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DISCRIMINANT FUNCTION FOR CLASSIFICATION OF GENUINE AND COUNTERFEIT NAIRA NOTES

I. A. Iwok[1] and A.W. Akpan[2]

Department of Mathematics/Statistics, University of Port-Harcourt,
P.M.B.5323, Port-Harcourt, Rivers State; Nigeria.
[1]E-mail: ibywok@yahoo.com
[2]E-mail: abwilliams77@yahoo.com

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ABSTRACT

The aim of this work was to construct a model for the detection of counterfeit and genuine naira currency in Nigeria. Measurements based on three variables (mass, thickness and tensile stress) were obtained from twenty pieces of fake and genuine ₦1000 notes. A derivation of the appropriate measure of the separation of the two groups was made. Discriminant analysis technique was used to construct a discriminant function. Test for the significance of differences between the mean values of the variables for each of the two groups (counterfeit and genuine) of the ₦1000 notes was also carried out. Findings of the research revealed that the naira notes are to be classified as counterfeit or genuine according to the model: $Z = 440.3007X - 858.8366Y + 147.5228Z$ *such that* $z > z_0$ *where* z_0 *is the end point of classification. Hotelling's* T^2 *and Mahalanobis quantity were also computed. The test result showed that differences between the counterfeit and genuine naira notes in the mean values of mass (X), thickness (Y) and tensile stress (Z) are significant.*

Key words: Mass, thickness, tensile stress, Hotelling's T^2 and Mahalanobis quantity.

INTRODUCTION

Money is a medium of exchange issued by a government or other public authority in the form of coins of gold, silver, metal or paper bills, used as the measure of the value of goods and services (Liky, 2009). The official currency of a country referred to as National Currency gives the country an identity. The Naira (sign: ₦; code: NGN) is the currency of Nigeria. The Central Bank of Nigeria (CBN) is the sole issuer of money throughout the Nigerian Federation. The naira was introduced in 1973, when the country decimalized its monetary system and substituted the naira for the pound (Holton, 2010)

The Naira notes are protected by a number of security features to enable the recognition of genuine notes. The distinguishing features which can immediately be recognized by touch and visibility are raised print, the security thread and the watermark. Other areas such as the portrait, lettering and the denominational numerals on the front and the back are embossed. The raised print provides the facility while the security thread which ordinarily looks broken but is not when held against the light, has "CBN" in small lettering printed on both sides of the notes. The Naira notes are also protected against photocopying. Some features of the notes such as the serial number turns from black to green under ultraviolet light.

The currency notes issued in Nigeria are in: ₦5, ₦10, ₦20, ₦50, ₦100, ₦200, ₦500 and ₦1000 denominations.

On the 1000 naira notes being selected for this study, there is a subtle shiny strip running down the back of the note. It is a shimmery gold colour showing 1000 naira. The triangular shape in the middle of the front of the note changes its color from green to blue when tilted. The main feature on the front is the engraved portraits of Alhaji Aliyu Mai-Bornu and Dr. Clement Isong, former governors of the Central Bank of Nigeria.

To counterfeit means to imitate something (Wikipedia, 1995). Counterfeit money is currency that is produced without the legal sanction of the state or government and in deliberate violation of that country's laws. Counterfeit paper money is the most popular product counterfeited. The counterfeiting of money is usually attacked aggressively by governments worldwide. Counterfeiting is a crime and is therefore punishable by law. The reason why counterfeiting is treated as such a serious crime is because money is so important to the society. Money is the oxygen of the economy. If people cannot trust that the money they carry is authentic, then it gets much harder to buy things and the economy slows down. When one gets caught counterfeiting, the punishment can be unbelievably harsh.

Traditionally, anti-counterfeiting measures had been in existence to allow non-experts to easily spot forgeries. Recently, however, there has been a discovery of new tests that could be used on U.S. Federal Reserve Notes to ensure that the bills are authentic. These tests are done using intrinsic fluorescence lifetime. This allows for detection of counterfeit money because of the significance in difference of fluorescence lifetime when compared to authentic money.

