Phase II, three points were above the upper control limit. The MYT decomposition technique was used to derive the model for the four machines which shows the contribution of each machine independently as well as the relationships among the machines. Sample 9 and 30 were out of control due to the deviation between the variance-covariance structure of machine 1 and 2. Also, machine 4 was observed to be out-of-control in sample 75. Removing the effect of the machines that caused the out-of-control signal brought the process to a state of control. The R software was used for the analysis with the aid of MSQC package downloaded throught the Comprehensive R Archive Network (CRAN).

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