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CONVERGENCE OF HYBRID METHODS FOR SOLVING NON-LINEAR PARTIAL DIFFERENTIAL EQUATIONS.

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ABSTRACT

This paper is concerned with the numerical solution and convergence analysis of non-linear partial differential equations using a hybrid method. The solution technique involves discretizing the non-linear system of PDE to obtain a corresponding non-linear system of algebraic difference equations to be solved at each time level. Several values of mesh size chosen, the results show that the numerical results approach the exact solution as the mesh size tends to zero; which confirmed convergence of the scheme.

INTRODUCTION

The scheme developed in this project is applied to solving megnetogydrodynamic (MHD) non-linear system of partial differential equations. The methodology is applied to discretize and solve the system.

According to Chaudhary et al (2013), an MHD model governing an unsteady free convection flow of an incompressible fluid through a vertical channel formed by two parallel plates moving with equal velocity but in opposite directions is given by

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = g\beta(T - T_d) + g\beta^*(C - C_d) + v\frac{\partial^2 u}{\partial y^2} - \frac{v}{k}u - \sigma\frac{B_0{}^2 u}{\rho}$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = \frac{k}{\rho^C p}\frac{\partial^2 C}{\partial y^2} + \frac{s}{\rho^C P}(T - T_d)$$

$$\frac{\partial C}{\partial t} - v_0 \frac{\partial C}{\partial y} = D\frac{\partial^2 c}{\partial y^2} + D_\ell \frac{\partial^2 T}{\partial y^2}$$

where $g,\ \beta,\ \beta^*,\ D,\ S,\ D_\ell,\ \rho,\ v_0,\ \sigma,\ and\, C_p$ are all known values (constants), for this particuar system of equations. $T_d, and\ C_d$ are temperature and concentration of plate at distance $y = d$, respectively.

Mathematical Formulation

Originally, the system is subject to boundary conditions

$$u = U + L_1\frac{\partial u}{\partial y},\ T = T_0 + \in (T_0 - T_d)e^{iwt}, c = C_0 + \in (C_0 - C_d)e^{iwt}, at\ y = 0;$$

$$u = -U + L_1 \frac{\partial u}{\partial y},\ T = T_d \quad c = C_d,\ at\, y = d;$$

where $L\left(\frac{2-m_1}{m_1}\right)L;\ L$ is the mean free path, and m_1 is Maxwell's reflection coefficient.

after non-dimensionalization by Chaudary et al (2013), the governing equations transform to

$$\frac{w}{\lambda}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = Gr\lambda\theta + Gc\lambda C + \frac{1}{\lambda}\frac{\partial^2 u}{\partial y^2} - \frac{\lambda}{k}u - \frac{M^2}{k}u$$

$$\frac{w}{\lambda}\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\Pr\lambda}\frac{\partial^2 u}{\partial y^2} + \lambda S\theta$$

$$\frac{w}{\lambda}\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sr\lambda}\frac{\partial^2 u}{\partial y^2} + \frac{S_0}{\lambda}\frac{\partial^2 \theta}{\partial y^2}$$

with corresponding boundary conditions

$$u = 1 + h_1\frac{\partial u}{\partial y},\quad \theta = 1 + \in e^{it},\ C = 1 + \in e^{it} \quad at\ \ y = 0$$

$$u = -1 + h_1\frac{\partial u}{\partial y},\ \theta = \theta,\ C = 0 \ \ at\ \ y = 1$$

From the governing equations, this defines a one-dimensional MHD flow.
$\Omega = \{y; 0 \le y \le 1\}$ are of three subsets $-\ j = 1, 1 < j < n, j = n$.
Let $\Delta t =$ time step, and $\Delta y =$ mesh length (stencil size). Suppose we choose n discrete nodes internally, the mesh length is defined by

$$\Delta y = \frac{1}{n+1}$$

for this particular system, since non-dimensionalized length $= 1$.

