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TESTING FOR EQUALITY OF MEANS WITH EQUAL AND UNEQUAL VARIANCES

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ABSTRACT

In this paper, we are interested in comparing the conventional t –test with the proposed t – test for testing equality of means with unequal and equal variances. Here, we proposed harmonic mean of variances as an alternative to the pooled sample variance when there is heterogeneity of variances. Two sets of secondary data were obtained from Agricultural Development Project (KWADP) and the Ministry of Agriculture in Ilorin, Kwara State to demonstrate the two test statistics used and the results show that the proposed t – test statistic is found to be appropriate than the conventional t – test statistic when we have unequal variances but the conventional t – test perform better when we have equal variances.

Keywords: equality of means, t – test statistic, homogeneity of variances, harmonic mean of variances, heterogeneity of variances.

INTRODUCTION

Statistical hypothesis testing is a method of making statistical decision using experimental data (Fisher, 1925; Lehmann and Joseph; 2005) and Abidoye; 2012)). One use of hypothesis testing is in deciding whether experimental results contain enough information to cast doubt on conventional wisdom or not (Lehmann and Joseph, 2005). In statistical methods, decisions are almost always made using null hypothesis (Lehmann and Joseph, 2005). The null hypothesis is often the reverse of what the experimenter actually believes. It is put forward to allow the data to contradict it (Abidoye, 2012). This paper primarily concerns itself with the application of test hypothesis with directional alternatives. This has application in many fields. For example, in medicine it may be of interest to study the viral loads of three groups of patients that were diagnosed and treated for a particular viral disease by different methods. (Yahya and Jolayemi, 2003). In Agricultural research where the interest is to investigate the effectiveness of certain brands of fertilizer meant for a particular crop, there might be a pre – conceived belief that certain brand(s) are more effective than others (Abidoye , 2012), Abidoye et. al (2016a, 2016b, 2016c)).

Adegboye and Gupta (1986) discussed testing equality of means under common but unknown variance (σ^2) using ordered alternative with strict inequality. Bartholomew (1959) considered testing k normal variates having same mean against the alternative hypothesis $H_1 : \mu_1 \neq \mu_2 \neq ... \neq \mu_k$.

Gupta et. al. (2006) in consideration of multivariate mixed models, suggested that the distributional assumptions of the errors are not required but assumed that the random sample is from large population of levels. Cochran (1964) investigated the test of equality of means in Behrens – Fisher problem and compared his test with the test developed by Benerjee (1960) and McCullough et. al (1960). Levene (1960) proposed a test criterion for testing equality of variances for specified significance level.

Kupolusi and Adebola (2017) applied a developed procedure by (Abidoye; 2012), Abidoye et.al; 2013a), Abidoye et.al; 2013b) and Abidoye *et. al;* 2007)) to test the hypothesis: $H_0 : \mu_1 = \mu_2 = ... = \mu_g$ against alternative $H_1 : \mu_1 \neq \mu_2 \neq ... \neq \mu_g$, at least two μ_i s are not the same where the error term $e_{ij} \sim N(0, \sigma_i^2)$ $i = 1,2,...g$.

METHODOLGY

Proposed Test Statitic for Unequal Variance

We are interested in developing a suitable test procedure to test the hypothesis

$H_0 : \mu_i = \mu$ against

$H_1 : \mu_1 > \mu_2 > \mu_3 > ... > \mu_g$ i.e

$$H_1 : \mu_i - \mu_{i+1} > 0 \qquad (2.1)$$

(Abidoye et. al., 2013a), 2016a) and (Abidoye,2012)

where $Y_{ij} \sim N(\mu_i, \sigma_i^2)$ $\quad i = 1,2,..., g$

and $j = 1,2,.., n_i$

i.e heterogeneity of variances (under the situation when the groups of variances are not equal).

