Mathematical literacy teachers' engagement with contextual tasks based on personal finance

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This article reports on a study carried out with a group of 108 practising Mathematical Literacy (ML) teachers who participated in an Advanced Certificate in Education (ACE) programme. The purpose of the qualitative study was to identify and describe the teachers' varying levels of engagement with mathematics tools and resources. The teachers were given questions based on financial mathematics as part of a routine assessment, including questions based on other aspects of the module. Their written responses to the selected test items were analysed. Thereafter 13 teachers were interviewed individually with the purpose of confirming or disconfirming the categories identified by the initial script analysis. The analysis identified varying levels of skill in using the mathematics and contextual resources and tools. The study also found that success in the items required flexible participation in both the mathematics and contextual domains.

Keywords: mathematical literacy, ACE, mathematics of finance, situative perspective, local community of mathematical literacy practice, direct problem, inverse problem, process-object duality

Introduction and literature review

In South Africa, authorities are most concerned that our past education has resulted in very low levels of numeracy in our adult population. International studies show that South African learners' performance in mathematical literacy test items is very poor when compared to other counties (Soudien, 2007) In response to this widespread problem, one of the interventions from the Department of Education was to introduce the subject Mathematical Literacy (ML) as a fundamental subject in the Further Education and Training (FET) band in order to help develop numeracy skills among South African citizens. ML seeks to produce learners who are participating citizens, contributing workers and self-managing people (DoE, 2003). Its purpose is not for learners to do more mathematics, but more application and to use mathematics to make sense of the world. The Department of Education (DoE 2003:9) defines ML as follows:

> *Mathematical literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday solutions and to solve problems.*

Curriculum documents emphasise that in ML, context and content should be inextricably intertwined in any teaching and learning situation:

> *When teaching and assessing Mathematical Literacy, teachers should avoid teaching and assessing content in the absence of context. At the same time teachers must also concentrate on identifying and extracting from the context the underlying mathematics or 'content' (DoE, 2007:7).*

The stipulation about the relationship between content and context offers us as mathematics educators an exciting opportunity to deepen our own understanding about how students engage with mathematics concepts which are embedded in real-life contexts.

Assessment at school level in ML is guided by the ML assessment taxonomy (DoE, 2007:27-28), which specifies 4 levels in the hierarchy. Venkat, Graven, Lampen and Nalube (2009) have criticised the taxonomy for a number of reasons, one of which is that "combining content (in terms of facts and procedures) and context oriented complexity within a single hierarchy appears to suggest that both these aspects become more complex together". This is in contrast with the case of the UK subject Functional Mathematics where the categories suggest that these two aspects can vary independently of each other (Venkat *et al*., 2009:46). Berger, Bowie and Nyaumwe (2010) argue that the Mathematics assessment taxonomy (DoE, 2008), like other taxonomies, highlights an intrinsic difficulty of conflating cognitive level with mathematical activity. Berger *et al*. (2010:30) state that the taxonomy assumes that cognitive level increases with the type of mathematical activity, an assumption which they question. In the taxonomy, memorisation is the lowest cognitive level, followed by routine procedures, complex procedures and finally problem-solving. It is hoped that this article, while examining students' responses to items set within a particular context, will add insight into the issue concerning the relationship between context, mathematics content and mathematical activity and the use of the taxonomy in categorising assessment items.

It is important to note that the use of contexts in ML is different from the ways in which it is used in mathematics assessment. Contexts in ML demand a greater "real-life" authenticity. The emphasis of ML is on life-related applications of mathematics; the purpose is for learners to use mathematics in order to make informed decisions in everyday life. This should be done by taking contexts in real life (like the billing systems of different cellphone providers) and using mathematics to explore the meaning and implications of the information, thereby helping them make more informed decisions. It is expected that, by studying these life-related applications, learners will inculcate the habit of seeking to be more informed before they make decisions.

The decision to introduce ML in all secondary schools resulted in various challenges. There were few mathematics teachers available, who understood the subject and who were willing to facilitate the subject with the FET learners. The Faculty of Education at the University of KwaZulu-Natal (UKZN) responded to the challenge by offering teachers the opportunity to enrol for an ACE (ML) to upgrade or retrain themselves to teach ML. In this article, both the words "teacher" and "student" are used to describe our participants because they are practising ML teachers who are part-time university students.