MATERIALS AND METHODS

Following Bazuaye and Tamuno (2015) the discretization is obtained as follows;

$$\frac{w}{\lambda}\left(\frac{u_j^{l+1} - u_j^l}{\Delta t}\right) - \tfrac{1}{2}\left(\frac{u_{j+1}^{l+1} - u_{j-1}^{l+1}}{2\Delta y} + \frac{u_{j+1}^l - u_{j-1}^l}{2\Delta y}\right) = G\tau\lambda\frac{1}{2}(\theta_j^{l+1} + \theta_j^l) + Gc\lambda\tfrac{1}{2}\left(C_j^{l+1} + C_j^l\right) +$$

$$\tfrac{1}{\lambda}\tfrac{1}{2}\left(\frac{u_{j+1}^{l+1} + 2u_j^{l+1} - u_{j-1}^{l+1}}{(2\Delta y)^2} + \frac{u_{j+1}^l - u_{j-1}^l}{(2\Delta y)^2}\right) - \left(\frac{\lambda}{k} + \frac{M^2}{k}\right)\tfrac{1}{2}\left(u_j^{l+1} + u_j^l\right)$$

Moving everything to the LHS, this gives

$$\frac{w}{\lambda}\left(\frac{u_j^{l+1} - u_j^l}{\Delta t}\right) - \tfrac{1}{2}\left(\frac{u_{j+1}^{l+1} - u_{j-1}^{l+1}}{2\Delta y} + \frac{u_{j+1}^l - u_{j-1}^l}{2\Delta y}\right) - G\tau\lambda\frac{1}{2}(\theta_j^{l+1} + \theta_j^l) + Gc\lambda\tfrac{1}{2}\left(C_j^{l+1} + C_j^l\right) -$$

$$\tfrac{1}{\lambda}\tfrac{1}{2}\left(\frac{u_{j+1}^{l+1} + 2u_j^{l+1} - u_{j-1}^{l+1}}{(2\Delta y)^2} + \frac{u_{j+1}^l - 2u_j^l - u_{j-1}^l}{(2\Delta y)^2}\right) + \left(\frac{\lambda}{k} + \frac{M^2}{k}\right)\tfrac{1}{2}\left(u_j^{l+1} + u_j^l\right) = 0$$

Let $\alpha_1 = -\frac{\lambda\Delta t}{4w\Delta y},\ \alpha_2 = -\frac{G\tau\lambda^2\Delta t}{2w}, \alpha_3 = \frac{Gc\lambda^2\Delta t}{2w}, \alpha_4 = -\frac{\Delta t}{2w(\Delta t)^2}, \alpha_5 =$

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$\frac{\lambda\Delta t}{2w}\left(\frac{\lambda}{k}+\frac{M^2}{k}\right)$, and equate the resultant finite difference scheme to F_1,

$$F_1=\left(u_j^{l+1}-u_j^l\right)+\alpha_1\left(u_{j+1}^{l+1}-u_{j-1}^{l+1}+u_{j+1}^l-u_{j-1}^l\right)+\alpha_2\left(\theta_j^{l+1}+\theta_j^l\right)+$$
$$\alpha_3\left(C_j^{l+1}+C_j^l\right)+\alpha_4\left(u_{j+1}^{l+1}+2u_j^{l+1}-u_{j-1}^{l+1}+u_{j+1}^l+2u_j^l-u_{j-1}^l\right)+\alpha_5\left(u_j^{l+1}+u_j^l\right)$$

$$\frac{w}{\lambda}\left(\frac{\theta_j^{l+1}-\theta_j^l}{\Delta t}\right)-\tfrac{1}{2}\left(\frac{\theta_{j+1}^{l+1}-\theta_{j-1}^{l+1}}{2\Delta y}+\frac{\theta_{j+1}^l-\theta_{j-1}^l}{2\Delta y}\right)=\frac{1}{P\tau\lambda}\frac{1}{2}\left(\frac{\theta_{j+1}^{l+1}+2\theta_j^{l+1}\theta_{j-1}^{l+1}}{(\Delta y)^2}+\frac{\theta_{j+1}^l+2\theta_j^l-\theta_{j-1}^l}{(\Delta y)^2}\right)+$$
$$\lambda S\tfrac{1}{2}\left(\theta_j^{l+1}+\theta_j^l\right)$$

Moving everything to the LHS, this gives

$$\frac{w}{\lambda}\left(\frac{\theta_j^{l+1}-\theta_j^l}{\Delta t}\right)-\tfrac{1}{2}\left(\frac{\theta_{j+1}^{l+1}-\theta_{j-1}^{l+1}}{2\Delta y}+\frac{\theta_{j+1}^l-\theta_{j-1}^l}{2\Delta y}\right)-\frac{1}{P\tau\lambda}\frac{1}{2}\left(\frac{\theta_{j+1}^{l+1}+2\theta_j^{l+1}\theta_{j-1}^{l+1}}{(\Delta y)^2}+\frac{\theta_{j+1}^l+2\theta_j^l-\theta_{j-1}^l}{(\Delta y)^2}\right)-$$
$$\lambda S\tfrac{1}{2}\left(\theta_j^{l+1}+\theta_j^l\right)=0$$

