# Trainee teachers: Changes in approach to developing a "connected" understanding of mathematics

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## Abstract

*The mathematics curriculum in the United Kingdom (UK) has undergone radical changes, placing a particular focus on mental computation. Pupils are not taught written methods of computation until they are able to add and subtract any pair of two digit numbers mentally. Teacher education programmes in the UK have required adaptation to reflect these changes by developing beginning teachers' understanding of "connectedness" of mathematical thinking. Drawing on data from one institution in West London, this article explores the development of connected thinking. Results suggest that, without specific intervention geared to the development of such thinking, trainee teachers' mental mathematics understanding is likely to be at variance with connected mathematics thinking.*

**Keywords:** Mathematics; subject knowledge; "connected" understanding; mental strategies

## Introduction – the National Numeracy Strategy

The National Numeracy Strategy (Department for Education and Employment (DfEE) 1998) was introduced into British schools in 1999 because of concerns that British schools compared unfavourably with those in other countries (DfEE, 1998). A study by the Basic Skills Agency found that results by British adults in a mathematics test were inferior to those from six other countries (Basic Skills Agency, 1997). A Task Group was set up in 1997 to review research and theory and thereby to explore the possibility of raising mathematical achievement. It led to the establishment of the National Numeracy Strategy (Brown, Askew, Baker, Denvir & Millet, 1998).

### A new emphasis on mental mathematics

Mental mathematics was emphasised in the original National Numeracy Strategy as a result of highly influential research taking place in the Netherlands (Beishuizen, 1997). The emphasis placed on developing mental calculation was radical in its implications and effects. For example the National Numeracy Strategy (DfEE, 1998) stated that children should not be taught a standard method of written calculation until they were able to "add or subtract reliably any pair of two digit numbers in their heads" (DfEE, 1998, 7). However, although young children were expected to use oral methods, the use of pencil and paper was not prohibited. Informal jottings were to be encouraged and in the early years children were to be taught to record answers to problems. As children got older and began to use larger numbers, informal jottings could be used to assist their mental calculations.

Furthermore children were now expected to deal with whole numbers. Until the implementation of the National Numeracy Strategy (DfEE, 1998), children were taught to split numbers into tens and units and then add or subtract them separately. The stance adopted in the National Numeracy Strategy (DfEE, 1998) was based on new evidence (Beishuizen, 1997). This suggested that children's understanding of the number system developed more effectively when they thought of numbers as a whole because they were more likely to make good use of estimation and approximation techniques.

The emphasis on mental capability was one of the biggest changes introduced in the National Numeracy Strategy and a considerable amount of work has since been undertaken to develop this aspect of children's mathematical understanding (Anghileri, 1999; Beishuizen, 1999; Treffers & Beishuizen, 1999; Thompson, 1997).

This work has led Beishuizen (1997) to point out some of the dangers inherent in the reliance on mental calculation. He argues that there is a difference between doing mental arithmetic *in* your head, and doing mental arithmetic *with* your head. He suggests that mental recall (i.e. memorising number facts) is done in the head, whereas the mental strategies that lead to understanding are done with the head. Partly because results can be arrived at quickly, there is a danger, when talking about daily mental work (known and encouraged in the National Numeracy Strategy (DfEE, 1998) as the 'mental and oral starter') of focusing on doing work in the head, emphasising the procedural at the expense of understanding.

## The preparation of teachers

Beishuizen's (1997) research implies that there is an on-going requirement within teacher education to promote and assess the development of trainee teachers' own knowledge and understanding of mental mathematics as they prepare to meet the obligations and expectations of the National Numeracy Strategy. His interest in the distinction between working in the head and with the head is mirrored, in the teacher education context, by concepts such as 'relational' understanding (Skemp, 1989) and, 'connected knowledge' (Ma, 1999; Davis, 2001) which have emerged from general research on the knowledge requirements of teachers of mathematics conducted over the last two decades.