The study reported in this article was carried out with a group of 108 ML teachers who were studying towards an Advanced Certificate in Education (ACE). The assessment items under scrutiny were part of a module that focused on exploring mathematical applications in real-life contexts. As part of the module, the teachers studied a unit on personal finance, including aspects of bond repayments, transfer duties, simple and compound interest, and loan repayments. This article explores their engagement with two tasks based on the context of transfer duties.

The tasks under scrutiny are the transfer duty formulae used to calculate the transfer duty payable to the government when one buys a house. The costs that are payable are described in different levels and can be explained as an example of a piecewise function, where each piece is defined by a separate rule or formula over a specified domain. Luthuli (2000) wrote an account of real-life applications of such piecewise functions by describing how one could use integer-valued functions to derive formulae to describe them. However, this article does not intend to examine the specific algebraic formulae that can be used to represent the transfer duty rule.

Theoretical framework

This research is underpinned by a situated learning perspective – a broad set of understandings which conceptualise the learning process as changes in participation in socially organised activity (Lave, 1988). Learning is located in increased access of learners to participation rather than in the acquisition of structure. Situative perspectives focus on the social and contextual nature of knowledge and emphasise the notion that much of what is learnt is specific to the situation in which it is learnt. This perspective is compatible with a study of students' engagement with ML tasks which involve the use of mathematics to solve problems arising from a real-life context. Consistent with the situative perspective, Greeno

(1991) describes a conceptual domain in mathematics as an environment with resources at various places in the domain (instead of the usual view of a subject matter domain as a structure of facts, concepts, principles, procedures, and phenomena that support the cognitive activities of knowing, understanding, and reasoning). To know the domain is to know one's way around the environment and also includes the ability to recognise, find and use those resources productively (Greeno, 1991:175). This perspective implies that learning can be described as the increasingly skilled use of tools and resources in the domain.

Our perspective is that ML is a subject that entails the use of mathematical tools and resources together with those from the contextual domain in order to solve mathematical problems which must be interpreted in the context. We therefore find it useful to examine the group of ACE (ML) teachers as constituting a community that can be described as a Local Community of ML Practice (LCMLP), in a similar manner that Watson and Winbourne (2008) described an LCMP as a local community of mathematical practice. Similar to Watson and Winbourne's notion of LCMP, the ACE was structured with these goals in mind: students are engaged in ML problem-solving activities; there is public recognition of developing competence during class discussions and in assessments; the students work together towards the achievement of a common understanding; in each class there are shared ways of language, values, and tool-use; the class is constituted by the active participation of both students and tutors, and students consider themselves engaged in the ML activities. An important aspect of their participation is the communication with tutors and other students. However, unlike the LCMP described by Watson and Winbourne (2008), our LCMLP involves participation within two domains, namely a contextual domain and the mathematics domain, because of the nature of ML. This implies that the objects, tools and resources used to participate in the LCMLP are drawn from both domains.

Within any community, individual learning is an important aspect of the dynamics between the participants. In fact, Bowers, Cobb and McClain (1999) demonstrated that the relationship between classroom practices and individual learners' learning is reflexive. That is, learners contribute to the development of practices within the classroom community; these practices, in turn, constitute the immediate context for their learning. Similarly, Sfard introduces the term *commognition*, a combination of communication and cognition, and emphasises "that interpersonal communication and individual thinking are two facets of the same phenomenon" (Sfard, 2008:xviii). An individual's cognitive understanding is enhanced by communicating with other learners. As newcomers communicate with more experienced participants, they begin to use the tools and resources of the domains more appropriately; this enhances their participation as their practices become endorsed by the community. We argue that communication by text is an important element of the communication within the LCMLP and that written responses of participants serve as an important indicator of their varying skills in the use of necessary tools, thus also serving as one (but not the only) indicator of participation within the LCMLP. The written responses allow us to make inferences about the levels of the use of tools and resources found in the contextual and mathematics domains.

Our central research question is: What do the teachers' responses to ML assessment tasks based on financial mathematics reveal about their skills in using tools and resources drawn from the contextual and mathematics domains?