Let $\alpha_6=-\frac{\Delta t}{2wP\tau(\Delta y)^2}$, *and* $\alpha_7=-\frac{\lambda^2 S\Delta t}{2w\Delta\lambda}$ and equate the resultant finite difference scheme to F_2,

$$F_2=\left(\theta_j^{l+1}-\theta_j^l\right)+\alpha_1\left(\theta_{j+1}^{l+1}-\theta_{j-1}^{l+1}+\theta_{j+1}^l-\theta_{j-1}^l\right)+\alpha_6\left(\theta_j^{l+1}+2\theta_j^{l+1}-\theta_{j-1}^{l+1}+\right.$$
$$\left.\theta_{j+1}^l+2\theta_j^l-\theta_{j-1}^l\right)+\alpha_7\left(\theta_j^{l+1}+\theta_j^l\right)$$

Discretizing third equation;

$$\frac{w}{\lambda}\left(\frac{c_j^{l+1}-c_j^l}{\Delta t}\right)-\tfrac{1}{2}\left(\frac{c_{j+1}^{l+1}-c_{j-1}^{l+1}}{2\Delta y}+\frac{c_{j+1}^l-c_{j-1}^l}{2\Delta y}\right)-\frac{1}{Sc\lambda}\frac{1}{2}\left(\frac{c_{j+1}^{l+1}+2c_j^{l+1}+c_{j-1}^{l+1}}{(\Delta y)^2}+\frac{c_{j+1}^l+2c-c_{j-1}^l}{(\Delta y)^2}\right)+$$
$$\frac{S0}{\lambda}\frac{1}{2}\left(\frac{\theta_{j+1}^{l+1}+2\theta_j^{l+1}-\theta_{j-1}^{l+1}}{(\Delta y)^2}+\frac{\theta_{j+1}^l+2\theta_j^l-\theta_{j-1}^l}{(\Delta y)^2}\right)$$

Moving everything to the LHS, this gives

$$\frac{w}{\lambda}\left(\frac{c_j^{l+1}-c_j^l}{\Delta t}\right)-\tfrac{1}{2}\left(\frac{c_{j+1}^{l+1}-c_{j-1}^{l+1}}{2\Delta y}+\frac{c_{j+1}^l-c_{j-1}^l}{2\Delta y}\right)-\frac{1}{Sc\lambda}\frac{1}{2}\left(\frac{c_{j+1}^{l+1}+2c_j^{l+1}-c_{j-1}^{l+1}}{(\Delta y)^2}+\frac{c_{j+1}^l+2c-c_{j-1}^l}{(\Delta y)^2}\right)-$$
$$\frac{So}{\lambda}\frac{1}{2}\left(\frac{\theta_{j+1}^{l+1}+2\theta_j^{l+1}-\theta_{j-1}^{l+1}}{(\Delta y)^2}+\frac{\theta_{j+1}^l+2\theta_j^l-\theta_{j-1}^l}{(\Delta y)^2}\right)=0$$

Let $\alpha_8=-\frac{\Delta t}{2wSc(\Delta y)^2}$, *and* $\alpha_9=-\frac{So\Delta t}{2w(\Delta\lambda)^2}$ and equate the resultant finite difference scheme to F_3,

$$F_3=\left(C_j^{l+1}-\theta_j^l\right)+\alpha_1\left(C_{j+1}^{l+1}-C_{j-1}^{l+1}+C_{j+1}^l-C_{j-1}^l\right)+\alpha_8\left(C_{j+1}^{l+1}+2C_j^{l+1}-C_{j-1}^{l+1}+\right.$$
$$\left.C_{j+1}^l+2C-C_{j-1}^l\right)+\alpha_9\left(\theta_{j+1}^{l+1}+2\theta_j^{l+1}-\theta_{j-1}^{l+1}+\theta_{j+1}^l+2\theta_j^l-\theta_{j-1}^l\right)$$