Skemp (1989) argued that understanding can be both relational and instrumental and if the aim is to develop relational understanding in children (that is, knowing both what to do and why), then teachers of mathematics must have relational understanding too. Ball (1990) claimed that mathematics teachers need knowledge of the nature and discourse of mathematics enquiry and that their knowledge needs to be correct, connected and meaningful. Askew defined primary mathematics teachers' development, in terms of their "appreciation of the multifaceted nature of mathematical meaning" (Askew, Brown, Rhodes, William & Johnson, 1997, 93).

Ma (1999) compared American and Chinese teachers and found that the Chinese education system encourages learning where problems are approached in a number of ways and where there is an expectation that any given result will be mathematically justified. She described this as 'profound' learning that is 'deep, broad and thorough' (Ma, 1999, 121) and which produces connected knowledge. Connecting with more conceptually powerful ideas produces depth, connecting with concepts of similar power produces breadth and thoroughness is 'the capability to "pass through" all parts of the field – to weave them together' (Ma, 1999, 121).

Davis (2001) agrees. Arguing that connectedness is an integral part of mathematics classrooms, he claims that 'addition cannot be grasped without realising its relationship with subtraction and the way in which it operates within the set of natural numbers, the integers and ultimately the set of real numbers' (Davis, 2001, 136). However he goes further when he suggests that 'the connectedness of this discipline extends beyond the links between mathematical ideas as such. There are relationships to empirical concepts. For instance, we cannot exhaust the 'meaning' of subtraction merely by specifying the sets of numbers to which this operation may be applied, its relationship to addition, and so on. Something must also be said about the way in which it may be modelled in the 'real world'. It can be illustrated by means of the physical removal of objects from a group and by the physical comparison of one group of objects with another (Davis, 2001, 137).

Logically, mental mathematics cannot be exempt from these ambitions. The implicit assumption is that trainee teachers should develop the facility to work relationally (Skemp, 1989) or 'connectedly' (Ma, 1999) with regard to mental mathematics. In the absence of other dedicated research on the nature of teachers' understanding of mental mathematics, this paper describes work undertaken in one university to test that assumption.

## The project

The incentive for the development project may therefore be traced to the requirements of the National Numeracy Strategy mediated by the implications of research into relational understanding and connected knowledge. The intention, during the first year of the project, was to map the extent and development of trainee teachers' relational or connected understanding of mental mathematics during the one-year course of training. The results led to the adaptation of the mathematics programme of study and this was used with a new cohort of trainee teachers during the second year of the project. During that second year, qualitative data on trainees' habitual responses were gathered continuously during normal teaching sessions.

## Year one – the cohort

The subjects were the 170 trainee teachers on a primary one-year Postgraduate Certificate of Education teacher training course at a London University. All participants had a first degree (or equivalent) in a subject directly relevant to the national curriculum in primary schools. In addition to gain entry to the programme, trainees were required to have GCSE grade C or above (or equivalent) and to pass a basic mathematics test at interview. The mathematics test included basic numeracy questions, requiring trainees to identify prime numbers and find percentages, together with problems involving reasoning and proof. Test results showed that the cohort represented a broad range of ability and previous experience of mathematics. Many students had not studied mathematics since school while others had only recently gained their GCSE in mathematics.

## The course

The course began with two weeks' observation and task-focused experience in primary schools. Trainees then spent twelve weeks on a teaching programme within the university. This included modules on subject knowledge and pedagogic knowledge in the National Curriculum's core subjects of mathematics, English and science. Trainees then completed a six-week teaching practice block in a primary school, followed by a further four weeks of university-based training. The final school placement of nine weeks was tapered to allow trainees to undertake an equivalent teaching timetable to that expected of a newly qualified teacher.

Within the mathematics subject knowledge modules trainees were progressively introduced to the strategies taught to pupils in school, as outlined in the National Numeracy Strategy (DfES, 1998). In the first seminar on calculation, the mental methods for addition were introduced, followed by mental methods for subtraction. In the succeeding seminar, written strategies for addition were covered, followed by written strategies for subtraction. This format was also used for multiplication and division. These sessions were taught to groups of thirty trainees, in a classroom environment, with little time available for application of the strategies.

