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RIDGE TRACE AS A BOOST TO RIDGE REGRESSION ESTIMATE IN THE PRESENCE OF MULTICOLLINEARITY

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ABSTRACT

Multicollinearity often causes a huge interpretative problem in linear regression analysis. The ridge estimator is not generally accepted as a vital alternative to the ordinary least squares (OLS) estimator because it depends on unknown parameters. In any specific application of ridges regression, there is no guarantee that the sample estimate is a member of the class of more accurate estimates. This paper therefore reveals the importance of ridge trace to boost ridge regression estimate in the presence of multicollinearity. We observed from our sample analysis that the use of ridge trace produced a model close to the principal component regression which was used as a check model in solution to an ill-conditioned regression.

Key words: Multicollinearity, Ridge Regression, Shrinkage Parameter, Ridge Trace

INTRODUCTION

The problem of multicollinearity is being able to separate the effects of two (or more) variables on a response variable. If two variables are significantly related, it becomes impossible to determine which of the variables accounts for variance in the dependent variable. High interpredictor correlations will lead to less stable estimate of regression weights. The relationship can be problematic if the regression weights variability obscures some functional relationship of interest to a researcher. It becomes very difficult to identify the separate effects of the variables involved precisely.

The ridge regression was originally suggested by Hoerl (1962) as a procedure for investigating the sentivity of least-squares estimates based on data exhibiting near-extreme multicollinearity, where small perturbations in the data may produce large changes in the magnitude of the estimated coefficients. Unfortunately, the ridge estimator is not generally accepted as a vital alternative to the ordinary least squares (OLS) estimator because it depends on unknown parameters. In any specific application of ridges regression, there is no guarantee that the sample estimate is a member of the class of more accurate estimates. The efficacy of ridge estimation often depends upon the estimate of prior information the researcher has regarding the population model. In this our work, we mainly deal on the use of ridge trace to capture the approximate shrinkage parameter in a ridge regression model. In section 2, we introduced the concept of ridge regression. We briefly describe how the shrinkage parameter of a ridge regression works. In section 3, we present an example using data from CBN bulletin which showed the presence of multicolinearity using VIF, condition index and variance proportion. Then we report how to obtain an efficient ridge trace. We also compared the result of the ridge estimates with that of ordinary least squares and the principal component which was used as a check model in section 4.

Ridge Regression

Consider the centered and scaled multiple regression model.

Let $\overline{X}_j = \sum_{i=1}^{n} X_{ij} \Big/ N$ j = 1,……….,k

and let

$\lambda_i, i = 1,................k$ be the eigenalues of X^1X.

The ridge regression estimate of the coefficients $\beta_j, j = 1,..............., k$ are given by

$$\hat{\beta}(\kappa) = (X^{'}X + \kappa I)^{-1} X^{'}y.$$

………………………….. (2.1)

where $\kappa > 0$ is a constant, often called the shrinkage or biasing parameter and usually assumes values between 0 and 1. When $\kappa = 0$, the ridge regression estimates reduce to the usual least squares estimates. In order to make ridge regression operational a value for κ must be selected. Further, the corresponding least squares estimates of coefficient in the multiple regression model are

$$\left.\begin{aligned} \hat{\beta}_{0(\kappa)} &= \hat{\beta}_{j(\kappa)} \sqrt{\frac{S_{yy}}{S_{jj}}} \\ \text{and} \quad \hat{\beta}_{0(\kappa)} &= \overline{Y} - \sum_{j=1}^{k} \hat{\beta}_{j(\kappa)} \overline{X}_j \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (2.2)$$

j= 1,…….,k

The expected value of $\hat{\beta}(\kappa)$ is given by

$$E\{\hat{\beta}^*(\kappa)\} = (X^1X + \kappa I)^{-1} X^1 X\beta.$$

…………………. (2.3)

