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An analytical framework for mathematics teacher education from a critical perspective

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Abstract

Whereas there has been considerable advancement in the last few decades with regard to theories and practices in mathematics education from a critical perspective, very little is known about what it means to prepare teachers for such approaches. In this article I undertake a retrospective, reflexive analysis of my praxis as a teacher educator over the past decade, particularly when introducing an innovation such as project work to prospective primary mathematics teachers within what may be referred to as a social cultural political approach to a mathematics curriculum. Drawing on theoretical methodological tools developed for researching mathematics education from a critical perspective, I reinterpret these for building an analytical framework for mathematics teacher education from the same perspective – as an imagined praxis, an actual praxis and an arranged praxis. I then discuss the qualities that connect and transform these teacher education praxes and conclude with reflections on the consequences for student teachers' learning and actions when one of these dominates a teacher educator's curriculum.

Keywords: Critical mathematics education; praxis; mathematics teacher education; preservice teacher education; project work in mathematics

Introduction

Mathematics education from a critical perspective explores and deepens the link between mathematics education in its broadest sense and concerns about democracy, equity and social justice. A critical mathematics education is not a particular kind of mathematics education, but refers rather to a diversity of perspectives that may be brought to bear on mathematics education (Skovsmose & Nielsen, 1996) from social, historical, cultural, economic, political and other dimensions. To this end it draws on a broad landscape of theory and practice within recent developments in mathematics education. I bring this work into teacher education programmes for prospective primary school teachers through what I have termed a social, cultural, political approach to the mathematics curriculum (Vithal, 1997, 2003). Such an approach integrates a critical perspective in that it attempts to go beyond a concern for developing mathematical

knowledge, skills, attitudes and values, to being able to act in the world with fairness and justice on the basis of the competences produced. Drawing on a range of areas such as ethnomathematics, critical mathematics education, as well as issues of race, class, gender and other aspects of diversity (Vithal, 2003) such an approach engages in a direct way the relation between mathematics education and society. The current South African mathematics curriculum reforms have increased the imperative to consider mathematics education from a critical perspective. The new curriculum embeds a particular ideological and value orientation that explicitly implores teachers to teach and to respect human rights and other constitutional ideals, in attempting to heal the divisions of an apartheid past (Department of Education, 2002).

A social, cultural, political curriculum approach may be realised through a number of different practices (Vithal 2003). In this article I focus on one particular practice: that of project work. Project work has been researched and elaborated with respect to a critical perspective in mathematics education (Mellin-Olsen, 1989; Skovsmose, 1994; Vithal, Christiansen & Skovsmose, 1995; Christiansen, 1996; Nielsen, 1999; Niss, 2001; Vithal, 2003, 2004). Inspired especially by project work theories and practices in the Scandanavian countries (e.g. Olesen & Jensen, 1999), I have introduced project work to prospective primary school teachers as a means of realising a social cultural political approach to the school mathematics curriculum that integrates a critical perspective (Vithal, 2006, 2003, 2004, 1997; Vithal *et al.*, 1997). Within this conception project work is theorised as a research-like activity. It is problem-oriented in that the project ideas to be investigated or explored arise from the interests and concerns of learners and is jointly directed with teachers, but where the teacher's role is that of facilitator and supervisor developing mathematical competences, connecting them across disciplinary boundaries with possibilities for acting on the learning gained. Such specific experiences show learners how mathematics is connected to the real world, and its role and function in society. In this article, the project work by student teachers provides a means and a lens for analysing teacher education itself. The focus is on how the teacher educator enables student teachers to understand and realise a critical mathematics pedagogy. Elsewhere detailed descriptions and analyses of student teachers' engagement with project work have been provided (Vithal, 2006, 2003, 2004, 1997; Vithal *et al.*, 1997) but here it is the teacher educator that is being put under scrutiny – my thoughts and actions – rather than that of the student teachers.

Mathematics teacher education: Theory, practice and praxis

If teachers are to be inducted into a mathematics education that integrates a critical perspective, the question is how are its related theory, practices, intentions and expectations to be developed and maintained within a mathematics teacher education pedagogy? It may be argued that there should be some resonance between the pedagogy which teachers are expected to enact as a consequence of their participation in education and training programmes, and the pedagogy enacted by the teacher educators themselves to induct teachers into particular perspectives and practices they advocate. This imperative is especially relevant when referring to a critical mathematics pedagogy if teacher educators are not to risk flouting in practice, the very values and ideals that they are propagating in their theory.

