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PARAMETER RANKING OF STOCK MARKET DYNAMICS: A COMPARATIVE STUDY OF THE MATHEMATICAL MODELS OF COMPETITION AND MUTUALISTIC INTERACTIONS

[1]E. N Ekaka-a and [2] N. M. Nafo
[1]Department of Mathematics/Statistics
University of Port Harcourt,
Port Harcourt, Rivers State, Nigeria

[2]Department of Mathematics/Computer Science,
Rivers State University of Science and Technology,
Port Harcourt, Nigeria

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ABSTRACT
In this paper, we study a parameter ranking of stock market dynamics for two types of complex interactions in the context of stock markets over a trading period using a Lotka-Volterra system of non-linear first order ordinary differential equations. We started this study by describing the model under consideration and pointing out a few of the complexities inherent in our proposed model. Using the existing literature, we selected a set of best-fit model parameters and initial conditions which form the basis for our parameter ranking. Our parameter ranking or sensitivity analysis permits us to make realistic conclusions about the relative importance of individual parameters. Our approach also permits us to gain some useful insights into the ability of the model to reflect what is obtainable within the stock markets.

***Key words**: Sensitivity analysis, competition interaction, mutualistic interaction*

INTRODUCTION
The notion of both sensitivity analysis at a point and over a time interval are dynamic areas of numerical mathematics. In the application of this technique in analyzing stock market data, the work of Okoroafor and Osu (2009) on the Aba Stock Exchange forms the first foundation study in Nigeria. In their work, an empirical optimal portfolio selection model was developed and analyzed using an appropriate probability distribution law. Although this pioneering research is based on a selection model, it did not address the complex interaction between two populations of investors especially when one investor is prone to invest and the other is less prone to invest.

One of the key limitations in the work of Okoroafor and Osu (2009) concerns the lack of which parameter should be efficiently estimated and which should be taken as a rough estimate. The interesting portfolio selection model proposed by these experts is yet to capture the derivation of a deterministic mathematical model which can fully describe the following features of a dynamical system such as complex interactions in the context of stock markets over a trading period. The application of sensitivity analysis which we proposed in this pioneering study complements the pioneering study of Okoroafor and Osu (2009).

Outside the Nigerian stock markets, the concept of sensitivity analysis has been extensively applied to solve complex scientific problems [3], [15], [9], [2], [17], [13], [6], [18].

The application of the principle of sensitivity analysis is an important numerical concept in our study because it would be used to find those model parameters whose variation will have a biggest effect on the solution of the model equations. For the stock market interaction problems, sensitivity analysis can indicate which parameters need to be estimated most accurately and which need only be given as rough estimates. Hence, sensitivity analysis can guide effort in parameter estimation. Although the concept of the sensitivity of model parameters is not new, it can be recognized as an old need within the scientific community. One of the new methods of meeting this old need is the implementation of a technique of a sensitivity analysis over a time interval which we have proposed in this paper.

We know that norms serve the same purpose on vector spaces that absolute value does on the real line. The concept of a norm on a vector space and that of absolute value on a real line furnishes a measure of distance.

What then is sensitivity analysis over a time interval. It is simply the generic term for the changes in the output of an initial value problem due to changes in the data. How does this numerical method work? The numerical method of sensitivity or the principle of parsimony works in the following pattern:

(1) Write down some complicated interaction model
(2) Fix the values of each possible model parameter
(3) Take one parameter at a time, vary it and see how much this variation changes the solution
(4) If the variation of one parameter changes the solution a lot, then this parameter would be called a more sensitive parameter.
(5) On the other hand, if the variation of another parameter produces a small change in the solution, then this parameter would be called a less sensitive parameter.

We do not necessarily have to remove the less important parameters according to the hypothesis of the principle of parsimony. The notion of sensitivity analysis is a widely applied numerical method often being used in the study of biological, immunological and applied science problems ([3], [15], [9], [2], [11], [4], [17], [5], [1], [13], [6], [18]).

We remark that most of these applied sensitivity analysis are considered at a particular time when a solution is reasonably constant whereas our method as proposed in this paper will consider data points over a period of time.

Sensitivity analysis is a standard method for studying mathematical models especially if a further numerical simulation is required to analyse a particular research problem. It aims to find the dependency between model predictions and the particular set of parameter values being used ([10]). This knowledge can be useful in the study of other complex stock market interaction systems.