## Data collection

A test of mental mathematical competence was administered to the whole cohort when trainees had completed the university-based programme and the six-week placement in school. The test consisted of twenty problems involving the four number operations and incorporating different numbers of digits, decimals and fractions (shown in Table 1). During the introduction to the activity it was described as a mental test. However, trainees had a space in which to do jottings should they choose to do so. This was done to establish the preferred method of computation, to provide data on the methods and strategies used within and across items and to identify the frequency of use of multiple methods for individual items.

Trainees' use of formal or informal methods was of particular interest. Formal methods were defined as algorithms (strategies which do not require an understanding of the process, e.g. inverting one fraction and multiplying, for division by fractions, or decomposition for subtraction). Informal methods included rounding and adjusting, where calculations took into account the numbers involved and decisions were made about the most appropriate calculation. For example, item 5 (199 + 174) could be solved informally by rounding to 200, then adjusting (200 + 174 – 1).

Items were chosen so that their solution was relatively straightforward when using informal methods and rather more complicated when using an algorithm or standard written method mentally. For example item number 2 (1442 + 4739) was a relatively complicated calculation to perform mentally, using a formal algorithm, since it involved 'carrying' several times as shown in Figure 1 and the potential for errors was great. It was relatively straightforward when using informal partitioning methods.

$$\begin{array}{ccccc} & 1 & 4 & 4 & 2 \\ \hline + & 4 & 7 & 3 & 9 \\ & {\scriptstyle 1} & & {\scriptstyle 1} & \\ \hline & 6 & 1 & 8 & 1 \end{array}$$

**Figure 1: Formal algorithm for item 2**

This was the justification for the study's assumption that trainees who solved the items mentally (without using written working) used an informal method but it was recognised that this included cases where trainees relied on 'known facts' to which they had instant recall through memory.

# Results

**Table 1: Responses and incidence of written and informal work by item**

| | | | Results link to attempted questions (not necessarily correct) | | Results link to those solutions which were correct | | |
|---|---|---|---|---|---|---|---|
| Item number | Item | % of students with correct solutions | % of students **attempting** each question using written work | % of informal methods out of those who **attempted** using written methods | % of correct solutions without using written work out of those who got it correct | % correct solutions using informal written methods out of those who got it correct | % correct solutions using some sort of informal method |
| 1 | 95 + 46 | 95 | 40 | 43 | 57 | 16 | 73 |
| 2 | 1442 + 4739 | 89 | 87 | 20 | 9 | 16 | 25 |
| 3 | $\frac{1}{2} + \frac{3}{8}$ | 82 | 72 | 9 | 19 | 5 | 24 |
| 4 | 0.4 + 2.8 | 94 | 35 | 10 | 62 | 3 | 65 |
| 5 | 199 + 174 | 92 | 58 | 41 | 39 | 23 | 62 |
| 6 | 49 – 18 | 92 | 38 | 40 | 57 | 14 | 70 |
| 7 | 2002 – 2475 | 63 | 50 | 19 | 35 | 6 | 41 |
| 8 | $\frac{3}{4} - \frac{2}{3}$ | 64 | 78 | 3 | 6 | 0 | 6 |
| 9 | 2.58 – 1.29 | 77 | 77 | 14 | 14 | 8 | 22 |
| 10 | 7 x 8 | 92 | 6 | 100 | 86 | 5 | 92 |
| 11 | 32 x 20 | 93 | 50 | 78 | 47 | 36 | 83 |
| 12 | 155 + 156 | 93 | 52 | 38 | 46 | 18 | 64 |
| 13 | 52 x 34 | 63 | 94 | 45 | 0 | 22 | 22 |
| 14 | $\frac{2}{3} \times \frac{5}{8}$ | 48 | 73 | 9 | 2 | 4 | 6 |
| 15 | 3.4 x 4.9 | 31 | 78 | 26 | 1 | 4 | 5 |
| 16 | 8 ÷ 2 | 98 | 1 | 0 | 97 | 0 | 97 |
| 17 | 42 ÷ 7 | 98 | 9 | 82 | 90 | 8 | 97 |
| 18 | 200 ÷ 25 | 88 | 15 | 78 | 75 | 9 | 84 |
| 19 | $\frac{7}{8} \div \frac{1}{3}$ | 24 | 53 | 6 | 1 | 0 | 1 |
| 20 | 1.2 ÷ 0.2 | 49 | 30 | 7 | 32 | 13 | 45 |

