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CONVERGENCE CHARACTERIZATION OF THE BORDER STEADY-STATE SOLUTION OF TWO INTERACTING LEGUMES

[1]E. N. Ekaka-a, [2]N. M. Nafo, [3]C. Ugwu, and [4]I. A. Agwu,

[1]Department of Mathematics and Statistics,
University of Port Harcourt,
Port Harcourt, Nigeria

[2]Department of Mathematics and Computer Science,
Rivers State University of Science and Technology,
Port Harcourt, Nigeria.

[3]Department of Computer Science,
University of Port Harcourt,
Port Harcourt, Nigeria

[4]Department of Mathematics,
Abia State Polytechnic, Aba, Nigeria

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ABSTRACT

In this paper, we investigated the process and extent of studying the convergence of the steady-state solutions of a mathematical model of two interacting legumes (cowpea and groundnut) growing within the same uncontaminated environment. The convergence process was conducted using some standard numerical procedures. The result obtained shows that after repeated simulations, the unstable steady-state solution converged. Using our technique we found that the convergence of the steady-state solution (3.2599, 0) is reached when the value of the final time is 750.

Key words. Best-fit parameters, agricultural data, 1-norm, 2-norm, infinity-norm.

INTRODUCTION

Given the data on the growth of legumes ([3]), it is a challenging scientific problem to construct a mathematical model for the convergence of the unstable steady-state solutions of any two interacting legumes within an uncontaminated environmental setting ([1], [2], [8], [9], [11], [13]). In our previous study, we have selected the precise values of the deterministic competition model between two legumes of cowpea and groundnut over a growing season in days ([4], [7], [12], [16]). For this system of continuous nonlinear first order ordinary differential equations, the model parameters are $a = 0.0225$, $b = 0.006902$, $c = 0.0005$, $d = 0.0446$, $e = 0.01$ and f = 0.0133. It is very clear that this system of model equations has four steady-state solutions namely (0, 0), (0, 3.3534), (3.2599, 0) and (3.1908, 0.9543). The trivial steady-state solution is unstable because its calculated eigenvalues have two positive values of 0.0225 and 0.0446 whereas the steady-state solution (0, 3.3534) is clearly unstable because its eigenvalues are -0.0446 and 0.0208. The steady-state solution (3.2599, 0) is unstable having two eigenvalues of −0.0.0225 and 0.0120. The only unique positive steady-state solution (3.1908, 0.9543) is stable because its eigenvalues are -0.0234 and -0.0113.

The numerical challenge at this sophisticated level of analysis is to investigate the process and extent of convergence of the three unstable steady-state solutions as well as attempting to stabilize the only stable steady-state solution. The convergence method will be defined and discussed next.

METHODOLOGY

The aim of this paper is to stabilize a nonlinear system of first order ordinary differential equations of the form

$$\frac{dN_1(t)}{dt} = F\,(N_1(t),\ N_2(t)) \tag{2.1}$$

$$\frac{dN_2(t)}{dt} = G(N_1(t), N_2(t)). \tag{2.2}$$

with the initial conditions $N_1 = N_{10} > 0$ and $N_2 = N_{20} > 0$.

The arbitrary steady-state solution (N_{1e},N_{2e}) is unstable, that is, the point (N_1,N_2) is not convergent to (N_{1e},N_{2e}) when t tends to infinity. How do we stabilize the unstable steady-state solution? Following [2],[5], [6],[10],[14], [15], the process of stabilizing a mathematical model of a population system is conducted using three standard procedures namely find the linearized problem about (N_{1e},N_{2e}), next find a positive definite matrix P_i from the Riccati equation and apply the P_i matrix in the nonlinear equation to check if (N_1,N_2) is convergent to (N_{1e},N_{2e}). The next stage in our algorithm is implemented following these steps: put the steady-state solution (N_{1e},N_{2e}) which we want to stabilize; choose m = 0 for the unstable case and m = 1 for the stable case; choose different initial values for a different steady-state solution if this choice is realistic; choose a different final time *Tfinal* for a different steady-state solution; choose a time step k; choose the number of loops $M = \frac{Tfinal}{k}$; construct a feedback control; solve the nonlinear system and construct the subplots which will show the convergence behaviour of the uncontrolled and controlled solution trajectories.