According to a Central Bank of Nigeria article ‘Why Naira May Not Be Easily Counterfeited’, ‘the fortified security features on naira notes have made it difficult for counterfeiters to successfully clone the currency’ (www.nairaland.com, 2010). But, does the man on the street know these security features?

A currency may be cloned, identically replicated, but is still fake because it duplicates the serial number. However, the cost of attaining such an authentic replica (especially in lower denomination bills) is almost the cost of producing the real notes. That is the reason counterfeiters concentrate on the highest available denomination besides the fact that it amounts to higher monetary value against lower denominations. The highest denomination of the Nigerian naira is the ₦1000 note. Observantly, almost all cases of counterfeiting have been of a particular denomination (₦1000 notes). This is because, the loss encountered by victims of counterfeit naira notes is greater with ₦1000 note than any other denomination. For instance a trader who sales a ₦100 worth of items for a counterfeit ₦1000 note losses ₦900 cash (change given to the counterfeiter in exchange of the fake ₦1000 note) whereas, with a fake ₦500 note he losses ₦400 cash. Thus, the counterfeiters make more gains with fake ₦1000 notes (highest denomination).

The ₦1000 note is clearly the most counterfeited naira denomination and in consideration of the huge losses suffered by

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victims of counterfeit naira note, it is most appropriate to select the ₦1000 note for this study to guide the public on the distinction between fake and genuine naira notes.

Most times the counterfeit is so close to the original such that when they are mixed together it becomes very difficult to identify or to separate the fake from the real notes. More so, the perpetrators of this criminal act capitalize on the ignorance of the uneducated rural dwellers by distributing the counterfeit notes in bits in the rural communities. It is only when the counterfeit notes get to the bank that they are identified. The issue gets more complicated when there is an inside involvement within the bank such that ATM machines are sometimes deliberately stuffed with fake notes.

The purpose of this research work is to develop a model that can be used to classify the Naira notes and by extension, any other currency into counterfeit and original. Multivariate techniques will be employed and the method to be used in the classification is the Discriminant function.

According to Abdi (2007), the subject of multivariate deals with the statistical analysis of data collected on more than one response variable. These variables may be correlated with each other, and their statistical dependence makes multivariate analysis somewhat different in approach and considerably more complex than the corresponding univariate analysis which involves only one response variable.

Discriminant analysis on the other hand is a multivariate technique that is concerned with the separation of distinct sets of objects (Johnson and Wichern, 1992). This involves finding 'discriminants' whose numerical values are such that the groups are separated as much as possible.

Discriminant function analysis is used to determine which continuous variables discriminate between two or more naturally occurring groups (Poulsen and French, 2013). This work attempts to investigate which variables discriminate between original and counterfeit notes. Data was collected on three variables associated with the naira note (₦1000) namely: mass, area, and tensile stress (strength of material).

MATERIALS AND METHODS

Definition of Relevant Terms

(i) *Mass* (m)

Mass is the material content of an object which is measured in kilogram (kg).

(ii) *Force* (F)

This has to do with a pull or push on a body and is measured in Newton (N).

Mathematically,

$$\text{Force} = \text{Mass} \times \text{Acceleration.}$$

(iii) *Acceleration Due to Gravity* (g)

This is the time rate of change of velocity. It acts in a downward direction and is measured in meter per square second (m/s^2).

(iv) *Weight* (W)

Weight is the action or influence of a gravitational pull on a body. It is a force and so is measured in Newton. It is the product of mass and acceleration due to gravity.

Mathematically,

$$\text{Weight} = \text{Mass} \times \text{Acceleration due to gravity}$$

(v) *Area* (A)

The extent of the surface of all or part of a solid measured in the square units is referred to as its area. The ₦1000 note used to obtain the data for this study is rectangular in shape. Thus the area is calculated by multiplying together the length and width of the rectangle.

(vi) *Tensile Stress* (T)
Tensile stress is the force per unit cross-sectional area measured in kilograms per square meter that a body can resist before it ruptures. It is a measure of the strength of the material used in making the substance or object (Microsoft Encarta, 2009).