## Discussion – item type, choice of strategies and success

The relationship between the type of item, successful completion and reliance on written work, whether formal and informal, was shown to be complex. Item 13 (52x34) and item 15 (3.4x4.9) (Table 1) illustrate this complexity. Sixty-three per cent of trainees correctly answered item 13 and 94% of those who attempted it, used some written work. Of those who chose to use written working, almost half used informal methods. The most popular informal method was one which involved some element of partitioning; this was done in a variety of ways, but the most frequent choice was (52x30)+(52x4). This item prompted the largest range of informal methods. Below is an example of the types of strategies employed.

((34 x 100) ÷ 2) + (34 x 2)
((52 x 10) x 3) + (52 x 4)
(10 x 34 x 5) + 34 + 34
((52 x 20) + 520) + (52 x 4)

Five per cent of the cohort used the grid method for this item, but not always with success.

| x | 50 | 2 |
|---|---|---|
| 30 | 1500 | 60 |
| 4 | 200 | 8 |

In this case the errors, which included entering incorrect numbers in the operant cells and entering 150 in the first cell rather than 1 500, were not rectified through the application of checking procedures.

Of the 37% of trainees who were unable to solve this problem accurately, the most frequent error – the incorrect application of the distributive law – implied a lack of understanding of the relationship between the operation of multiplying in relation to the numbers:

52 x 34 = (50 x 30) + (2 x 4)

The same errors were displayed for item 15 (3.4 x 4.9). Only 31% of the trainees were able to solve this problem and very few did so without using written working. In total 78% tried to use written work, almost 75% of whom tried to use a formal algorithm such as that below:

| | | | | |
|---|---|---|---|---|
| | 3 | . | 4 | |
| X | 4 | . | 9 | |
| | 1 | | 3 | |
| | 3 | . | 0 | 6 |
| 1 | 3 | . | 6 | 0 |
| 1 | 6 | . | 6 | 6 |

Just under half of the trainees using this method gave 166.6 as an answer, implying a lack of connection between the operation of multiplying and place value and little recognition to the advisability of using alternative methods e.g. estimation, to check.

The most common informal strategy was to rewrite the problem without the decimal point, in effect multiplying both numbers by ten. However, many trainees divided the answer by ten rather than a hundred again suggesting a lack of connection between the operation and place value. One trainee wrote that she did not know how to complete this calculation accurately, but changed it to 3 ½ multiplied by 5, to provide an approximate solution.

As with the previous item, partitioning was a popular choice of strategy, but once again, the connection between the operation and the numbers was not fully appreciated and the distributive law was incorrectly applied.

$3.4 \times 4.9 = (3 \times 4) + (0.4 \times 0.9)$

The four items (3, 8, 14 and 19) which prompted the greatest use of formal written work (algorithms) were those involving fractions.

For three out of four of the fraction problems, approximately three quarters of the trainees used a formal written method. The fraction problem which prompted the least amount of written work (53%) was a problem involving division of fractions (19: $\frac{7}{8} \div \frac{1}{3}$). However, this was clearly the problem that trainees found the most difficult; only 24% were able to solve it although they had successfully solved fraction problems involving addition and subtraction. The most common error was to invert the wrong fraction, resulting in an inverted solution. Six trainees added comments to their test paper noting that they were unable to remember which fraction to invert, or that they could not remember the rule for dividing fractions. During overheard conversations after the test trainees had confessed that they did not know whether or not 'to turn upside down and multiply', which fraction to invert and whether they needed to find a common denominator.