Hence, the discretization to the system of partial differential equations are

$$F_1=\left(u_j^{l+1}-u_j^l\right)+\alpha_1\left(u_{j+1}^{l+1}-u_{j+1}^{l+1}+u_{j+1}^l-u_{j-1}^l\right)=\alpha_2\left(\theta_j^{l+1}+\theta_j^l\right)+$$

$$\alpha_3\left(C_j^{l+1}+C_j^l\right)+\alpha_4\left(u_{j+1}^{l+1}+2u_j^{l+1}-u_{j-1}^{l+1}+u_{j+1}^l+2u_j^l-u_{j-1}^l\right)+\alpha_5\left(u_j^{l+1}+u_j^l\right)$$

$$F_2=\left(\theta_j^{l+1}-\theta_j^l\right)+\alpha_1\left(\theta_{j+1}^{l+1}-\theta_{j-1}^{l+1}+\theta_{j+1}^l-\theta_{j-1}^l\right)=\alpha_6\left(\theta_{j+1}^{l+1}+2\theta_j^{l+1}-\theta_{j-1}^{l+1}+\right.$$
$$\left.\theta_{j+1}^l+2\theta_j^l-\theta_{j-1}^l\right)+\alpha_7\left(\theta_j^{l+1}+\theta_j^l\right)$$

$$F_3=\left(C_j^{l+1}-C_j^l\right)+\alpha_1\left(C_{j+1}^{l+1}-C_{j-1}^{l+1}+C_{j+1}^l-C_{j-1}^l\right)=\alpha_8\left(C_{j+1}^{l+1}+2C_j^{l+1}-C_{j-1}^{l+1}+\right.$$
$$\left.C_{j+1}^l+2C-C_{j-1}^l\right)+\alpha_9\left(\theta_{j+1}^{l+1}+2\theta_j^{l+1}-\theta_{j-1}^{l+1}+\theta_{j+1}^l+2\theta_j^l-\theta_{j-1}^l\right)$$

To implement the Newton-Raphson iterative algorithm, we obtain the Jacobian matrix $J(j)$ for each jth node by finding the first partial derivatives of $F_1, F_2\, and\, F_3$ with respect to $u_j^{l+1},\ \theta_j^{l+1}\, and\, C_j^{l+1}$, since these are the nodal values we seek to obtain from a current time level, l.

$$J_{11}(j)=\frac{\partial F_1}{\partial u_j^{l+1}}=1+2\alpha_4+\alpha_5,\quad J_{12}(j)=\frac{\partial F_1}{\partial \theta_j^{l+1}}=\alpha_2,\quad J_{13}(j)=\frac{\partial F_1}{\partial \theta_j^{l+1}}=\alpha_3$$

$$J_{21}(j)=\frac{\partial F_2}{\partial u_j^{l+1}}=0,\quad J_{22}(j)=\frac{\partial F_2}{\partial \theta_j^{l+1}}=1+2\alpha_6=\alpha_7,\ J_{23}(j)=\frac{\partial F_2}{\partial \theta_j^{l+1}}=0$$

$$J_{31}(j)=\frac{\partial F_3}{\partial \theta_j^{l+1}}=0,\quad J_{32}(j)=\frac{\partial F_2}{\partial \theta_j^{l+1}}=2\alpha_9,\quad J_{33}(j)=\frac{\partial F_2}{\partial \theta_j^{l+1}}=1+2\alpha_8$$

Therefore Jacobian matrix for jth node is

$$J(j)=\begin{bmatrix} J_{11}(j) & J_{12}(j) & J_{13}(j)\\ J_{21}(j) & J_{22}(j) & J_{23}(j)\\ J_{31}(j) & J_{32}(j) & J_{33}(j)\end{bmatrix}=\begin{bmatrix}1+2\alpha_4+\alpha_5 & \alpha_2 & \alpha_3\\ 0 & 1+2\alpha_6+\alpha_7 & 0\\ 0 & 2\alpha_9 & 1+2\alpha_8\end{bmatrix}$$

Applying boundary condition $\zeta_0\ and\, \zeta_1$

Form the boundary conditions, this system is subject to Neumann boundary conditions, Dirichlet boundary conditions and Cauchy boundary conditions simultaneously. These conditions are applied to the difference equations for the boundary nodes, $j=1 and\ j=n$.