Financial mathematics involves calculations based on simple and compound interest, monthly bond instalments, transfer duties, and so on. It is an area rich in opportunities to help teachers understand and make more informed decisions about their lives. This article focuses on two tasks concerning transfer duties. We are also cognisant of the fact that, in our attempt to offer students the authenticity of tasks drawn from their real-life experiences, aspects of the authenticity disappear in the transformation that takes it from a real-life experience to a class-based task. In a real-life situation, decisions are based on extraneous factors which may not be recognised by the task designers or instructors. In a classroom or examination setting, the driving force is the desire to pass the assessment by providing responses judged to be what the examiner wants. Students' desire to obtain high marks, rather than to demonstrate the ability to use mathematics and contextual tools appropriately and accurately may influence their responses to tasks.

Methodology

A naturalistic inquiry with an emphasis on interpretive dimensions was used. The goal of the researcher is to understand reality (Cohen, Manion & Morrison, 2000). According to Cohen *et al*. (2000), the interpretive research paradigm assumes that people's subjective experiences are substantive and worthy of study. The sample of 108 students for the study was drawn from a group of students who are enrolled for the ACE (ML). Students were given questions based on financial mathematics as part of a routine classroom assessment which included questions based on other aspects of the module they were studying. This article reports on the 108 students' responses to two specific test items described below. The teachers were drawn from different backgrounds and took part in the programme on ML. To this end, the purpose of this small-scale study reported in this article is to explore the engagement with ML tasks based on the calculation of transfer duties. We argue that students' responses to specially designed test items can add to knowledge about their participation practices in a developing community of ML practice.

The analysis of the responses can be viewed as content analysis which could be used in the analysis of educational documents and throws "additional light on the source of communication, its author, and on its intended recipients, those to whom the message is directed" (Cohen *et al*., 2000:165). In addition, Neuman (2011:323) states that content analysis is "nonreactive" because those who are being studied are not aware of that fact; therefore, "the process of placing words, messages, or symbols in a text to communicate to a reader or receiver occurs without influence from the researcher who analyses its content".

A sample of 13 students was selected to be interviewed based on their responses to assessment items across the module. Individual interviews were held with the 13 students, with the purpose of confirming or disconfirming the categories identified by the initial script analysis. The purpose of the interviews was to find out more about why students responded in the manner in which they did.

We distinguish between direct and inverse problems (Groetsch, 1999) in this study. A direct problem is one which asks for an output, when given the input and the process. For an inverse problem, the output is given, and the problem could ask for the input or the process that led to the output. In this study the inverse problem asks for the input.

The test items presented in this article are described below. The first is a direct problem and the second an inverse one.

In order to analyse the mathematical demand of this task, we use the process-object duality notion to distinguish between the uses of the transfer duty rule. Sfard's (1991) process-object model[1] for the learning of mathematical concepts asserts that a concept can be conceived in two fundamentally different ways: operationally when a mathematical concept is viewed as a process, and structurally when a mathematical concept is viewed as an object. Sfard (1991) argues that the ability of learners to view a mathematical concept both as a process and as an object is indispensable for a full understanding of mathematics. Mason's (1989:2) description of "a delicate shift of attention [that] proceeds from seeing an expression as an expression of generality to seeing the expression as an object or property" captures the process-object movement of thought. The suggested solutions discussed in Table 1 allow us to disentangle to some extent what we mean by contextual tools and the use of mathematical tools and resources. Contextual tools refer to the context-specific rules, language, terminology, objects or visual mediators and reasoning used by those who interact within these contexts. The description of the context-specific rule with the three options is the contextual tool. Skill in using the tool can be inferred by whether the person can identify the correct option and use the rule to calculate the transfer duty that is payable on specific houses. Carrying out this procedure for Question 1 is not particularly mathematically demanding and demonstrates a routine-driven use (Sfard, 2008:182) of the rule, requiring the calculation of a percentage on a subtraction. The rule is applied in the form in which it is presented and the user needs to substitute the input and obtain an output.

However, Question 2 requires a more sophisticated object-driven use (Sfard, 2008:182) of the same rule. In order for a rule, process, manipulation or transformation to be used in further processes, it needs a shift of attention by the student so that it is perceived as an object or entity that can be operated upon. Such a shift will enable a student to view the entity (transfer duty payable) as the result of, but separate from the process that produced it. Only when the entity is encapsulated/reified (Dubinsky, 1991; Sfard, 2008:44, 170; Gray &Tall, 1991:173) into an object (the result of the transfer duty rule) can the person carry out further transformations on the rule. Thereafter the solution of the inverse transfer duty problem involves a transformation of the rule, which entails an object-driven use of the rule.