A common error for the problem that involved multiplying fractions was the practice of finding a common denominator, and then multiplying or dividing the numbers. For example, a typical solution for item 14 ($\frac{2}{3}$ x $\frac{5}{8}$) is shown below.

$$\frac{2}{3} \times \frac{5}{8} = \frac{16}{24} \times \frac{15}{24} =$$

Although at this stage the calculation was not incorrect, few trainees starting with this strategy went on to complete the problem. Two trainees continued to attempt to solve the problem, but made mistakes in calculating 16x15 or 24x24.

Across all the items where written work was used, in 75% of cases a formal algorithm was the most popular choice. The exceptions were those items involving knowledge of multiplication tables (items 10, 11, 17 and 18, see Table 1.) When a formal algorithm was not used, the most popular strategies were partitioning, the grid method, rounding and adjusting, use of knowledge of place value, and doubling. Thirty-two per cent of the cohort used partitioning to solve item 13 (52 x 34) and a further 5% used the grid method. For item 12 (155 + 156) 29% of the cohort used a formal algorithm (such as that shown in Figure 1) and 13% used doubling, either by doubling 150, then adding 11, or doubling 155 and adding 1.

## Conclusions of year 1 and implications for year 2

The results and analysis of the test of trainee teachers' competence and their procedural preferences as revealed by formal and informal written methods provided very little evidence of relational or connected thinking. For example the use of multiplication and addition as if they were interchangeable involved a denial of the fundamental connection between these and other operations. Confusion about the inversion of fractions algorithm directly challenged the internal consistency of the multiplication and division operations. The significant differences and evidence of success associated with different operations suggested that the trainee teachers experienced and perceived mathematics, not as a whole, but as an assortment of disparate procedures. This provided support for the suggestion made by Frank (1990) and Foss and Kleinsasser (1996), on the basis of their studies of teachers of mathematics, that many teachers viewed mathematics as a collection of fragmented facts, procedures and right and wrong answers.

The National Numeracy Strategy (DfEE, 1998) identified the following two skills, among others, as a necessary element in the development of numeracy:

- Remembering number facts and recalling them without hesitation
- Drawing on a repertoire of mental strategies to work out calculations

The results from some of the questions in this study shed some light on trainee teachers' use of rote memory to recall number facts. Some of the most commonly correctly solved problems in the competence test were also the ones which prompted the least amount of written work. These were items 10 (7 x 8), 16 (8 ÷ 2) and 17 (42 ÷ 7). At least 90% of the trainees were able to solve these problems correctly without written working. For the purpose of this research, it was assumed that no written work implied the use of informal methods. However, in the case of these questions it is possible that trainees relied on the recall of facts memorised in primary school. All of the problems could have been solved using knowledge of multiplication tables without recourse to invented or informal methods and without making connections between multiplication and its inverse operation, division.

So, since they were successful, did it matter that trainee teachers were not using informal methods? The evidence from item 20 (1.2 ÷ 0.2) suggests that it did. Although the basic number fact required to solve the problem was extremely simple, only 49% of trainees were able to solve it correctly. This implies that for all but the most basic problems, rote knowledge of number facts is insufficient for their solution.

The year one test provided examples of the use of what have been defined as 'formal' algorithms. There was also evidence that participants may have used some 'informal' methods as a set of rules – hence creating a new algorithm. The grid method was one such example. The subject knowledge element of the postgraduate course included an introduction to these 'informal' methods, and whilst it was clear that trainee teachers were able to use this method correctly, during the test, there were several situations where it was incorrectly applied.

The results presented here, and in more depth in Ineson (in press) highlighted the need to address the way in which calculation was covered on the course, in order to promote relational and conceptually connected understanding through mental mathematics. On the basis of the results, it was difficult to be as confident as Murphy (2006) about the development of trainee teachers' relational or connected mathematics thinking through the practice of teaching. Trainees had had substantial school experience before taking the test and the results gave at least some cause for concern, suggesting that a difference may exist between the rhetoric of pedagogical skills for relational teaching and what Murray (2006) referred to as the necessary underpinning reconstruction of trainee teachers' own subject understanding. The implication was that the calculation element of the course needed to make provision for the development of a 'fuller, deeper understanding of the number system and number operations and relations, and the way different interpretations of these interconnect' (Askew *et al*., 1997, 93).