Data Collection
The data for this work was collected first hand in person by taking measurements of mass, thickness, length and width of each of the randomly selected counterfeit and genuine ₦1000 notes. The micrometer screw gauge, electronic balance and meter rule were used to measure respectively the thickness, mass and length/width of genuine and counterfeit ₦1000 naira notes randomly selected for this study.

Data Preparation
Measurement of mass (X) taken in gram were converted to kilogram while thickness (Y) measured in millimeter were converted to meter (the SI units for length and mass respectively). Also, the area computed in square centimeter was converted to square meter.

The area, weight and tensile stress (Z) of the ₦1000 notes were computed as follows:

Area (m^2) = Length × Width

Weight (N) = Mass (kg) × Acceleration due to gravity (10m/s^2)

Tensile Stress (N/m^2) = $\frac{Weight}{Area}$

The data collected are displayed in Table1.

The Discriminant Function
Discrimination functions are linear combinations of variables that best separate the groups (Rencher, 2002).

We define a linear combination of p independent variables ($X's$) as

$$Z = a_1X_1 + a_2X_2 + \cdots + a_pX_p \qquad (1)$$

Thus for the two groups under consideration, we have

$$Z_{i1} = a_1X_{i11} + a_2X_{i21} + \cdots + a_pX_{ip1}$$

$$Z_{i2} = a_1X_{i12} + a_2X_{i22} + \cdots + a_PX_{ip2}$$

for the i^{th} individual in groups I and II.

The values of the $a's$ in the discriminant equation are chosen so as to maximize the separation between the two groups denoted by D.

The separation between groups is expressed in terms of the difference between the means of the discriminant functions for the two groups.

Now, suppose we define the group means $\bar{Z}_1$ and $\bar{Z}_2$ as

$$\bar{Z}_1 = a_1\bar{X}_{11} + a_2\bar{X}_{21} + \cdots + a_p\bar{X}_{p1} \qquad (2)$$

and

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$\bar{Z}_2 = a_1\bar{X}_{12} + a_2\bar{X}_{22} + \cdots + a_p\bar{X}_{p2}$ (3)
then,
$D = \bar{Z}_1 - \bar{Z}_2$ is the quantity to be maximized.

According to Fisher (1954), the maximization criterion is equivalent to maximizing the ratio

$$\frac{D^2}{SS_W} \qquad (4)$$

where SS_W is the within groups sum of squares. For simplicity, let $SS_W = W$.

Now, in the present situation, if we define

$z_{i1} = Z_{i1} - \bar{Z}_1$ and $z_{i2} = Z_{i2} - \bar{Z}_2$ for the two groups; then

the sum of squares can be expressed in matrix form as

$$\sum_{i=1}^{n_1} z_{i1}^2 = \underline{a}'\underline{S}_1\underline{a} \text{ and } \sum_{i=1}^{n_2} z_{i2}^2 = \underline{a}'\underline{S}_2\underline{a}$$

So that
$$W = \sum_{i=1}^{n_1} z_{i1}^2 + \sum_{i=1}^{n_2} z_{i2}^2 = \underline{a}'(\underline{S}_1 + \underline{S}_2)\underline{a} = \underline{a}'\underline{S}\underline{a} \quad (5)$$

where, $\underline{S}_1$, $\underline{S}_2$ and $\underline{S}$ are the group I covariance matrix, group II covariance matrix and
the pooled covariance matrix respectively.

Let $D = a_1d_1 + a_2d_2 + \cdots + a_pd_p$

where $d_j = \bar{X}_{j1} - \bar{X}_{j2}$ denote the difference between group means on variable X_j so that in matrix form, we have
$D = \underline{a}'\underline{d}$.