Node $j=1$; Subject boundary condition ζ_0

$$F_1(u_1^{l+1}-1)+\alpha_1(u_2^{l+1}-\zeta_0^{l+1}u(0)+u_2^1-\zeta_0^1u(0))+\alpha_2(\theta_1^{l+1}+\theta_1^l)+\alpha_3(C_1^{l+1}+C_1^l)+$$
$$\alpha_4(u_2^{l+1}+2u_1^{l+1}-\zeta_0^{l+1}u(0)+u_2^1+2u_1^l-\zeta_0^1u(0))+\alpha_5(u_1^{l+1}+u_1^l)$$

$$F_2=(\theta_1^{l+1}-\theta_1^l)+\alpha_1(\theta_2^{l+1}-\zeta_0^{l+1}\theta(0)+\theta_2^1-\zeta_0^l\theta(0))+\alpha_6(\theta_2^{l+1}+2\theta_1^{l+1}-\zeta_0^{l+1}\theta(0)+$$
$$\theta\tfrac{1}{2}+2\theta_1^l-\zeta_0^l\theta(0))+\alpha_7(\theta_1^{l+1}+\theta_1^l)$$

$$F_3=(C_1^{l+1}-C_1^l)+\alpha_1(C_2^{l+1}-\zeta_0^{l+1}C(0)+C_2^1-\zeta_0^1C(0))+\alpha_8(C_2^{l+1}+2C_1^{l+1}-\zeta_0^{l+1}C(0)+$$
$$C\tfrac{1}{2}+2C_1^l-\zeta_0^1C(0))+\alpha_9(\theta_2^{l+1}+2\theta_1^{l+1}-\zeta_0^{l+1}\theta(0)+\theta\tfrac{1}{2}+2\theta_1^l-\zeta_0^l\theta(0))$$

Since the jth Jacobian matrix has constant entries, the Jacobin matrix remains the same. That is

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$$J(1)=\begin{bmatrix} J_{11}(j) & J_{12}(j) & J_{13}(j) \\ J_{21}(j) & J_{22}(j) & J_{23}(j) \\ J_{31}(j) & J_{32}(j) & J_{33}(j) \end{bmatrix}=\begin{bmatrix} 1+2\alpha_4+\alpha_5 & \alpha_2 & \alpha_3 \\ 0 & 1+2\alpha_6+\alpha_7 & 0 \\ 0 & 2\alpha_9 & 1+2\alpha_8 \end{bmatrix}$$

Node $j=n$; Subject boundary condition ζ_1

$$F_1(u_n^{l+1}-u_n^l)+\alpha_1(\zeta_1^{l+1}u(1)-u_{n-1}^{l+1}+\zeta_1^l u(1)-u_{n-1}^l)+\alpha_2(\theta_n^{l+1}+\theta_n^l)+\alpha_3(C_n^{l+1}+C_n^l)+$$
$$\alpha_4(\zeta_1^{l+1}u(1)+2u_n^{l+1}+2u_n^{l+1}+\zeta_1^l u(1)-2u_n^l-u_{n-1}^l+\alpha_5(u_n^{l+1}+u_n^l)$$
$$F_2(\theta_n^{l+1}-\theta_n^l)+\alpha_1(\zeta_1^{l+1}\theta(1)-\theta_{n-1}^{l+1}+\zeta_1^l\theta(1)-\theta_{n-1}^l)+\alpha_6(\theta_n^{l+1}\theta(1)+$$
$$2\theta_n^{l+1}-\theta_{n-1}^{l+1}+\zeta_1^{l+1}\theta(1)+2\theta_n^l-\theta_{n-1}^l)+\alpha_7(\theta_n^{l+1}+\theta_n^1)$$
$$F_3=(C_n^{l+1}-C_n^l)+\alpha_1(\zeta_1^{l+1}C(1)-C_{n-1}^{l+1}+\zeta_1^l C(1)-C_{n-1}^l)+\alpha_8(\zeta_1^{l+1}C(1)+$$
$$2C_n^{l+1}-C_{n-1}^{l+1}+\zeta_1^{l+1}C(1)+2C_n^l-C_{n-1}^l)+\alpha_9(\zeta_1^{l+1}+\theta(1)+2\theta_n^{l+1}-$$
$$\theta_{n-1}^{l+1}+\zeta_1^l\theta(1)+2\theta_n^l-\theta_{n-1}^l)$$

NUMERICAL ILLUSTRATIONS AND ANALYSIS OF RESULTS

In this work, four numerical illustrations are considered. These numerical examples are drawn from examples treated by earlier researchers published in reputable international journals. This is to ascertain the validity of the results obtained from the scheme in comparison with others. Perspective views of the solution domains are shown, besides the numerical results, to enhance visualization. Besides global errors associated with the scheme for individual problems are plotted to ascertain the convergence of the scheme.