That is

$H_0 : \mu_i = \mu \quad \forall_i$ Vs

$H_1 : \min(\mu_i - \mu_{i+1}) > 0 \quad \forall_i$

The unbiased estimate of $\min(\mu_i - \mu_{i+1})$ =

$$\min(\bar{Y}_i - \bar{Y}_{i+1}) = \text{Y} \qquad (2.2)$$

$V(Y) = Var[\min(\bar{Y}_i - \bar{Y}_{I+1})]$

$$\frac{\sigma_i^2}{n_i} + \frac{\sigma_{i+1}^2}{n_{i+1}} \qquad (2.3)$$

but

$$Y = \min(\bar{Y}_i - \bar{Y}_{i+1}) \sim \lambda N(\mu_i - \mu_{i+1}, \frac{\sigma_i^2}{n_i} + \frac{\sigma_{i+1}^2}{n_{i+1}})$$

See Abidoye (2012) (2.4)

where

$$\lambda = g(1 - \Phi(\bar{Y}_i - \bar{Y}_{i+1}))^{g-1} \quad , \quad 0 < \lambda < 1 \qquad (2.5)$$

and λ is a scaling factor.

From equation (2.4), S_p^2 will be appropriate to use as the estimate if the group variances are equal i.e $\sigma_i^2 = \sigma_{i+1}^2$. But in a situation where the group variances are not equal, that is, $\sigma_i^2 \neq \sigma_j^2$ for at least one $i \neq j$. S_p^2 (Pooled variance which is the weighted mean of variances) cannot be used. In this regard, the harmonic mean of the variances rather than the pooled or mean of the variances is hereby proposed as $\sigma_H^2 = \left(\frac{1}{g} \sum \frac{1}{\sigma_i^2} \right)^{-1}$

where $\hat{\sigma}_H^2 = S_H^2$ which is the alternative estimator (S_H^2) that best estimates all σ_i^2 .

From equation (2.3)

$$Var(Y) = \frac{\sigma_i^2}{n_i} + \frac{\sigma_{i+1}^2}{n_{i+1}}$$

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$\doteq \sigma_H^2\left(\frac{1}{n_i}+\frac{1}{n_{i+1}}\right)$ see Abidoye 2012 and Abidoye et.al (2013a) (2.6)

The test statistic for the hypotheses set in equation (2.1) is therefore

$t^{\bullet} = \frac{\lambda Y}{Z}$ See Abidoye (2012) and Abidoye et. al (2013b) (2.7)

where

$Y = \min(\bar{Y}_i - \bar{Y}_{i+1})$ (2.8)

and

$$Z = \sqrt{S_H^2\left(\frac{1}{n_i}+\frac{1}{n_{i+1}}\right)} \quad (2.9)$$

Then, the null hypothesis is rejected if

$$P(t_r^{\bullet} = \lambda \frac{Y}{Z} > t_0^{\bullet}) = P(t_r^{\bullet} = \lambda t_r > t^{\bullet})$$

$$= P(t_r > \frac{t^{\bullet}}{\lambda}) < \alpha \quad (2.10)$$

where

$$t_r = \frac{\left|(\bar{Y}_{i+1} - \bar{Y}_i)\right|}{S_H\left(\frac{1}{n_i}+\frac{1}{n_{i+1}}\right)^{\frac{1}{2}}} \quad (2.11)$$

The Procedure for Conventional T - Test Statisic

We set the conventional t – test statistic to test the hypothesis

$H_0 : \mu_i = \mu$ against

$H_1 : \mu_1 > \mu_2 > \mu_3 > \ldots > \mu_g$ i.e

$H_1 : \mu_i - \mu_{i+1} > 0$ (2.12)

where $Y_{ij} \sim N(\mu_i, \sigma_i^2)$ $i = 1,2,\ldots,g$ and $j = 1,2,..,n_i$ (Abidoye, 2012) , Abidoye et. al ,2013b) and Abidoye et. al ,2013a)).

and the error term is $e_{ij} \sim N(0,\sigma^2)$ $i = 1,2,\ldots,g$ and $j = 1,2,\ldots,n_i$ with constant variance.

Then $X_{ij} \sim N(\mu_i, \sigma_i^2)$ and set $\sigma_i^2 = \sigma_{i+1}^2$. where X_{ij} are the observed values.