In this context, a sensitivity analysis is a general procedure which entails changing parameter values and observing the corresponding changes in the model prediction. It seems easy to describe this procedure, but conducting a detailed sensitivity analysis in any specific case is a dauting task. Problems may include

(1) How much to vary each parameter by
(2) What combinations of parameter values are acceptable
(3) What values of the explanatory variables to use
(4) Which model parameters when varied will have the biggest or smallest effect on the solutions.
(5) How to interpret the results.

For the recent research in modelling stock market data, see ([16], [14], [8]).

This paper is organized into the following sections. Section 2 deals with the mathematical formulation of two interacting populations of investors over a trading period in days. Section 3

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is focused in defining the concept of sensitivity analysis and its practical application in stock market competition data, while section 4 will apply the technique of sensitivity analysis over a time interval to analyse a mutualistic stock market interaction.

MATHEMATICAL FORMULATION

If y(t) and z(t) are two populations of investors during a trading period *t* in days, see also [8]. The models of competition interaction and mutualistic interaction have the following forms.

(2.1)
$$\frac{dy}{dt} = y(t)(a - \beta_1 y(t) - \gamma_1 z(t)),$$

(2.2)
$$\frac{dz}{dt} = z(t)(d - \beta_2 z(t) - \gamma_2 y(t)),$$

(2.3)
$$\frac{dy}{dt} = y(t)(a - \beta_1 y(t) + \gamma_1 z(t)),$$

(2.4)
$$\frac{dz}{dt} = z(t)(d - \beta_2 z(t) + \gamma_2 y(t)),$$

Here the nonnegative constants a, d, β_i, γ_i, i = 1, 2 are given respectively, as the intrinsic growth rate, the intra-species competitive parameter and the inter-species competitive parameter. The model equations of competition have four steady states namely

$$y = 0, \quad z = 0,$$

$$y = 0, \quad z = \frac{d}{\beta_2},$$

$$y = \frac{a}{\beta_1}, \quad z = 0,$$

$$y = \frac{a\beta_2 - d\gamma_1}{\beta_1\beta_2 - \gamma_1\gamma_2}, \quad z = \frac{d\beta_1 - a\gamma_2}{\beta_1\beta_2 - \gamma_1\gamma_2}$$

For the purpose of this paper, we will study the parameter ranking of stock market dynamics of competition and mutualistic interactions.

CALCULATION OF SENSITIVITY ANALYSIS: Competition Interaction

Following our recent data ([14], [8]) in which a = 0.0373, d = 0.03, β_1 = 0.0014, β_2 = 0.0005, γ_1 = 0.0005 and γ_2 = 0.0.0009.

The starting investment values for the two populations of investors are 10 thousand naira and 15 thousand naira respectively. Here we choose the trading interval of T = 180 days in the range T = 0 : 5 : 180.

In this present analysis, the sensitivity values of the intrinsic growth rate a of the first population of investors over a long time interval are presented in the table below:

Table 1: Sensitivity analysis: cumulative percent change of a model parameter a = 0.0373

norms of solutions	***ODE45* sensitivity analysis of a variation of a**						
type of norm	0.05a	0.1a	0.2a	0.3a	0.4a	0.5a	0.6a
1-norm	145.84	143.23	136.68	128.64	118.54	105.57	90.05
2-norm	115.12	113.31	108.67	102.83	95.28	85.35	73.19
∞ - norm	200.60	199.21	195.18	188.99	179.46	164.77	144.33

Table 2: Sensitivity analysis: cumulative percent change of a model parameter a = 0.0373

norms of solutions	***ODE45* sensitivity analysis of a variation of** a						
type of norm	$0.7a$	$0.8a$	$0.9a$	$0.95a$	$1.01a$	$1.05a$	$1.1a$
1-norm	71.52	50.11	25.61	13.31	2.84	13.55	26.53
2-norm	58.40	41.05	21.02	10.93	2.33	11.14	21.79
∞ - norm	116.99	82.96	42.48	22.03	4.66	22.16	42.99

A similar set of sensitivity values over the same interval for the competition interaction has been for the model parameter of inter-specific coefficient for the first population of investors. Our results are presented below:

Table 3: Sensitivity analysis: cumulative percent change of a model parameter c = γ_1 = 0.0005

norms of solutions	***ODE45* sensitivity analysis of a variation of** c = γ_1 = 0.0005						
type of norm	0.05c	0.1c	0.2c	0.3c	0.4c	0.5c	0.6c
1-norm	64.95	62.13	56.32	50.25	43.89	37.28	30.37
2-norm	53.35	51.11	46.47	41.58	36.45	31.05	25.38
∞ - norm	97.89	94.50	87.25	79.30	70.63	61.15	50.79