Table 1: Test items

<table>
<tr><td colspan="2">Context: The formula that is used to calculate the transfer duty, payable by a new home owner, is as follows:
• For a purchase price of R0-R500 000, the transfer duty is 0%.
• For a purchase price of R500 001 to R1 000 000, the transfer duty is 5% on the value above R500 000.
• For a purchase price of R1 000 001 and above, the transfer duty is R25 000 + 8% of the value above R1 000 000.</td></tr>
<tr><td>Question</td><td>Solutions and comments</td></tr>
<tr><td>1. Calculate the transfer duty payable on a house that is valued at R895 000.</td><td>Direct problem: Input (purchase price) given, asked to find output (transfer duty).
• Identify second option as relevant to the amount of R895 000.
• Calculate that R895 000 is more than R500 000 by the amount of R395 000 (R895 000 – R500 000).
• Find 5% of R395 000= R19 750</td></tr>
<tr><td>My friend paid transfer duty of R45 280 on the house that she bought.
2. How much did her house cost?</td><td>Inverse problem: Output (transfer duty) is given, asked to find input (purchase price).
Set up of equation: One solution would be to let the amount above R1000 000 be P. Then R45 280 = R25 000 + 8% of P
• Solution of equation:
R20 280 = (8/100)P
P = R253 500
• Cost of house = 1 000 000 + R253 500 = R1 253 500
Another acceptable solution would be a series of calculations which systematically “undo” the operation in the original rule.</td></tr>
</table>

Thus basic skill in using the context rule in ML tasks can be judged by whether a person can appropriate a routine-driven use of the rule, in this case by correctly calculating the output of the context rule for particular inputs as in Question 1. Formulating an equation to solve an inverse problem where the output is known and the input is unknown demands a more mathematically sophisticated use of the rule by drawing on mathematics tools and resources such as equation and the percentage concept. The demonstration of such skill is evidence of what we term an object-driven use of the context-specific rule. A shift from using the contextual tool in a routine-driven to object-driven manner depends on the participants’ skill in using available mathematical resources.

Results

This section discusses the results of both Question 1 and Question 2.

Question 1

As described in the preceding section, the solution required a routine-driven use of the context-specific rule. The responses to Question 1 were classified as *correct*, *nearly correct* or *misinterpretation of the context-specific rule*. These categories are described below.

Correct or nearly correct

For this question there were 87 (81%) students whose responses were classified as correct because they correctly calculated the transfer duty payable on the house. Two students correctly identified the first step, but did not successfully complete the calculation, because of slips with additional zeros or subtraction errors.

Misinterpretation of the context-specific rule

We classified twelve responses as a misinterpretation of the context rule. There were several types of misinterpretation.

Table 2: Summary of students who misinterpreted the rule

Eight students found 5% of the full price: House price = R895 000 R895 000 x 5% = R44 750	Two students found 8% of the amount over R500 000 895 000 – 500 000 = 395 000 8% of 395 000 = 31 600	Two students made other errors, e.g. 100 000 – 895 000=105 000 5% of 105 000 = 5 250

The eight responses of column 1 suggest that the students did not fully understand the instruction "*the transfer duty is 5% on the value above R500 000"*. This context-specific rule indicates first that 5% is the applicable tariff per cent for those amounts above R500 000, and secondly that the 5% is calculated on the difference between the full price and R500 000. When asked why he took 5% of the full amount, one student mentioned "I thought that this is this 5% ... Ngicabanga ukuthi 895 ufalo [I think that it falls] under 5%". This shows that the student only considered that part of the instruction which indicated the tariff per cent. A second student explained similarly that, since R895 000 was above R500 000, she thought she needed to use the figure of 5% in her calculation. Her explanation only considered part of the context-specific rule.

The two students in column 2 misinterpreted the rule by confusing options 2 and 3.

Two students in column 3 made fundamental errors. In the one instance, the student took 5% of the difference between R1 000 000 and R895 000, showing that, although he opted for the relevant option, he applied the rule incorrectly. The student commented during the interview: "Yes, R895 000, it is closer to R1 000 000 than R500 000, so we have to find R1 000 000 – R895 000". This shows a misinterpretation of the context-specific rule.

Table 3 summarises the above discussion.