The unbiased estimate of $\min(\mu_i - \mu_{i+1})$ = $\min(\bar{Y}_i - \bar{Y}_{i+1})$ = Y (2.13)

and

$V(Y) = Var[\min(\bar{Y}_i - \bar{Y}_{I+1})]$

$\frac{\sigma_i^2}{n_i} + \frac{\sigma_{i+1}^2}{n_{i+1}}$ but $\sigma_i^2 = \sigma_{i+1}^2$

$$= \sigma^2\left(\frac{1}{n_i}+\frac{1}{n_{i+1}}\right) \quad (2.14)$$

but

$$Y = \min(\bar{Y}_i - \bar{Y}_{i+1}) \sim N\left(\mu_i - \mu_{i+1}, \sigma^2\left(\frac{1}{n_i}+\frac{1}{n_{i+1}}\right)\right) \quad (2.15)$$

Distribution of Pooled Variance

The pooled variance is defined as S_p^2 which represents series of equal group variances. Abidoye (2012) , Abidoye et. al (2013b) and Abidoye et. al (2013a), showed that the sample distribution of S_p^2 is approximated by the chi – square distribution.

where $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$

Abidoye (2012)

$$Y = \min(\bar{Y}_i - \bar{Y}_{i+1}) \sim N(\mu_i - \mu_{i+1}, S_P^2(\frac{n_i + n_{i+1}}{n_i n_{i+1}}))$$
(2.16)

Consequently, the test statistic for the hypotheses set in equation (2.12) is

$t = \frac{Y}{Z}$ (Abidoye, 2012) , Abidoye et. al, 2013b) and Abidoye et. al 2013a)) (2.17)

where

$$Y = \left(\bar{X}_i - \bar{X}_{i+1}\right) \quad (2.18)$$

and

$$Z = \sqrt{S_P^2\left(\frac{1}{n_i} + \frac{1}{n_{i+1}}\right)} \quad (2.19)$$

Now p- value = $P(t = t_r > t_{cal})$ (2.20)

where t_r is regular t – distribution and r is the appropriate degrees of freedom for the t – test . (Abidoye, 2012), Abidoye et. al, 2013b) and Abidoye et. al, 2013a).

$$t_r = \frac{Y}{Z} \sim t_{(\alpha, n_1+n_2-2)}$$
(2.21)

Then equation (2.21) can be written as

$$t_r = \frac{\left(\bar{Y}_{i+1} - \bar{Y}_i\right)}{S_P\left(\frac{1}{n_i} + \frac{1}{n_{i+1}}\right)^{\frac{1}{2}}} \sim t_{(\alpha, n_1+n_2-2)} \quad (2.22)$$

DATA ANALYSIS

The data used in this study are secondary data, collected primarily by Kwara Agricultural Development Project (KWADP), Ilorin, Kwara State, Nigeria.

Table 1: The yield of maize (in Tonnes/hectare) in four different locations in Kwara Agricultural Development Project, Ilorin, Kwara State.

Years	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Zone A 1	1.3	0.9	1.0	0.5	1.0	0.9	0.9	1.2	1.24	1.2
Zone B 2	0.6	0.5	0.7	0.4	0.7	0.8	0.6	0.6	0.6	0.68
Zone C 3	0.9	0.7	0.8	0.6	0.9	0.9	0.7	0.8	0.8	0.9
Zone D 4	2.8	2.9	2.62	3.14	2.6	2.6	2.80	2.80	2.80	2.7

Table 2: Spss Result Output On Test Of Homogeneity Of Variance

	Levene STATISTIC	df_1	df_2	P- value
	7.017	3	36	0.001

By the application of Levene test of equality of variances in the Table 3.2 (Levene's Statistic = 7.017, p – value =0.001), the variances differ significantly from location to location. (Abidoye, 2012), Abidoye et. al ,2013b) and Abidoye et. al ,2013a)).

The hypothesis to be tested is

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$$H_0 : \mu_A = \mu_B = \mu_C = \mu_D vs H_1 : \mu_A < \mu_B < \mu_C < \mu_D$$

$\sigma_i^2 \neq \sigma_{i+1}^2$ (Abidoye, 2012), Abidoye et. al., 2013b) and Abidoye et. al., 2013a).