Thus, (4) can equivalently be expressed as:

$$\lambda = \frac{D^2}{W} = \frac{(\underline{a}'\underline{d})^2}{\underline{a}'\underline{S}\underline{a}}$$

The next task is to maximize λ. This is achieved by differentiating λ with respect to $\underline{a}$ and setting it equals to zero. Thus, we have

$$\frac{\partial\lambda}{\partial\underline{a}} = \frac{W\left(\frac{\partial D^2}{\partial\underline{a}}\right) - D^2\frac{\partial W}{\partial\underline{a}}}{W^2} = \frac{\frac{\partial D^2}{\partial\underline{a}}}{W} - \left(\frac{D}{W}\right)^2\frac{\partial W}{\partial\underline{a}} = 0$$

which gives the result

$$\left(\frac{W}{D}\right)\underline{d} = \underline{S}\underline{a}$$

$$\Rightarrow \underline{a} = \left(\frac{W}{D}\right)\underline{S}^{-1}\underline{d}$$

Since $\left(\frac{W}{D}\right)$ cannot affect the proportionality in the elements of $\underline{a}$, it is set equal to one for convenience sake. Hence,

$$\underline{a} = \underline{S}^{-1}\underline{d}\,.$$

The a_i's $(i = 1,2,\ldots,p)$ can now be substituted in (1) to obtain the discriminant equation.

RESULT
From the data of table 1 below, we obtain from group I(original note):

Table1: Mass, Thickness and Tensile Stress of Genuine and Counterfeit₦1000 Notes

	Original ₦1000 notes (Group 1)			Counterfeit ₦1000 notes (Group 2)		
S/NO	Mass(kg) X_1	Thickness(m) Y_1	T/Stress(N/m^2) Z_1	Mass(kg)X_2	Thickness(m)Y_2	T/Stress(N/m^2)Z_2
1	0.00117	0.00021	0.01012812	0.00119	0.00027	0.010507726
2	0.00115	0.00019	0.00963633	0.00105	0.00022	0.009334993
3	0.00120	0.00019	0.01018849	0.00124	0.00023	0.010192339
4	0.00129	0.00020	0.01060332	0.00116	0.00027	0.010312945
5	0.00137	0.00019	0.01156118	0.00137	0.00026	0.012685185
6	0.00130	0.00018	0.01118568	0.00112	0.00024	0.010023268
7	0.00128	0.00019	0.01079622	0.00128	0.00020	0.010864952
8	0.00131	0.00018	0.01112243	0.00107	0.00021	0.009771689
9	0.00128	0.00018	0.01115468	0.00101	0.00019	0.009099099
10	0.00123	0.00019	0.01037449	0.00128	0.00020	0.010589890
11	0.00106	0.00020	0.00899754	0.00109	0.00021	0.009754788
12	0.00125	0.00018	0.01054319	0.00123	0.00022	0.010509228
13	0.00128	0.00019	0.01072566	0.00122	0.00022	0.010159893
14	0.00126	0.00018	0.01097944	0.00124	0.00019	0.010734072
15	0.00130	0.00018	0.01125346	0.00125	0.00020	0.011261261
16	0.00117	0.00019	0.00993125	0.00114	0.00018	0.010000000
17	0.00119	0.00018	0.01003711	0.00113	0.00019	0.010044444
18	0.00124	0.00018	0.01025896	0.00120	0.00019	0.010526316
19	0.00121	0.00018	0.01014335	0.00114	0.00019	0.010133333
20	0.00122	0.00019	0.01042379	0.00117	0.00021	0.010195190
Total	**0.02476**	**0.00375**	**0.21004468**	**0.02358**	**0.00429**	**0.206700611**

Mean mass $(\bar{X}_1)$ = 0.001238

Mean thickness $(\bar{Y}_1)$ = 0.000188

Mean tensile strength $(\bar{Z}_1)$ = 0.010502.

$$\underline{S}_1 = \begin{pmatrix} 0.00000001 & -0.00000001 & 0.00000004 \\ -0.00000001 & 0.00000001 & 0.00000001 \\ 0.00000004 & 0.00000001 & 0.00000039 \end{pmatrix}$$

Similarly for group II (counterfeit note), we have :

Mean mass $(\bar{X}_2)$ = 0.001179

Mean thickness $(\bar{Y}_2)$ = 0.000215

Mean tensile strength $(\bar{Z}_2)$ = 0.010335.