Numerical Illustration

Consider the two-dimensional coupled non-linear Bursger's equations,

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\text{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$
$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\text{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

with a solution domain

$$\Omega=\{(x,y);0\le x\le 0.5,0\le y\le 0.5\}$$

Subject to initial conditions

$$u(x,y,0)=\sin\pi x+\cos\pi y;\quad v(x,y,0)=x+y$$

and boundary conditions

$u(0,y,t)=\cos\pi y$;
$u(0.5,y,t)=1+\cos\pi y$
$v(0,y,t)=y$;
$v(0.5,y,t)=0.5+y$
$u(x,0,t)=1+\sin\pi x$;
$u(x,0.5,t)=\sin\pi x$
$v(x,0,t)=x$;
$u(x0.5,t)=x+0.5$

We are obtain solutions to the governing equations at time t = 0.625 and Reynolds number Re = 50 and 500.

Numerical Solution Using the Proposed Scheme

For this problem, the solution domain $\Omega=\{(x,y);0\le x\le 0.5,0\le y\le 0.5\}$ is discretized uniformly using 20×20 grids. That is, suppose $m\,and\,n$ are the internal nodes in the $x-and\,y-$directions respectively; then $m=n=19$.Since the computational domain $\Omega=\{(x,y);0\le x\le 0.5,0\le y\le 0.5\}$ defines a square geometry such that $L_x = L_y = 0.5$, the uniform mesh size $\Delta x \times \Delta y$ is defined by

$$\Delta x = \Delta y = \frac{L_x}{m+1}, \qquad = \frac{0.5}{20} = 0.025$$

A time step, $\Delta t = 0.0001$ is used. Therefore time levels required to obtain numerical solutions of the system at time $t = 0.625$ is $\ell_{0.625} = \frac{0.625}{\Delta t} = \frac{0.625}{0.0001} = 6250$ time levels

The 20×20 grids discretization results in 361 internal nodes, as obtained in section 4.1, with total number of nodes in and on boundary of solution domain = 441. The boundary conditions for this problem are all Cauchy.

In other to effectively solve the resulting 7220×722 nonlinear system of finite difference equations, which are to be solved for each of the 6250 time level to obtain results for time $t = 0.625$ with a time step $\Delta t = 0.0001$, MATLAB program is written to implement the scheme.

Table (5.2a) Numerical solutions; $\mathrm{Re} = 50, \Delta x = \Delta y = 0.05, \Delta t = 0.0001\, at\, t = 0.625$

$u(x, y, 0.625)$; $\mathrm{Re} = 50$

(x, y)	**Jain and Holla**	**Bahadir A. R.**	**Srivastava et al**	**Project Result**
(0.1,0.1)	0.97258	0.96688	0.97146	0.97146
(0.3,0.1)	1.16214	1.14827	1.15280	1.15282
(0.2,0.2)	0.86281	0.85911	0.86307	0.86307
(0.4,0.2)	0.96483	0.97637	0.97981	0.97982
(0.1,0.3)	0.66318	0.66019	0.67316	0.66316
(0.3,0.3)	0.77030	0.76932	0.77230	0.77230
(0.2,0.4)	0.58070	0.57966	0.58180	0.58180
(0.4,0.4)	0.74435	0.75678	0.75856	0.75856

$v(x, y, 0.625)$; $\mathrm{Re} = 50$

(x, y)	**Jain and Holla**	**Bahadir A. R.**	**Srivastava et al**	**Project Result**
(0.1,0.1)	0.09773	0.09824	0.09869	0.09869
(0.3,0.1)	0.14039	0.14112	0.14158	0.14159
(0.2,0.2)	0.16660	0.16681	0.16754	0.16755
(0.4,0.2)	0.17397	0.17065	0.17110	0.17113
(0.1,0.3)	0.26294	0.26261	0.26378	0.26378
(0.3,0.3)	0.22463	0.22576	0.22654	0.22657
(0.2,0.4)	0.32402	0.32745	0.32851	0.32852
(0.4,0.4)	0.31822	0.32441	0.32500	0.32506

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Table (5.2b) Numerical solutions; $\mathrm{Re} = 500, \Delta x = \Delta y = 0.05, \Delta t = 0.0001 \; at\, t = 0.625$