Table 4: Sensitivity analysis: cumulative percent change of a model parameter c = γ_1 = 0.0005

norms of solutions	***ODE45* sensitivity analysis of a variation of** c = γ_1 = 0.0005						
type of norm	0.7c	0.8c	0.9c	0.95c	1.01c	1.05c	1.1c
1-norm	23.18	15.71	7.98	4.02	0.81	4.07	8.19
2-norm	19.44	13.22	6.74	3.39	0.68	3.45	6.95
∞ - norm	39.53	27.31	14.13	7.18	1.46	7.39	15.00

Table 5: Sensitivity analysis: cumulative percent change of a model parameter d = 0.03

norms of solutions	***ODE45* sensitivity analysis of a variation of** d = 0.03						
type of norm	0.05d	0.1d	0.2d	0.3d	0.4d	0.5d	0.6d
1-norm	114.38	112.50	108.07	102.53	95.56	86.76	75.67
2-norm	94.67	93.38	90.26	86.23	80.99	74.16	65.27
∞ - norm	153.39	152.53	150.12	146.34	140.34	131.67	118.56

Table 6: Sensitivity analysis: cumulative percent change of a model parameter d = 0.03

norms of solutions	***ODE45* sensitivity analysis of a variation of** d = 0.03						
type of norm	0.7d	0.8d	0.9d	0.95d	1.01d	1.05d	1.1d
1-norm	61.81	44.72	24.13	12.49	2.59	13.31	27.38
2-norm	53.81	39.29	21.38	11.12	2.32	11.92	24.58
∞ - norm	99.79	74.09	40.74	21.23	4.43	22.72	46.63

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CALCULATION OF SENSITIVITY ANALYSIS: Mutualistic Interaction

Following our recent data ([14], [8]) in which a = 0.0373, d = 0.03, β_1 = 0.0014, β_2 = 0.0005, γ_1 = 0.0005 and γ_2 = 0.0.0009.

The starting investment values for the two populations of investors are 10 thousand naira and 15 thousand naira respectively. Here we choose the trading interval of T = 180 days in the range T = 0 : 5 : 180.

In this present analysis, the sensitivity values of the intrinsic growth rate d of the second population of investors over a long time interval are presented in the table below:

Table 7: Sensitivity analysis: cumulative percent change of a model parameter a = 0.0373

norms of solutions	***ODE45* sensitivity analysis of a variation of** a					
type of norm	$0.05a$	$0.1a$	$0.2a$	$0.3a$	$0.4a$	$0.5a$	$0.6a$
1-norm	120.86	116.16	105.57	94.44	82.60	69.78	56.75
2-norm	89.03	85.46	77.40	69.01	60.15	50.65	41.04
∞ - norm	119.53	114.08	102.19	90.26	78.06	65.28	52.59

Table 8: Sensitivity analysis: cumulative percent change of a model parameter a = 0.0373

norms of solutions	***ODE45* sensitivity analysis of a variation of** a						
type of norm	$0.7a$	$0.8a$	$0.9a$	$0.95a$	$1.01a$	$1.05a$	$1.1a$
1-norm	43.23	29.29	14.16	7.54	1.59	7.61	14.88
2-norm	31.16	21.05	7.61	5.39	1.14	5.43	10.59
∞ - norm	39.69	26.71	13.28	6.94	1.43	6.82	13.30

We have calculated the following sensitivity values over the same time interval for the mutualistic interaction for the parameters c and d respectively for the first and second population of investors. Our results are presented below:

Table 9: Sensitivity analysis: cumulative percent change of a model parameter $c = \gamma_1 = 0.0005$

norms of solutions	***ODE45* sensitivity analysis of a variation of** $c = \gamma_1 = 0.0005$						
type of norm	0.05c	0.1c	0.2c	0.3c	0.4c	0.5c	0.6c
1-norm	116.84	114.00	107.78	100.73	92.66	83.33	72.44
2-norm	91.68	89.59	84.98	79.72	73.63	66.53	58.14
∞ - norm	139.71	136.83	130.43	123.03	114.37	104.11	91.76

Table 10: Sensitivity analysis: cumulative percent change of a model parameter $c = \gamma_1 = 0.0005$

norms of solutions	***ODE45* sensitivity analysis of a variation of** $c = \gamma_1 = 0.0005$						
type of norm	0.7c	0.8c	0.9c	0.95c	1.01c	1.05c	1.1c
1-norm	59.51	43.89	24.58	13.08	2.84	15.05	32.58
2-norm	48.06	35.69	20.17	10.79	2.36	12.55	27.37
∞ - norm	76.61	67.59	33.00	3.93		21.14	46.64