Computation Using Proposed t – test Statistic

Computation on maize: From the data above the following summary statistics were obtained:

Zone A: $\bar{Y}_A = 1.014, S_A^2 = 0.056, n_A = 10$

Zone B: $\bar{Y}_B = 0.618, S_B^2 = 0.0126, n_B = 10$

$Zone C:$ $\bar{Y}_C = 0.8, S_C^2 = 0.0111, n_C = 10$

$Zone D:$ $\bar{Y}_D = 2.776, S_D^2 = 0.027, n_D = 10$

Arranging the means in order we have

$\bar{Y}_D = 2.776,\quad \bar{Y}_A = 1.014,\quad \bar{Y}_C = 0.80,\quad \bar{Y}_B = 0.618$

Consider the differences given below;

$\bar{Y}_D - \bar{Y}_A = 2.776 - 1.014 = 1.62$

$\bar{Y}_A - \bar{Y}_C = 1.014 - 0.80 = 0.214$

$\bar{Y}_C - \bar{Y}_B = 0.80 - 0.618 = 0.182$

Then

$\min(\bar{Y}_{i+1} - \bar{Y}_i) = (\bar{Y}_C - \bar{Y}_B) = 0.182$

$g = 4$

$$n = \sum_{i=1}^{4} n_i = 40, \quad S_H^2 = \left(\frac{1}{g} \sum \frac{1}{S_i^2} \right)^{-1}, \quad S_H = 0.1335$$

$$t^{\bullet} = \frac{\min(\bar{Y}_{i+1} - \bar{Y}_i)}{S_H \sqrt{\left(\frac{1}{n_{i+1}} + \frac{1}{n_i} \right)}} = \frac{0.182}{0.1335 \sqrt{\frac{1}{10} + \frac{1}{10}}} = 3.0484$$

$$p - value = p(t^{\bullet} = \lambda t_r > t_{cal}) = p(t_r > \frac{t_{cal}}{\lambda})$$

$$= p(t_r > 9.6805)$$

$$= 1 \times 10e^{-8} < 0.05$$

We reject H_0 and conclude that the mean yields of maize in all the four zones were indeed ordered as D, A, C, B

Computation Using Conventional t – test Statistic

In the above data set, $n_i = 10$, g = 4, $n = \sum_{i=1}^{2} n_i = 20$,

$$S_p^2 = \frac{(n_A - 1)S_A^2 + (n_B - 1)S_B^2 + (n_C - 1)S_C^2 + (n_D - 1)S_D^2}{n_A + n_B + n_C + n_D - 4} = 6.3031$$

The hypothesis to be tested is

$$H_0 : \mu_A = \mu_B = \mu_C = \mu_D \text{ Vs } H_1 : \mu_A < \mu_B < \mu_C < \mu_D ; \sigma_i^2 = \sigma_{i+1}^2$$

$$t^* = \frac{\overline{X}_A - \overline{X}_B}{S_P \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim t_r$$

$$= \frac{-1.430}{2.5106\sqrt{\left(\frac{1}{10} + \frac{1}{10}\right)}} = \frac{-1.430}{1.1228}$$

= - 1.2736

Now p- value = $P(t^* = t_r > t_{cal}) = P(t_r > t_{cal})$

$$= P(t_r > 1.2736)$$

= 0.08789 > 0.05

We do not reject H_0 and conclude that the mean yields of maize in the four zones are not significantly different.

Also in this paper, we made use of another data, obtained from the Ministry of Agriculture in Ilorin, Kwara State which were secondary data, covering the period 1998 – 2007.

Table 3: Yields of Sorghum for ten years (1998 – 2007).

Years	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Zone A 1	7.9	10.9	12.0	15.2	20.0	19.9	12.9	14.6	17.36	15.82
Zone B 2	18.45	17.78	16.83	20.42	16.27	18.83	12.52	20.88	19.43	15.32

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Zone C 3	21.4	13.34	12.97	14.26	18.45	15.39	16.73	9.87	7.35	19.45
Zone D 4	9.38	10.11	14.45	13.14	22.48	12.39	11.55	15.44	9.72	10.53

Table 4: Spss Result Output on Test of Homogeneity of Variance

	Levene STATISTIC	df_1	df_2	P- value
	0.710	3	36	0.552

By the application of Levene test of equality of variances to data in Table 3.2, Levene's Statistic = 0.710, p – value = 0.552, the variance are the same from location to location.