By using this defined algorithm, we have been able to stabilize the unstable steady-state solutions for this system of model equations. Our contributions are presented and discussed in the next section of this paper.

DISCUSSION OF RESULTS

In this section, we will present and discuss the convergence of the border steady-state solution (3.2599, 0). The full stabilization of this steady-state solution is a challenging problem. We will attempt for the first time to study the extent of stabilizing (3.2599, 0) in which the cowpea legume will survive at its carrying capacity value of 3.2599 while the groundnut legume will be driven into extinction. The ecological survival of the fittest pattern of this present steady-state solution is the opposite of the first border steady-state solution. The stabilization of this second border steady-state solution poses difficult challenging issues which we have attempted for the first time to successfully stabilize.

For the first case of this simulation, we have considered the initial data (4, 4) and the step length $k = 0.01$. Our analysis has revealed that for this choice of simulation parameters, the steady-state solution (3.2599, 0) cannot be stabilized when the *Tfinal* values are 10, 20, 30, 40 and 50. For each stabilizing point, the first co-ordinate specifies the converging value of the cowpea population while the second co-ordinate specifies the converging value of the groundnut population. For example, when the value of the final time is 10, the converging point is (0.3440, 0.3538). For

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other final time values such as 20, 30, 40 and 50, the converging points are (0.2837, 0.3612), (0.2076, 0.3704), (0.1107, 0.3822) and (−0.0146, 0.3975).

For this simulation parameters, our contributions show that the chosen steady-state solution cannot converge.

For another initial data such as (10, 4) and k = 0.01, when the values of the final time are 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100, our calculated converging points are (4.9757,−0.2071), (4.5434,−0.1550), (4.2446,−0.1190), (4.0289,−0.0929), (3.8684,−0.0736), (3.7462,−0.0588), (3.6515,−0.0473), (3.5770,−0.0383), (3.5180,−0.0312) and (3.4707,−0.0255).

In another scenario when the values of the final time are 100, 200, 300 and 400 using the initial data (10, 10) and the step length of 0.01, the steady-state solution (3.2599, 0) starts to indicate some evidence of convergence. The convergence of this steady-state solution for other variations of the final time is displayed in the next table:

Examples	**Convergence of (3.2599, 0)**		
no	*Tfinal*	N_{1e}	N_{2e}
1	500	3.2536	0.0008
2	510	3.2547	0.0006
3	520	3.2556	0.0005
4	530	3.2563	0.0004
5	540	3.2569	0.0004
6	550	3.2574	0.0003
7	560	3.2579	0.0002
8	570	3.2582	0.0002
9	580	3.2585	0.0002
10	590	3.2587	0.0001
11	600	3.2589	0.0001
12	610	3.2591	0.0001
13	620	3.2592	0.0001
14	630	3.2594	0.0001
15	640	3.2595	0.0001
16	650	3.2595	0.0000438
17	660	3.2596	0.0000358
18	670	3.2597	0.0000292
19	680	3.2597	0.0000237
20	690	3.2597	0.0000192
21	700	3.2598	0.000015365
22	710	3.2598	0.000012219
23	720	3.2598	0.0000096076
24	730	3.2598	0.00000743910
25	740	3.2599	0.00000563869
26	750	3.2599	0.000004143890

Table 1. Convergence of the steady-state solution (3.2599, 0)

CONCLUSION

In this paper, our major contribution in this complex simulation analysis is that the steady-state solution (3.2599, 0) can be considered as fully stabilized after 26 repeated simulation runs. For this interaction between two types of legumes where the cowpea population will survive at its carrying capacity value of 3.2599 as the groundnut population tends to extinction, by using our technique of feedback control which is one of the current numerical techniques of stabilizing a mathematical model of a population system, we have successfully stabilized the unstable steady-state solution (3.2599, 0). We would expect our present contribution to provide useful insights in the ecological functioning and stabilization of two interacting legumes which are sources of economic livelihood and sustainable development.

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