Table 3: Summary of students' responses to Question 1

	Response	Number
Question 1	Correct or close to correct	89 (82%)
	Misinterpreted context rule	12
	No attempt	7
	Total	108

The mathematics (calculation of 5% of a value) was not demanding, requiring merely a procedural application of the percentage concept after identifying the amount that had to be operated upon. In summary, for this question, the majority of the students (82%) were able to use the context-specific rule appropriately in order to calculate the transfer duty that was due, displaying a routine-driven use of the rule. However, 19 students were unable to carry out the procedure in a routine-driven manner. The engagement of twelve students, who found it difficult to make an attempt, was restricted because of a misinterpretation of the context rule which prevented them from performing the routine. Their misinterpretation of the rule also hampered their responses to Question 2, with 4 of them presenting an incorrect equation and the others presenting no equation. Hence, none of these students were able to work out Question 2 correctly.

Some students indicated that their difficulties with the transfer duty rule were caused by their unfamiliarity with the real-life context of transfer duty. One student remarked: "I was not used to [this question] … I did not understand it". These comments reveal a dilemma faced by mathematics educators. To fulfil the mandate of ML, students need opportunities to engage in real-life contexts, yet when the context is unfamiliar, their engagement is limited. In this group, the transfer duty was covered in their lectures and calculations were also discussed in class; however, it seems that this exposure was still insufficient for the twelve students (and perhaps the seven students who did not respond).

Question 2

This question required a more sophisticated use of the transfer duty rule, in a way that can be described as an object-driven use of the tool. The responses to this question are used to typify varying levels of mathematical skill in using the context tool. These are labelled as *non-recognition of mathematical demand*; *incorrect set-up of equation, object-driven use but with calculation errors*, and *object-driven use of context rule.*

Varying levels of skills

The data allows us to distinguish between varying levels of mathematical skills in the use of the tools. Knowing the domain in a situated cognition perspective includes the ability to identify, find and use the appropriate resources skilfully (Greeno, 1991:175).The four blocks below illustrate that, as we move to the right, the responses indicate increasingly skilled use of the procedure starting from a non-recognition of the mathematics resources that were needed and moving towards being able to use the tools in a mathematically productive – that is, in an object-driven manner.

Low skills Non-recognition of mathematical demand	→	Intermediate Identified need for set-up of equation, but set up incorrect equation	→	Object-driven with errors Set up the equation appropriately, but did not compute the input correctly	→	Object-driven use of contextual tool
Responses of 13 students (12%) who did not set up an equation		Responses of 15 students (14%) who set up an inappropriate equation		Responses of 14 students (13%) who set up an appropriate equation but made computation errors		Responses of 59 students (55%) who set up and correctly solved the appropriate equation

Figure 1: Indications of increasing mathematical skill in using the transfer duty rule

As pointed out earlier, Question 2 is an inverse problem and requires the identification of the unknown, an appropriate set-up of an equation, substitution into the equation, and the solution of the equation.

Non-recognition of mathematical demand

The 13 students described in Category 1 did not recognise the mathematical demand because they were unable to distinguish between a direct and an inverse problem. Below are three students' responses indicating different reasons why they did not set up the equation.

Table 4: Examples of responses considered as non-recognition of mathematical demand

Student A	Student B	Student C
1000 000 – 45 280 = R954 720 + 8% 1 000 000) = R1034,720	R1 000 000	Property value = R45 280 x 5% + R500 000 x/100 = 45 280/100 × (100 × 0.05) x = 226 400

First, Student A did not understand the question, although he understood the transfer duty rule because he was able to work out the first question. This student first subtracted the transfer duty from R1 million and then added 8% of R1 million to the result. His interview responses are translated for clarity: "I was told her house cost R1 000 000 and I was given transfer duty" showing that he thought the price of the house was R1 000 000. After some discussion, he asked: "Are they asking what her house cost?", showing that he did not originally understand the question because of his language skills. He said: "… I write so slowly so sometime I run out of time … that's why I can't do certain problems right". This student did not set up an equation because he did not understand the instruction.

Student B, who wrote an estimated amount of R1 000 000, explained that he was optimistic after answering the first question but when he moved to the second question, things changed:

> *I was stuck and ... then maybe my problem was understanding the question correct and I must say I am not poor in maths ... I realize what I did was wrong I just estimated the value and just decided to write 1million ... So I need to know a correct formula that why everything was wrong if you look at my answer there is no formula there is nothing you can write. You draw a formula, you must correct substitute, you must know which are you looking for, it can be an unknown value than you need to work out in order to get the [answer] correct. So once there is no formula everything was totally wrong.*

This excerpt conveys the student's frustration in trying to identify a formula he could use to work out the unknown value and he expresses why he resorted to merely estimating an answer of R1 million. He knew that he was being asked for the house price but he could not identify the mathematical resource needed to solve the problem.