Table 11: Sensitivity analysis: cumulative percent change of a model parameter d = 0.03

norms of solutions	***ODE45* sensitivity analysis of a variation of** d = 0.03						
type of norm	0.05d	0.1d	0.2d	0.3d	0.4d	0.5d	0.6d
1-norm	119.33	114.67	104.65	93.76	82.07	69.68	56.68
2-norm	88.12	85.32	77.61	69.29	60.44	51.13	41.44
∞ - norm	120.50	114.99	103.34	91.27	78.99	66.29	53.39

Table 12: Sensitivity analysis: cumulative percent change of a model parameter d = 0.03

norms of solutions	***ODE45* sensitivity analysis of a variation of** d = 0.03						
type of norm	0.7d	0.8d	0.9d	0.95d	1.01d	1.05d	1.1d
1-norm	43.14	29.14	14.74	7.41	1.49	7.48	15.03
2-norm	31.43	21.15	10.66	5.35	1.07	5.38	10.80
∞ - norm	40.28	27.05	13.62	6.92		6.79	13.61

DISCUSSION

In this study, we observe that when the parameter a is varied by 5 percent for the cases of competition interaction and mutualistic interaction, our sensitivity analysis will produce the following cumulative percentage changes in the solution trajectories over a chosen time interval:

(1) 145.84 using the 1-norm, 115.12 using the 2-norm, and 200.60 using the infinity norm for the competition interaction (see Table 1, first column).

(2) 120.86 using the 1-norm, 89.03 using the 2-norm, and 119.53 using the infinity norm for the mutualistic interaction (see Table 7, first column).

Next, when the parameter d is varied by 5 percent for the cases of competition interaction and mutualistic interaction, our sensitivity analysis will produce the following cumulative percentage changes in the solution trajectories over a chosen time interval:

(1) 114.38 using the 1-norm, 94.67 using the 2-norm, and 153.39 using the infinity norm for the competition interaction (see Table 5, first column).

(2) 119.33 using the 1-norm, 88.12 using the 2-norm, and 120.50 using the infinity norm for the mutualistic interaction (see Table 11, first column).

Similarly, when the parameter c is varied by 5 percent for the cases of competition interaction and mutualistic interaction, our sensitivity analysis will produce the following cumulative percentage changes in the solution trajectories over a chosen time interval:

(1) 64.95 using the 1-norm, 53.35 using the 2-norm, and 97.89 using the infinity norm for the competition interaction (see Table 3, first column).

(2) 116.84 using the 1-norm, 91.68 using the 2-norm, and 139.71 using the infinity norm for the mutualistic interaction (see Table 9, first column).

In summary, when the model parameters a, d, and c are varied a little, a bigger effect is produced for the parameter a than the parameter d and c. In this scenario, a is classified as an important model parameter for the case of competition interaction irrespective of the type of the norms of the difference of solution trajectories.

For the case of mutualistic interaction, when the model parameters a, d, and c are varied a little, the model parameters a and d can be classified as equally important than parameter c when the 1-norm difference of solution trajectories is implemented. Using the 2-norm, the parameters a, d, and c are equally important

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whereas c can be classified as an important parameter than parameters a and d when the infinity norm is implemented.

On the basis of the impact of individual norms, our cumulative percentage effects of varying parameter 'a' a little for the competition interaction are bigger than the corresponding cumulative percentage effects for the mutualistic interaction irrespective of the norms of solutions being implemented. A similar observation can be seen when the parameter d is varied a little. In the opposite pattern, when the parameter c is varied a little, the cumulative percentage effects for the mutualistic interaction will outweigh the corresponding cumulative percentage effects for the competition interaction irrespective of the norms of solutions being implemented.

CONCLUSION

On the basis of our sensitivity analysis method over a time interval upon using the 1-norm for the competition interaction, we have found that the model parameter a needs to be estimated most accurately while the model parameters d and c should be given as rough estimates.

For the mutualistic interaction, we have found that the model parameters a and d need to be estimated most accurately while the parameter c should be given as rough estimates upon using the 1-norm.

Therefore, sensitivity analysis over a time interval has the capability to guide a further effort in parameter estimation of stock market interaction system. Although the notion of a sensitivity analysis is not new, but the method we have applied to solve the problem proposed in this study is a novel technique. The results which we have achieved in this study are expected to aid a further stochastic analysis of this deterministic stock market model formulation.

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