## Year two – changes to the course

In year two an introductory whole cohort lecture on calculation was followed by a series of seminars to provide an opportunity to explore calculation in more depth. Trainees were routinely encouraged to assume the position of pupils in the schools in which they would be teaching in order to adopt a new perspective on what they had previously been taught. With the new cohort of trainees, the approach to developing subject knowledge of calculation strategies adopted in year one was also changed in the ways described below.

### Trainees were encouraged to estimate solutions prior to thinking about strategies

Trainees reported that they felt uncomfortable about solving VAT problems (requiring the calculation of 17.5% of any number) without the aid of a calculator. Having already been introduced to and used practical resources to illustrate what happens when numbers are multiplied and divided by 10, 100, 1000 etc. they were asked to estimate what the answer was likely to be and to record the steps taken to reach it, Groups vied with one another to achieve the easiest way of estimating the answer in a crowded shopping environment. A popular choice of strategy is illustrated in Figure 2.

Finding 10% of 50 is straightforward:

$$\frac{10}{100} \text{ x } 50 = \frac{1}{10} \text{ x } 50 = \text{a tenth of 50, or } 50 \div 10 = 5$$

Once this has been calculated, finding 5% and then 2½% is achieved by halving 10%, and then halving the answer. 17 ½ % is the sum of 10, 5% and 2 ½ %.

**Figure 2: Finding 17 ½ % of £50**

The habit of estimating the solution became a game in which trainees competed to find the simplest way to 'get a rough answer'. In feedback from trainees after their first block school experience, they commented that this was also a technique which they adopted in the classroom. They also reported on the successful adoption of the rounding approach in their classrooms, where pupils were encouraged to round to the nearest ten or hundred to work out an approximate solution. One trainee used the mnemonic: "Rain drops are crystal clear" to encourage pupils to read, decide on operation, approximate, calculate and check.

## Trainees were encouraged to suggest multiple methods for individual items.

In all seminar sessions involving calculation, trainees were required to begin by estimating the solution. The next step was to identify multiple strategies, involving, for example, variation in operations and partitioning of numbers as is illustrated in the following example. Trainees regularly worked in small groups justifying their methods to other members of the group and to the whole cohort.

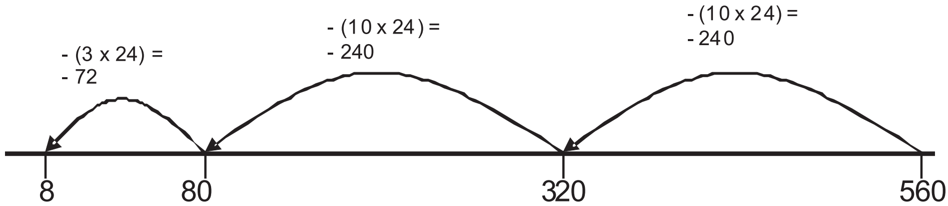
A typical example is 131x 4. The following strategies were suggested by trainees:

- 131 x 4 = (100 x 4) + (30 x 4) + (1 x 4)
- 131 x 4 = double 131, then double again
- 131 x 4 = ((131 x 10) / 2) – 131

## The four number operations were dealt with simultaneously.

The four number operations (addition, subtraction, multiplication and division) were covered together, during seminars. A variety of problems were chosen for each session, with the specific aim of involving all four operations. Murphy (2007) compared British and Dutch approaches to calculation noting that, in Holland, Beishuizen's (1999) Empty Number Line, for example, had been implemented in a holistic way.

During the sessions, the Empty Number Line was used to demonstrate the connections between operations. For example the following illustrated its use in tackling 560 ÷ 24.



This strategy used repeated subtraction to take 'chunks' of 24s away from 560. In total 23 'chunks' of 24 were taken away, leaving a remainder of 8, thus highlighting the links between operations.