It has been argued that when taking a critical perspective, a resonance needs to be maintained in the approaches and processes across theory, practice and research (Skovsmose & Borba, 2004; Vithal, 2003). If this argument is accepted, then it follows that each provides conceptual tools and frameworks for the other. Therefore it may be possible to appropriate "methodological theoretical tools" used to investigate mathematics education from a critical perspective, to analyse practices associated with the particular theory – be they the practices of teachers or those of teacher educators. The theoretical methodological tools I draw on are those of "imagined hypothetical situation", "current actual situation" and "arranged situation"

developed for researching mathematics education theory and practice from a critical perspective (Vithal, 2003, 47-67; Skovsmose & Borba, 2004, 213-215). These are reinterpreted for mathematics teacher education. These tools provided a means for putting theory-practice relations under the spotlight, both in teacher education and in learner education. The "situations" recognise the thoughts and ideas of the researcher, the dominant current actual context in which she would need to work, and the changes she would need to negotiate and arrange to interrogate any new theory or practice. These methodological tools were developed for researching innovations and interventions that are, firstly, exploratory and not widely available in the educational system, and secondly, deemed especially relevant for researching a critical perspective in mathematics education. The specific argument being made in this paper is that teacher change (or lack thereof) in action and thinking must, in part, be explained with reference to teacher education pedagogies and programmes themselves (as opposed to what student teachers do and say); and that a framework for researching theory-practice relations could be re-interpreted to become a framework for the theory-practice itself, specifically within teacher education.

The need for such frameworks is evident from the fact that while teacher education has made enormous strides in understanding mathematics teachers and their work, a number of authors have alluded to the dearth in literature that puts teacher educators and their theories and practices under scrutiny (Adler, 2004; Zaslavsky, *et al.*, 2003; Jaworski, 2001). It begs the question of who researches the researchers – the teacher educators and professors of teacher education, and why do accounts of (student) teachers' actions and understandings seldom refer back to the qualities of their teacher education and training? In his review of research perspectives in mathematics teacher education, Lerman (2001) points to the problem of theoretical frameworks remaining largely implicit. For Adler (2003) the challenges of researching teacher education in the South African context are those of providing descriptions and the complexity of practice. However, more recently, greater scrutiny of mathematics teacher education practices and discourses are emerging (Davis, Adler & Parker, 2007; Ensor 2004).

Within mathematics teacher education that engages a critical perspective, the question is what perspectives and practices guide teacher educators as they develop curricular experiences to induct teachers into particular approaches, especially those that privilege concerns about social justice, equity, human rights and democracy. For example Atweh and Keitel (2007) have theorised a social justice agenda for mathematics education; and Gutstein (2003) has shown how the teaching and learning of mathematics can be organised toward social justice insights and actions in a Latino school. How is such an agenda to be brought into a teacher education curriculum if teachers are to be provided with opportunities to learn and take up such concerns in their schools and classrooms. The interrogation of teacher education practices and theories are especially crucial when expecting to prepare teachers to make large and radical shifts in knowledge, skills, attitudes, values and ways of being present in mathematics classrooms. This obligation is greater when developing a teacher education pedagogy for new or widely unavailable approaches in the education system, and when advocating a critical approach that the teacher educators may not have researched or experienced in the diversity that characterises any education system.

A retrospective reflexive account of aspects of my curricula for preservice primary mathematics teacher education over a decade highlights three key areas: *imagined praxis, actual praxis and arranged praxis*. I refer specifically to my experimentation with project work within that component of the programme in which student teachers were introduced to (among other approaches) what I have called social, cultural and political approaches to mathematics praxis. These are by no means mutually exclusive, but offer a framework for thinking about, and understanding our work as teacher educators.