The third response (student C) in Table 4 was classified as non-recognition of the mathematical demand. Although the student used an x (unknown) in the calculation, the unknown was the output of a series of calculations. Student C was unable to set up an equation to find the input. When asked to explain his strategy, he said that he "blundered" and felt he needed more practice at this kind of problem:

> *[I need] proper practice and to do more of the exercises that will make me familiar with these problems ... because if you don't give yourself time ... you fail ... in class I was understanding it very well and I told myself I have no problems here but I realized that I need to practice this.*

This shows that the student knew he was not familiar with the different types of problems and could not identify the difference between the two types of questions.

Incorrect set-up of equation

A second level of mathematical skill in using the context tool was evidenced by those students who recognised that an equation was needed but who set up an inappropriate equation. Table 5 presents three such examples.

Table 5: Examples of students who set up an incorrect equation

Student C (Incorrect equation) T.D = 5/100 × *x* R45 280 = 5*x*/100; [not all steps captured] 4 528 000/5 = 905 600 The property costs R905 600	Student D (Incorrect Equation) R45 280 = P +(8% of P + 25 000) [not all steps captured] P = 18 111 2,77	Student E (Incorrect Equation) 45 280 = 25 000 + 8% (x – 500 000) [not all steps captured] x = 253 000,00

Student D, who set up an equation, responded as follows when asked why she used 5%: "45 thousand is for the transfer duty … I thought that with R45 thousand I have to apply 5%". This response shows that she misinterpreted the option at which the transfer duty was calculated, revealing that she did not access a routine- driven use of the rule, which also prevented her from progressing to using the rule in an object-driven manner. Although she recognised the need for the set-up of an equation, her set-up was influenced by the interpretation of the rule as 5% of the cost of the house.

An example of a student who recognised the need for a reversal was Student E, who said: "To get exactly the same price, I think is the reversing of the transfer duty to get the actual price … it goes back to that it's a reversal thing". This student recognised the need for a reversal strategy but found it difficult to write the appropriate equation. It is worth noting that this student got Question 1 correct. Therefore, it is not the interpretation of the context rule that hindered her as much as the skill of setting up the appropriate equation to capture the information.

Object-driven use, but with calculation errors

Figure 1 reveals that there were 14 students who set up the correct equation; unfortunately, they did not carry out the mathematics procedures correctly. Below are two examples of such students' responses, showing an appropriate set-up of a relevant equation, but containing algebraic manipulation errors.

Table 6: Examples of students who made computation errors although they set up the correct equation

Student F 45 280 = 25 000 + 8%P 20 280 = 8P P = 2 535 P = 1 000 000 + 2 535 = R1 002 535	Student G 25 000 + 8% (x – 1000 000) = 45 280 25 000 + 8x – 8 000 000 = 45 280 25 000 + 8x = 8 000 000 – 45 280 8x = 7 930 000 x = 9 912 250

We regard these 14 students as demonstrating higher levels of skills in this instance, because they showed more facility with the available resources and tools than the students who responded at the previous two categories. They set up the relevant equation, but their slips or errors in algebraic manipulation let them down.

Object-driven use of rule

Clearly, the responses of the 59 students who recognised the problem as an inverse problem and then went ahead, set up and solved an appropriate equation correctly, demonstrate a highly skilled use of the rule with respect to this particular problem. There were many constraints to using the rule in the manner required, as described in the difficulties experienced by the other students. It required a procedural understanding of the rule as well as a deep understanding of the mathematics of equations and concept of percentage, although it is possible that a student who wrote the correct solution could have learnt of the procedure. An excerpt from an interview with one student who found the correct price reveals that she has a sound understanding of the mathematical resources and context tools:

> *Number one we have to check the transfer duty ... there are less than 25 000 because the house which is above 1000 000 has a transfer duty of 25 000 plus 8% ... now we are able to find out the real amount of the house and including the transfer duty. We have to find the number it above because this 25 to 45. This 45 280, that was the transfer duty of the house so we have to subtract the 25 000 which is 1000 000 of the transfer duty then after that you have to calculate 20 000 remain 280 [R25 280] remain from when you minus 25 000 then after that we multiply by this times 8% which gives is 253 500 then we take 1000 000 ... then the house total amount is 1 253 500".*

Her explanation [unedited] conveys her understanding of the systematic reversal of each step, first subtracting the R25 000 (corresponding to a price of R1 million) from the given amount of R45 280, then equating that R20 280 to 8% of the amount [above R1 million], calculating that amount as R253 000 and then finally adding the R1 million back to the calculation.