Computation on Proposed t – test Statistic

$H_0 : \mu_A = \mu_B = \mu_C = \mu_D$ Vs $H_1 : \mu_A < \mu_B < \mu_C < \mu_D ; \sigma_i^2 \neq \sigma_{i+1}^2$ (Abidoye ,2012) , Abidoye et. al,2013b), Abidoye et. al,2013a), Kupolusi and Adebola (2017)).

Computation on maize: From the data above the following summary statistics were obtained:

Zone A: $\overline{Y}_A = 14.66, S_A^2 = 33.06, n_A = 10$

Zone B: $\overline{Y}_B = 18.06, S_B^2 = 23.04, n_B = 10$

$ZoneC$: $\overline{Y}_C = 14.92, S_C^2 = 18.66, n_C = 10$

$ZoneD$: $\overline{Y}_D = 12.92, S_D^2 = 15.45, n_D = 10$

Arranging the means in order are:

$\overline{Y}_B = 18.06,$ $\overline{Y}_C = 14.92,$ $\overline{Y}_A = 14.66,$ $\overline{Y}_D = 12.92$

Consider the differences given below;

$$\overline{Y}_B - \overline{Y}_C = 18.06 - 14.92 = 3.14$$

$$\overline{Y}_C - \overline{Y}_A = 14.92 - 14.66 = 0.26$$

$$\overline{Y}_A - \overline{Y}_D = 14.66 - 12.92 = 1.74$$

Then

$$\min(\overline{Y}_{i+1} - \overline{Y}_i) = (\overline{Y}_C - \overline{Y}_A) = 0.26$$

$g = 4$

$$n = \sum_{i=1}^{4} n_i = 40, \quad S_H^2 = \left(\frac{1}{g} \sum \frac{1}{S_i^2} \right)^{-1}, \quad S_H = 20.8376$$

$$t^{\bullet} = \frac{\min(\overline{Y}_{i+1} - \overline{Y}_i)}{S_H \sqrt{\left(\frac{1}{n_{i+1}} + \frac{1}{n_i} \right)}} = \frac{0.26}{20.8376\sqrt{\frac{1}{10} + \frac{1}{10}}} = 0.0279$$

$$p - value = p(t^{\bullet} = \lambda t_r > t_{cal}) = p(t_r > \frac{t_{cal}}{\lambda})$$

$$= p(t_r > 0.0837)$$

$$= 0.652 > 0.05$$

We do not reject H0 and conclude that the mean yields of Sorghum in the four zones are not significantly different.

Using Conventional t – test Statistic

In the above data set, $n_i = 10$, g = 4, $n = \sum_{i=1}^{2} n_i = 20$,

$$S_p^2 = \frac{(n_A - 1)S_A^2 + (n_B - 1)S_B^2 + (n_C - 1)S_C^2 + (n_D - 1)S_D^2}{n_A + n_B + n_C + n_D - 4} = 22.553$$

The hypothesis to be tested is

$H_0 : \mu_A = \mu_B = \mu_C = \mu_D$ Vs $H_1 : \mu_A < \mu_B < \mu_C < \mu_D ; \sigma_i^2 = \sigma_{i+1}^2$ See (Abidoye ,2012) , Abidoye et. al ,2013b) and Abidoye et. al ,2013a)).

$$t^* = \frac{\overline{X}_A - \overline{X}_B}{S_P \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t_r$$

$$= \frac{0.26}{4.749\sqrt{\left(\frac{1}{10} + \frac{1}{10} \right)}} = \frac{0.26}{2.1238}$$

= 0.1224

Now p- value = $P(t^* = t_r > t_{cal}) = P(t_r > t_{cal})$

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$$= P(t_r > 0.1224)$$

$$= 0.0235 < 0.05$$

We reject H_0 and conclude that the mean yields of Sorghum in all the four zones are indeed ordered from D, A, C, B.

CONCLUSION

The result of testing for equality of variances shown in Table 3.1 indicate that variances are not equal and the proposed t-test statistic performs better than conventional t- test because test of homogeneity of variance are not the same and we reject H_0 for the proposed t - test. In the second data set, test of homogeneity as shown in Table 3.4 indicate that variances are equal and we do not reject H_0 which reveals that variances are the same. Therefore, conventional t – test performed better than proposed t – test because H_0 is rejected.

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