The mean square error of the ridge regression

$$E\{(\hat{\beta}^*(\kappa) - \beta)^{'}(\hat{\beta}^*(\kappa) - \beta)\} = \delta^2 \sum_{j=1}^{p} \lambda_j (\lambda_j + \kappa)^{-1} + \kappa^2 \beta^1 (X^1X + \kappa I)^{-2} \beta,$$

…………… (2.4)

Computational Result

For a typical numerical example we use a time series data from Central Bank of Nigeria (CBN) statistical bulletin ranging from 1970 - 2010 with one regressand and five regressors. The existence of multicollinearity in the OLS model is dictated by considering various diagonistics: Variance Inflation Factor (VIF), Variable Proportion and Condition Index. This is shown in the variance proportion table of table 1. From the table, it can be found that the VIF for X_1, X_2, and X_5 exceeded 10 which shows a collinearity problem. Also the table reveals that condition numbers $\phi = \lambda_{\max} / \lambda_{\min} > 1000$, i.e. 4.575/0.002996 = 1527.04. Therefore, the effect of multicollinearity should be considered from both the condition number and variance proportion.

Table 1: Collinearity diagnostic

			Variance Proportions						
Number	VIF	Eigenvalue	Condition Index	Intercept	X_1	X_2	X_3	X_4	X_5
1	-	4.575	1.00	0.00	0.00	0.00	0.01	0.00	0.00
2	70.612	1.117	2.024	0.04	0.04	0.00	0.08	0.00	0.00
3	75.102	0.199	4.793	0.25	0.00	0.00	0.00	0.00	0.00
4	1.719	0.09579	6.911	0.43	0.00	0.00	0.10	0.23	0.00
5	6.261	0.009505	21.940	0.02	0.59	0.53	0.20	0.01	0.00
6	156.178	0.002996	39.078	0.26	0.40	0.46	0.01	0.74	1.00

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To overcome the collinearity problem, we have applied the ridge regression to the data using Statgraphics 5.1 software. As earlier stated, the aim is to determine the efficient shrinkage parameter using the ridge trace. The standardized regression coefficients for the data and the corresponding ridge trace are shown in table 2 and 3 respectively.

Table 2: Standardized regression coefficients

Ridge Parameter	Exchange Rate	GDP	Inflation Rate	Interest Rate	Private Foreign Inv.
0.0	-2.75363	3.70997	-0.180514	0.0939964	-1.53871
0.005	-2.14057	2.41685	-0.269849	-0.000557	-0.77382
0.01	-1.72627	1.80086	-0.315118	-0.021205	-0.544243
0.015	-1.45020	1.42972	-0.342141	-0.028806	-0.434884
0.02	-1.25532	1.18002	-0.359695	-0.032720	-0.371198
0.025	-1.11088	1.00014	-0.371693	-0.035117	-0.329608
0.030	-0.99968	0.86426	-0.380155	-0.036887	-0.300348
0.035	-0.91147	0.75795	-0.386233	-0.038358	-0.278658
0.04	-0.83979	0.67248	-0.390631	-0.039673	-0.261941
0.045	-0.78039	0.60226	-0.393804	-0.040902	-0.248666
0.05	-0.73036	0.54355	-0.396061	-0.042079	-0.23787
0.055	-0.68764	0.49372	-0.397614	-0.043221	-0.228917
0.06	-0.65073	0.45091	-0.398619	-0.044337	-0.221371
0.065	-0.61852	0.41373	-0.399191	-0.045432	-0.214925
0.07	-0.59015	0.34114	-0.399414	-0.046508	-0.209354
0.075	-0.56497	0.35234	-0.399356	-0.047566	-0.204489
0.08	-0.54247	0.32671	-0.399067	-0.048607	-0.200204
0.085	-0.52223	0.30374	-0.398587	-0.049630	-0.196400
0.090	-0.50393	0.28306	-0.397951	-0.050636	-0.19300
0.095	-0.48731	0.26433	-0.397182	-0.051625	-0.189941
0.10	-0.47213	0.24729	-0.396304	-0.052596	-0.187174