Trainees felt that long division was notoriously difficult when trying to remember an algorithm and their immediate response was that the Empty Number Line made sense as a tool for mental calculation. When asked about any inspirational moments the trainees had encountered during the course, one trainee replied that the empty number line best represented that breakthrough. Another suggested that as the Empty Number Line is a tool which can be used to assist in any calculation, this would be of great help to many primary pupils as there were no 'rules' to remember.

Some trainees also admitted that, as a visual aid, the empty number line had helped them in preparing for the skills test, a compulsory element in attaining qualifying teacher status in the UK.

## Written and mental methods were introduced concurrently

Mental and written methods were covered simultaneously during seminars, to encourage the use of informal jottings and to demonstrate the relationship between standard written methods and mental strategies. For example, the long division problem discussed above in relation to the Empty Number Line was also recorded in the style to which trainees were accustomed.

| | | |
|---|---|---|
| 24 | 560 | |
| | -240 | (**10** x 24) |
| | 320 | |
| | -240 | (**10** x 24) |
| | 80 | |
| | - 72 | (**3** x 24) |
| | 8 | |
| | ≡ | 560 ÷ 24 = 23 remainder 8 |

## At every stage all possible sources of variation were explored

During the seminars on division by fractions trainees were asked to consider the following problem:

$$1\frac{3}{4} \div \frac{1}{2}$$

Before attempting to solve it they were asked to differentiate between

- dividing by one half and multiplying by one half (varying the operation);
- dividing by one half and dividing by two over 1 (varying the numerator/ denominator),

and, in each case, to represent the differences by creating a story to contextualise the problem (Ma, 1999). The merits of particular solutions (e.g. the inverting and multiplying algorithm as opposed to the contextualised stories) then became the focus of further discussion. Finally, the stories were assessed and the most effective solution to the original problem was identified by the whole cohort as the following:

I can eat one half of a pizza in one day; how long does it take me to eat one and three-quarters pizzas?

At this stage trainees were introduced to two further techniques which could be used to simplify the problem of dividing by more complex fractions. The first was a 'Fraction Wall' which consisted of a wall of equal strips, each divided into different fractions. Trainees used this to investigate how it could help to solve the problem above. The second technique used software, currently being developed by Tony Harries from Durham University in the UK, which demonstrates how to use equivalent fractions to compare fractions and to use this technique to show how division of fractions can be solved visually.

# Conclusions

During year 2 of the project, qualitative data were gathered continuously in the course of normal teaching sessions in order to provide more specific information on trainees' choice and application of strategies than had proved to be possible through the previous year's test results. The test had been useful in identifying whether or not trainees could solve the items but a closer analysis of trainees' choice of strategy and justification for those choices was required in year 2 in order to determine whether or not the characteristics of relational or connected thinking, as identified in the literature – use of multiple strategies, estimation and the justification of solutions – had become habitual.

A minority of trainees routinely reported during the first seminars that they had found the standard formal written methods efficient and straightforward to apply, and that it was difficult to understand why any other methods were taught in school. When invited to reflect on their experiences during seminars and in schools, there was general agreement that it felt as if they were 'learning backwards'.

The majority of trainees, on the other hand, as reported above, assessed the problems with a learnt algorithm from their childhood. They were more aware of whether or not their solution was justifiable. They used a variety of informal checking strategies and they recognised that justifying results mathematically was a crucial part of the process. Furthermore there were some reports of the successful transfer of these behaviours to their own classrooms.

This project has therefore shown that there is sufficient scope, within the mental mathematics specifications of the UK's National Numeracy Strategy, for the development of connected or relational understanding of mathematics among trainee teachers. However, a comparison between the results of years 1 and 2 of the project, although not conclusive, suggest that this is unlikely to occur without the conscious application of intervention programmes targeted at the development of trainee teachers' own and their pupils' connected or relational thinking through mental mathematics. To achieve the success reported in year 2 of the project, these programmes are likely to include elements based on the results of research, including the use of informal estimation and checking strategies, multiple methods of achieving a solution, using the four number operations interchangeably and exploring all sources of possible variation.

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