$$\underline{S}_2 = \begin{pmatrix} 0.00000011 & 0.00000002 & 0.00000006 \\ 0.00000002 & 0.00000003 & 0.00000001 \\ 0.00000006 & 0.00000001 & 0.00000056 \end{pmatrix}.$$

Thus,

$$\underline{d} = \begin{pmatrix} \bar{X}_1 & - & \bar{X}_2 \\ \bar{Y}_1 & - & \bar{Y}_2 \\ \bar{Z}_1 & - & \bar{Z}_2 \end{pmatrix} = \begin{pmatrix} 0.000059 \\ -0.000027 \\ 0.000167 \end{pmatrix} \quad ;$$

$$\underline{S} = \underline{S}_1 + \underline{S}_2 = \begin{pmatrix} 0.00000012 & 0.00000001 & 0.0000001 \\ 0.00000001 & 0.00000004 & 0.00000002 \\ 0.0000001 & 0.00000002 & 0.00000095 \end{pmatrix}.$$

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and

$$\underline{S}^{-1} = \begin{pmatrix} 9267931.9694 & -1848656.6428 & -936652.6990 \\ -1848656.6428 & 25634705.4474 & -345082.5733 \\ -936652.6990 & -345082.5733 & 1158491.4962 \end{pmatrix}.$$

Hence,

$$\underline{a} = \underline{S}^{-1}\underline{d} = \begin{pmatrix} 440.3007 \\ -858.8366 \\ 147.5228 \end{pmatrix}$$

and the resulting discriminant equation is

$$Z = 440.3007X - 858.8366Y + 147.5228Z.$$

From (2) and (3), we obtain the group means as:

$\bar{z}_1 = 2.932915431$ and $\bar{z}_2 = 1.859112794$.

So that

$$D = \bar{z}_1 - \bar{z}_2 = 2.932915431 - 1.859112794 = 1.073802637$$

$$\Rightarrow D^2 = 1.1531 \ ;$$

which is the Mahalanobis distance between the two groups.

We now need to test the significance of the quantity, D^2.

Applying the Hotelling's T^2 statistic, we have

$$T^2 = \frac{n_1 n_2}{n_1 + n_2} D^2 = \frac{20 \times 20}{20 + 20}(1.1531\,) = 11.531$$

Now,

$F = \frac{N-p-1}{(N-2)p} T^2$; where $N = n_1 + n_2 = 40$ $and\ p = 3$

$$\Rightarrow F = 3.6414$$

Since at α = 0.05, $F_{(p,N-p-1)0.05} = 2.87$; then we conclude that the differences between the ₦1000 notes (originals and counterfeits) in the mean values of mass (X), thickness (Y) and tensile stress (Z) are significant.

The end point for classification is:

$$z_0 = (\bar{z}_1 + \bar{z}_2)/2$$
$$= (2.932915431 + 1.859112794)/2 \approx 2.3960$$

Thus for all ₦1000 notes encountered, the value of z can be calculated by substituting their measurements of X, Y, Z in the discriminant function.

Since $\bar{z}_1 > \bar{z}_2$, the ₦1000 notes having their z - values more than z_0 are to be allocated to group I and those having z values lower than z_0 are assigned to group II.

That is, a ₦1000 notes with mass, thickness and tensile stress(X, Y, Z) is classified as genuine (original) if

$$Z = 440.3007X - 858.8366Y + 147.5228Z > 1.8960$$

Otherwise it should be classified as counterfeit (fake).

DISCUSSION

In essence, this work aimed at obtaining a function that can discriminate between original and fake Naira currency. Three parameters (mass, thickness and tensile stress) were being measured for the two sets of monies. A derivation of the appropriate measure of the separation of the two groups was made. Hotelling's T^2 and Mahalanobis quantity, D^2 were applied to accomplish the subject aim.On carrying out the test, the differences between the ₦1000 notes, fake and genuine in the mean values of mass, thickness and tensile stress were found to be significant. Hence, the discriminant function and classification rule were obtained.

As recommendation, operators of financial institutions should not only rely on the use of conventional instruments listed in the review for detection of fake currencies but should rather apply statistically tested techniques for more efficient and reliable results.

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