Table 3: The variance inflation factors for the ridge regression

Ridge Parameter	Exchange Rate	GDP	Inflation Rate	Interest Rate	Private Foreign Inv.
0.0	70.7783	75.5347	1.72408	6.2102	155
0.005	27.8143	27.9891	1.51895	2.68902	34.8
0.01	15.6637	15.4602	1.42732	2.07208	15.0
0.015	10.1791	9.9526	1.37151	1.84096	8.34
0.02	7.19287	6.99325	1.33111	1.71853	5.32
0.025	5.3782	5.20902	1.29862	1.63887	3.70
0.030	4.19035	4.0472	1.27072	1.57973	2.73
0.035	3.36936	3.24723	1.24577	1.53189	2.11
0.04	2.77774	2.67241	1.22286	1.49096	1.68
0.045	2.33701	2.24521	1.20146	1.45459	1.38
0.05	1.99969	1.91886	1.18123	1.42145	1.15
0.055	1.73562	1.66382	1.16196	1.39075	0.98
0.06	1.52492	1.46062	1.14349	1.36195	0.85
0.065	1.35401	1.29603	1.12573	1.33470	0.75
0.07	1.21341	1.16079	1.10858	1.30876	0.66
0.075	1.09628	1.04826	1.09198	1.28395	0.59
0.08	0.99762	0.95358	1.07590	1.26014	0.53
0.085	0.91370	0.87314	1.06029	1.23722	0.49
0.090	0.84167	0.84168	1.04511	1.21511	0.45
0.095	0.77937	0.77937	1.03034	1.19374	0.41
0.10	0.75087	0.6927	1.01596	1.17307	0.38

The variance inflation factor in table 3, measures how much the variance of the estimated coefficients is inflated relative to the case when all the independent variables are uncorrelated. A good value for ridge parameter from both tables is the smallest value after which the estimates change slowly. Thus from the tables above, we observe that the ridge parameter which stabilizes the ridge trace should be 0.030. This result is confirmed in figure 1a and 1b.

