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NUMERICAL SIMULATION OF INTERACTING FISH POPULATIONS WITH BIFURCATION

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ABSTRACT
This paper presents a numerical simulation of the complex interactions between two competing fish populations with bifurcation. Our first result shows that the co-existence steady-state solution will be stable when the birth rate of each fish population outweighs its death rate. Our second result shows that the stability of the co-existence steady-state solution of the interacting fish populations is lost when the birth and death rates are not changing. Our third result shows that the co-existence steady-state solution will be unstable when the death rate of the fish populations outweighs its birth rate. It is our expectation that these results will provide short-term and a relatively long-term insights in marine ecology.

INTRODUCTION
The interaction between two fish populations can be described by using a system of nonlinear first order ordinary differential equations (Loisel and Cartigny, 2009). In the context of modelling fish populations, an important model has been formulated by Dubey, Chandra and Sinha (2003) which tackles the problem of describing fishery resource within a reserve area. The implication of these modelling results for a fisheries management tool and planning has interesting insights in the work of Lauck et al. (1998), Houde (2002), Boncoeur et al. (2002) and Kar (2009).

Several types of sophisticated mathematical models under some distinct simplifying assumptions have also been implemented theoretically and analytically to describe the scientific problem of understanding the dynamics of interacting fish populations and its implication in the optimal management of renewable resources (Clark, 1990).

The rich application of mathematical models in theoretical ecology can provide a good reflection of their implementation in the understanding of the dynamic processes which are inherently involved in making practical applications (Haque, 2009). Executive bio-economic modeling of resources like fisheries has been conducted [Conrad and Clark 1987, Clark 1996, Anderson (2000), Gerber et al. 2003, Zhang et al. 2007, Das et al. 2009]

However, according to Khamis, Tchuenche, Lukka and Heilio (2011), over-fishing, the use of destructive fishing methods, pollution, and commercial aquaculture do have devastating consequences on marine biodiversity. In this scenario, sophisticated mathematical ecological model otherwise called the ecomathematical model has been successfully developed and validated to mitigate marine biodiversity to enhance marine reserve. This model formulation can be used for ecotourism purposes and also for tackling harvesting problems in prey-predator type of fishery (Kar and Chakraborly, 2009).

Despite these several mathematical modelling and mathematical analyses contribution in marine ecology, the application of important mathematical techniques such as the numerical simulation of interacting fish populations with bifurcation which is capable of providing further insights in fisheries management and marine biodiversity remain to be an open problem.

In this study, we are interested in using an analytical approach to study the interaction between two fish populations with bifurcation.

Mathematical Formulation (Loisel and Cartigny, 2009)

Following Loisel and Cartigny (2009), we consider the following modified assumptions by considering a fish population that lives in a zone characterized by a carrying capacity k=1. Hence, the following assumptions are made: The zone is split into two sub-zones with capacity to a respectively; in the first zone with capacity a and second zone with capacity 1-, fishing is allowed; there is a proportionate change in the varied parameters; the model parameters admit sensitivity analysis (Ekaka-a, 2009).

Following Loisel and Cartigny (2009), we shall consider the following non-autonomous system of first order ordinary differential equations.

$$\frac{dx_1(t)}{dt} = r_1 x_1 \ (t)\left(1 - \frac{x_1(t)}{a}\right)$$

$$\frac{dx_2(t)}{dt} = r_2 x_2(t)\left(1 - \frac{x_2(t)}{1-a}\right)$$

$x_1(0) = x_{10} > 0$

$x_2(0) = x_{20} > 0$

where

$r_1 = 0.4$, $r_2 = 0.05$, $a = 0.5$

Modified Model

$$\frac{dx_1(t)}{dt} = ks \ x_1 \ (t)\left(1 - \frac{x_1(t)}{a}\right)$$

$$\frac{dx_2(t)}{dt} = r_2 r_2 (t)\left(1 - \frac{x_2(t)}{1-a}\right)$$

where k and s are positive constants

Steady State Solution of the Modified Model

For the Modified model, the steady-state solutions are (0,0), (a,0), (0,1-a), (a,1-a). The first steady-state solution (0,0) is the trivial solution. The second unique steady-state solution will occur if we assume that $x_1e \neq 0$, $x_2e = 0$. The third and fourth unique steady-state solution will occur if we assume that $x_1e = 0$, $x_2e \neq 0$ as well as $x_1e \neq 0$, $x_{2e} \neq 0$

Characterization of Steady-State Solution of the Modified Model

In the absence of bifurcation, we have the following characterizations of the four unique steady-state solutions. (0,0) is unstable because λ_1 = k, $\lambda_2 = r_2$; (a,0) is unstable because λ_1 =- k, $\lambda_2 = r_2$; (0,1-a) is unstable because λ_1 =- k, $\lambda_2 =- r_2$; (a, 1-a) is stable because λ_1 =- k, $\lambda_2 = - r_2$.