Summary of responses to Question 2

Table 7 contains a summary of the responses discussed above.

Table 7: Summary of responses to Question 2

Correct	**59(55%)**
Set up an appropriate equation, but made computation errors	14
Set up an inappropriate equation	15
Did not set up an equation	13
No attempt	7
Total	108

Concluding remarks

In this article we presented a summary of 108 teachers' responses to two ML assessment items set within the context of transfer duty, one of which was a direct problem and the other an inverse problem. By considering the transfer duty rule as the contextual tool and the use of equations and the percentage concept as the mathematical tools and resources, we used Sfard's process-object theory to distinguish between a routine-driven and an object-driven use of the context rule. The written responses of the 108 students and the interview responses of 13 students were used to show varying levels of skill in the ways in which the mathematics tools and resources were activated to use the contextual tool.

It is evident from Tables 3 and 7 that the students found the first question easier than the second one. Eighty one per cent (81%) got the first question correct while only fifty-five per cent (55%) got the second one correct: a 26 percentage point difference. We have already noted that Question 1 is a direct problem (it required a substitution into a given rule), while Question 2 is an inverse problem (it required the set-up and solution of the input, of an equation). Both Question 1 and Question 2 required an application of the mathematical concept of per cent to a context-specific rule. However, success in Question 2 was dependant on a more sophisticated object-driven use than that of the routine-driven use (of the same context rule) for Question 1. The responses to Question 2 allowed us to distinguish between varying levels of engagement (with the context rule) displayed by the students. These levels of engagement were related to the varying use of the mathematical resources.

The analysis also revealed some complexities in trying to classify items at particular levels of the ML assessment taxonomy referred to earlier in this article. The two items would be classified at Level 2 of the ML assessment taxonomy because both involve a routine procedure using a familiar context. The context is deemed to be familiar because all the students encountered it during their coursework and classroom assessments. The procedure is considered routine because "all of the information required to solve the problem is immediately available to the student" (DoE, 2007:27). However, this analysis has shown that the two items differ in mathematical complexity although they are based on the same procedure. The solution to Question 2 required a different use of the same procedure used for Question 1. It supports the assertion by Venkat *et al*. (2009) that mathematical complexity and contextual complexity may vary independently of each other. In these two items, the contextual complexity was similar but the mathematical complexity differed.

It was found that students who performed poorly in Question 1 were hampered by their interpretation of the context-specific rule. This misinterpretation also limited their movement to an object-driven use of the rule as required for Question 2. Four of the students who misinterpreted the rule in Question 1 presented an incorrect equation and the others presented no equation for Question 2. Thus none of these students were able to work out Question 2 correctly. Students' participation practices can be constrained by their access to, and use of contextual tools and resources. They may then be limited to peripheral participation because of their inability to interpret and use the contextual rules, which may be attributed to their limited experience of transfer duties.

The article has also shown that flexible participation within and across both the contextual and mathematical domains was necessary for success in these ML tasks. Flexible participation in the two domains requires students to use resources from both the contextual and the mathematics domains in an intertwined manner. In fact, the relationship between practices in the contextual domain and practices in the mathematics domain can be considered reflexive. As practitioners become more skilled in the use of mathematics resources, their participation practices in the contextual domain are enhanced because their use of the mathematics tools and resources helps them engage more deeply with aspects of the context. Conversely, their participation in the contextual domain adds insight to the constraints and affordances of the mathematics resources and tools, thus enabling them to develop their skills in the mathematics domain.

Endnote

1. Sfard's process-object duality construct can be regarded as a contraction of the more comprehensive APOS (action, process, object, schema) theory of Dubinsky (1991), in that the former does not include actions and schemas. An action is an externally driven, repeatable physical or mental manipulation that transforms objects. According to the APOS framework, actions and processes are operations on previously established objects and each action needs be interiorised into a process and then encapsulated into an object before being acted upon by other actions/processes.

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