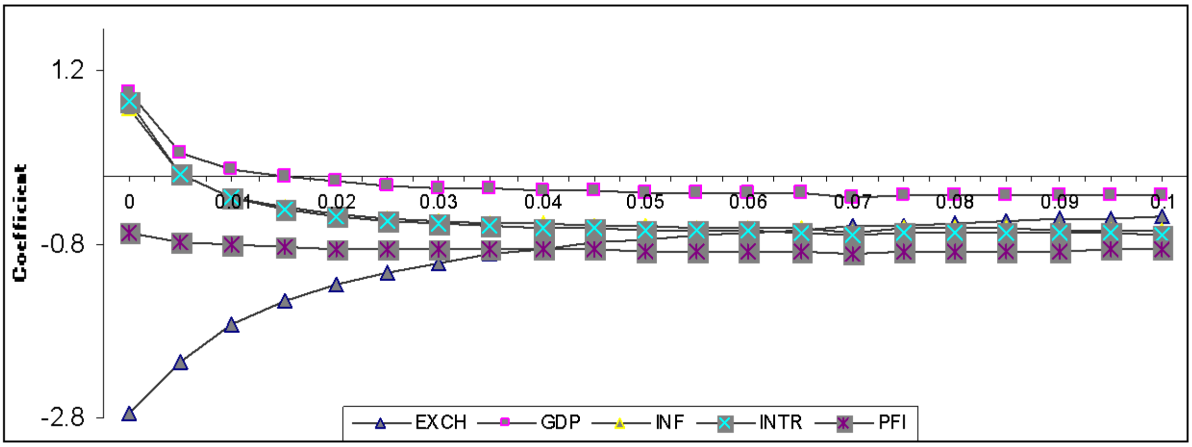

Fig.1a: Ridge Trace of estimate coefficient

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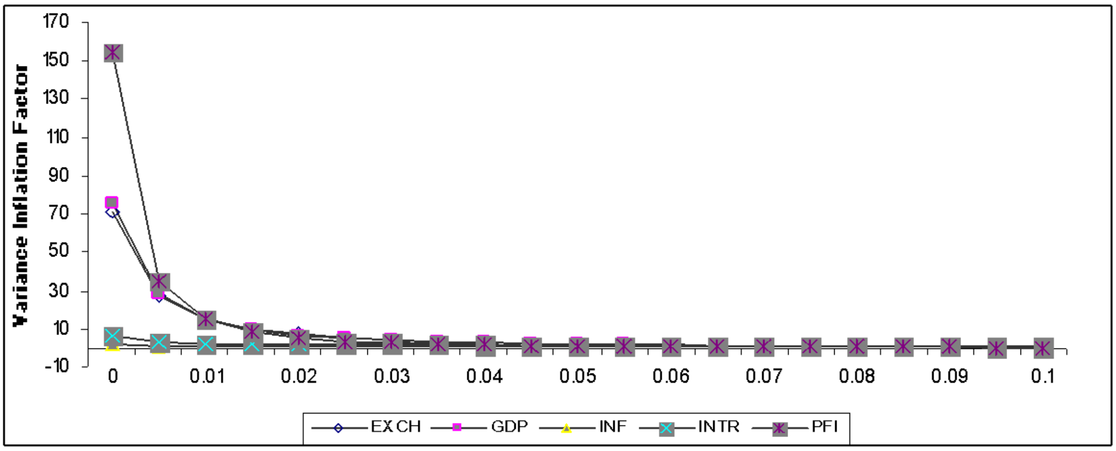

Fig.1b: Ridge Trace of VIF

The regression coefficients when $\kappa = 0.030$ are intercept $\beta_0 = 17949$, $\beta_1 = -1875.92$, $\beta_2 = 0.053897$, $\beta_3 = -826.321$ and $\beta_4 = -206.314$, and $\beta_5 = -1.53942$. Therefore, the ridge regression equation becomes

$$\hat{y} = -17949 - 1875.92x_1 + 0.053897x_2 - 826.321x_3 - 206.314x_4 - 1.53942x_5$$

The ridge regression model is then compared to principal component regression which serves as a check model.

Summary of the Regression Results:

Comparing the results from Ordinary Least Square (OLS), Ridge Regression (RR) and Principal Component Regression (PCR), we have

Ordinary Least Square (OLS)

$$BOP = 7704.27 - 4613.99EXR + 0.200878GDP - 413.9081INF + 621.94INT - 7.38001PFI$$

Ridge Regression (RR)

$$BOP = -17949 - 1875.92EXR + 0.053897GDP - 826.321INF - 206.31INT - 1.53942PFI$$

Principal Component Regression (PCR)

$$BOP = -17037.9 - 1880.701EXR + 0.08353GDP - 870.319INF - 208.291INT - 1.82186PFI$$

Table 3 illustrates the models of the multicollinearity problem

Table 3: OLS, RR and PCR Models

	β_0	β_1	β_2	β_3	β_4	β_5	R^2(%)	S_{yx}
OLS	7704.27	-4613.99	0.2009	-413.908	621.94	-7.38001	81.7	29069.4
RR	-17949.0	-1875.92	0.0539	-826.321	-206.31	-1.5394	63.2	25982.8
PCR	-17037.9	-1880.7	0.0835	-870.319	-208.291	-1.8219	54.4	32191.4

CONCLUSION

High interpredictor correlations will lead to less stable estimate of regression weights. The relationship can be problematic if the regression weights variability obscures some functional relationship of interest to a researcher. This study revealed the importance of ridge trace in obtaining an efficient value of the shrinkage parameter. The ridge regression and the Variance Inflation Factor plots (Fig.1a and 1b), clearly indicate the impact of multicollinearity in all the five predictor variables. From the result obtained, it was found that the value of the estimates from ridge regression using the ridge trace is close to that obtained using the principal component regression and both methods maintained the same signs. We therefore conclude that the use of ridge trace can boost the ridge regression method as a vital alternative to an ordinary least squares (OLS) estimator in solving the multicollinearity problem.

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