Bifurcation of the Steady State Solution (a, 1-a)

In this study, we are interested to study the bifurcation of the only unique positive steady-state solution which has two negative eigenvalues specified by λ_1=- k, $\lambda_2 = - r_2$. The changing patterns of the eigenvalues for this steady-state solution are displayed in the table below:

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s/n	Different Cases	λ_1 = -k	λ_2 = -r_2	Comment
1.	λ_1 <0, λ_2 <0	k>0	r_2>0	stable
2.	λ_1 <0, λ_2 =0	k>0	r_2=0	sitting on the cusp
3.	λ_1 =0, λ_2 <0	K=0	r_2>0	sitting on the cusp
4.	λ_1 =0, λ_2 =0	K=0	r_2=0	stability is lost
5.	λ_1 >0, λ_2 =0	K<0	r_2=0	sitting on the cusp
6.	λ_1 =0, λ_2 >0	K=0	r_2<0	sitting on the cusp
7.	λ_1 >0, λ_2 <0	K<0	r_2>0	unstable
8.	λ_1 <0, λ_2 >0	k>0	r_2<0	unstable
9.	λ_1 >0, λ_2 >0	K<0	r_2<0	unstable

DISCUSSION OF CORE RESULTS

Bifurcation result for the steady-state solution (a , 1-a) of the Modified Model

Case 1: λ_1 <0, λ_2 <0→ k>0, r_2>0, where k and r_2 are called the intrinsic growth rates for the two fish populations. In marine ecology, this important result clearly shows that the birth rate of each fish population will outweigh the death rate.

Case 4: λ_1 <0, λ_2 <0→ k=0, r_2=0. In this scenario, both the birth and death rates are not changing. Hence, the population size is constant.

Case 9: λ_1 <0, λ_2 <0→ k<0, r_2<0. In marine ecology, this result shows clearly that the birth rate of each fish population will not outweigh the death rates.

CONCLUDING REMARKS

In this study, we have found a few fundamental changes in the stability of the positive steady-state solution. The implications of our three core results are expected to provide useful insights to environmental marine ecologists working in marine ecology functioning and stability.

A few possible extensions of this present numerical stimulation study the bifurcation of the other calculated steady-state solutions and to carry out a study on the impact of other climate change factors such as rising sea level. Construction of an appropriate optimal controller to stabilize the three unstable steady-state solutions which we have calculated in this study. The capability of stabilization of these unstable steady-state solution will drive the fish populations from going into extinction which is detrimental and counter-sustainable development. Hence, the rigorous stabilization process will have substantial benefits in restoring marine bio-diversity and sustain local means of livelihoods within the Niger Delta fishing industry. This proposed idea will be one of our next analysis of interdisciplinary research subject to adequate funding.

Finally, the deterministic system of fish population model equations can be reformulated as a system of stochastic differential equations. The rigorous analysis of this extension is proposed as a further research topic which we could not tackled in this single study because of the difficulty inherent in analyzing complex stochastic models.

REFERENCES

B. Dubey, P. Chandra, P. Sinha. (2003). A model for fishery resource with reserve area, Non Linear Analysis. RWA (4) 625-637

C.W. Clark (1996), Marine reserves and the precautionary management of fisheries, J. Ecol. Appl. 6(2), pp. 369-370.

C.W. Clark, (1990). Mathematical Bioeconomics: The Optimal Management of Renewable Resources, 2nd ed., John Wiley and Sons. New-York

E.D. Houde, (2002). Testimony on the use of Marine Protected Areas (MPAs) as a Fisheries Management Tool, as a means of protect and restore Marine Ecosystem and as a Research Tool, Subcommittee on Fisheries Conservation, Wildlife and Oceans.

E.N. Ekaka-a (2009). Computational and Mathematical Modelling of Plant Species Interactions in a Harsh Climate, PhD Thesis, Department of Mathematics, The University of Liverpool and The University of Chester, United Kingdom, 2009.

J. Conrad and C. Clark (1987), Natural Resources Economics, Cambridge University Press, Cambridge, New York, Melbourne.

J. Boncoeur, F. Alban, O. Guyader, O. Thebaud, (2002). Fish, fishers, seals and tourists: Economics consequences of creating a marine reserve in a multi-species, multi-activity context, Natural Resources Modelling 15(4) 1-25.

L.G. Anderson (2000), Marine reserves, Conference of IIFET, 10-14 July, Corvallis, OR, USA.

L.R. Gerber, L.W. Botsford, A. Hastings, H.P. Possingham, S.D. Gaines, S.R. Palumbi, and S. Andelman (2003), Population models for marine reserve design: a retrospective and prospective synthesis, Ecol. Appl. 13(1) (Suppl.), pp.547-564.

Loisel, P and Cartigny, P (2009): How to Model Marine Reserves. Non-linear Analysis Real World Applications, (10) 1784-1796.

M. Haque (2009), Ratio-dependent predator-prey models of interacting population, Bull. Math. Biol. 71, pp. 430-452

R. Zhang, J. Sun, and H. Yang (2007), Analysis of a prey-predator fishery model with prey reserve, Appl. Math. Sci. 1(50), pp. 2481-2492.

S.A. Khamis, J.M. Tchuenche, M. Lukka, M. Heilio (2011), Dynamics of fisheries with prey reserve and harvesting, International Journal of Computer Mathematics, Vol. 88, No.8, pp. 1776-1802.

T. Das, R.N. Mukherjee, and K.S. Chaudhuri (2009), Harvesting of a prey-predator fishery in the presence of toxicity, Appl. Math. Model 33(5), pp.2282-2292.

T. Lauck, C.W. Clark, M. Mangel, G.R. Munro, (1998): Implementing the precautionary principles in fisheries management through marine reserves. Ecological Applications 8(1) 72-78.

T.K. Kar (2009), A model for fishery resource with reserve area and facing prey-predator interaction, J. Can. Appl. Math. Q. 14(4), pp.385-399.

T.K. Kar and K. Chakraborly (2009), Marine reserves and its consequences as a fisheries management tool, World J. Model Sim. 5(2), pp.83-95.