$u(x, y, 0.625)$; $\mathrm{Re} = 500$

(x, y)	**Jain and Holla**	**Bahadir A. R.**	**Srivastava et al**	**Project Result**
(0.150.1)	0.95691	0.96650	0.96870	0.96870
(0.3,0.1)	0.95616	1.02970	1.03200	1.03202
(0.10.2)	0.84257	0.84449	0.86178	0.84619
(0.2.0.2)	0.86399	0.87631	0.87814	0.87814
(0.1,0.3)	0.67667	0.67809	0.67920	0.67920
(0.3,0.3)	0.76876	0.79792	0.79947	0.79947
(0.15.0.4)	0.54408	0.54601	0.66036	0.54674
(0.2,0.4)	0.58778	0.58874	0.58959	0.58959

$v(x, y, 0.625)$; $\mathrm{Re} = 500$

(x, y)	**Jain and Holla**	**Bahadir A. R.**	**Srivastava et al**	**Project Result**
(0.150.1)	0.10177	0.09202	0.09043	0.09043
(0.3,0.1)	0.13287	0.10690	0.10728	0.10727
(0.10.2)	0.18503	0.17972	0.17295	0.18010
(0.2.0.2)	0.18169	0.16777	0.16816	0.16816
(0.1,0.3)	0.26560	0.26222	0.26268	0.26268
(0.3,0.3)	0.25142	0.23497	0.23550	0.23550
(0.15.0.4)	0.32084	0.31753	0.29019	0.31799
(0.2,0.4)	0.30927	0.30371	0.30419	0.30419

Numerical Study of Convergence

Using the Berger's equations in numerical illustration 1, different values of mesh sizes are used for a fixed time $t = 0.5,\ \Delta t = 0.001,\ \mathrm{Re} = 50$. The average global errors are computed in comparison with the exact solution given by Srivastava et al (2011) as

$$u(x, y, t) = \tfrac{3}{4} - \frac{1}{4(1 + e^{\mathrm{Re}(-4x+4y-t)/32})}$$

$$v(x, y, t) = \tfrac{3}{4} + \frac{1}{4(1 + e^{\mathrm{Re}(-4x+4y-t)/32})}$$

Table (4.3a) Mean global errors and mesh sizes

Global Error		
Mesh size	u	v
0.250000	0.001304	0.001303
0.166667	0.000709	0.000707
0.125000	0.000431	0.000428
0.100000	0.000289	0.000286
0.083333	0.000208	0.000205
0.071429	0.000158	0.000155
0.062500	0.000125	0.000122
0.555556	0.000102	0.000101
0.045455	0.000086	0.000086
0.041667	0.000074	0.000076
0.038462	0.000058	0.000069
0.035714	0.000052	0.000066
0.033333	0.000048	0.000069
0.031250	0.000045	0.000074
0.029412	0.000042	0.000083
0.027778	0.000040	0.000095

To study the convergence behaviour of the scheme from the time step perspective, different values of time steps $h = 0.1$, $\mathrm{Re} = 50$ at time $t = 0.5$, numerical illustration 1. Table (5.3b) and Fig. (5.3b) show the results.

Table (5.3b) Mean global errors and time step sizes

Global Error		
Mesh size	u	v
0.016667	0.000532	0.000693
0.008333	0.000350	0.000332
0.005556	0.000310	0.000321
0.004167	0.000297	0.000003
0.003333	0.000306	0.000296
0.002778	0.000289	0.000296
0.002381	0.000287	0.000296
0.002083	0.000287	0.000283
0.001852	0.000286	0.000282
0.001667	0.000286	0.000281
0.001515	0.000292	0.000281
0.001389	0.000285	0.000288
0.001282	0.000285	0.000281
0.001190	0.000284	0.000281
0.001111	0.000290	0.000281
0.001042	0.000284	0.000286
0.000980	0.000284	0.000281

This paper has investigated the numerical solution and convergence analysis of non-linear partial differential equations using hybrid method. The solution technique involves discretizing the nonlinear system of the PDE to obtain a corresponding nonlinear algebraic difference equation to be solved at each time level. Several values of meshsizes are chosen, the results show that the numerical result approach the exact solution as the meshsize tends to zero, which confirmed the convergence of the scheme. It also show that the new scheme is superior in terms of convergence to the work of Allen (2005) and Strivastava et al (2011)

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