I choose to use the term "praxis", firstly because it is well developed within critical education and research to refer to "the dialectical tension, the interactive, reciprocal shaping of

theory and practice" (Lather, 1986, 258). Praxis, for Freire (1997), brings together action and reflection, and is appropriate here because I am attempting a retrospective reflexive analysis of my own practices and theories. Praxis allows one to engage both the resonance and the dissonance of the relationship between theory and practice. Praxis as action-reflection is, in a sense, "theory in practice" – the doing of theory – and recognises that practice embeds and emanates implicitly or explicitly from theory. Praxis is not a neutral concept. It is associated and used with reference to emancipatory approaches to education that seek to align teaching and learning to democratic and socially just forms. While the praxis of teachers and schools has come under significant investigation in mathematics education, very little is known or written about how mathematics teacher educators produce a radical transformatory and empowering praxis in their own pedagogy, seeking to embody equity, diversity, anti- or non-racism, anti- or non-sexism, and social justice. How is mathematics teacher education pedagogy itself to enact democratic, open and fairer ways of teaching and learning within higher education institutions that both prepare student teachers for addressing the inequalities and injustices in schools, and also provide opportunities to address these in their own education and training in becoming teachers. Freire maintains that "no genuine learning can occur unless students are actively involved, through praxis in controlling their own education" (cited in Aronowitz, 1993, 9). Valuing teachers' vested interests and reasons for learning may be especially relevant in education systems characterised by diversity and large inequalities, making teacher education as praxis especially relevant. Bringing praxis, "understood as political practices informed by reflection" (Aronowitz, 1993, 9) into teacher education allows an analysis of the ideological underpinnings of the theories and practices of teacher education pedagogies and curricula, and of the social, political and cultural dimensions of the actions and reflections of teacher educators. South Africa's current socio-historical position and its concern with redress is made explicit and transparent by introducing imagined, actual and arranged praxis of project work into programmes for prospective teachers, and exploring the qualities characterising the connections between these and their transformation.

Imagined praxis

If teacher educators expect teachers to do their work significantly differently from what they have previously experienced or known, they need to find ways of 'lighting up' teachers' imagination to think and act differently. In this task, teacher educators bring their creative imagination to bear when they are introducing innovations or interventions. Teacher educators themselves interpret national curriculum reforms or theories and practices emerging from research to construct a teaching-learning environment for their student teachers to communicate their thoughts and ideas. What is brought into dialogue are the imagined mathematics classrooms of teacher educators with those of teachers, wherever they may be located in the diversity of the education system. Each has in mind notions of hypothetical learners in hypothetical teaching-learning environments. An imagined praxis allows for various kinds of practice-theory "thought experiments", which are crucial to begin any kind of change, especially those deemed radical by the people expected to change, such as project work in mathematics.

Typically, teacher educators present problems, issues, examples and contexts to prospective teachers who are expected to imagine themselves in their own classrooms working in ways explained and described by the teacher educator. The student teachers in my class, read and discussed different examples of project work from the literature (i.e. Skovsmose, 1994; Nielsen & Simoni, 1994; Paras, 1998; Nielsen *et al*., 1999) usually in groups, and engaged theoretical and practical ideas, both in mathematics content and pedagogy. They got to grips with the conceptual basis for project work: about how project work is problem-oriented; participant-directed; interdisciplinary; and based on the exemplarity principle (Vithal, Christiansen

& Skovsmose, 1995; Olesen & Jensen, 1999; Vithal, 2003, 2004). They discussed particular content knowledge and skills, learners' conceptual understanding of these, its representation in current mathematics curriculum reforms, and debated the possibilities and pitfalls in trying out these often imported ideas in a South African classroom. For example, even though student teachers could see the mathematics educational potential in implementing project work, they expressed concern about time for completing the expected school syllabus, the difficulties of employing interdisciplinary approaches within the confines of strong timetabling, and parents' acceptance of investigating home life conditions (Vithal 1997; Vithal *et al.*, 1997). The situation imagined is a hypothetical one, which attempts to take account of the diversity and inequalities of learners' and school contexts, and the theorising underpinning particular practices advocated.

An imagined praxis is in the main a discussion-oriented teacher education pedagogy. It could include observations of the innovative practice, but it is a dialogue-driven pedagogy which gives prospective teachers the opportunity to bring their sociological and pedagogical imagination (Skovsmose & Borba, 2004) to bear on constructing practices for implementation. Based on their critique of the projects they had read and discussed, student teachers brought their own dreams, hopes, backgrounds, experiences and knowledge about mathematics classrooms, curricula, schools and learners, and positioning as teachers to bear on their interpretation of the approach to develop hypothetical projects. Such imagined 'theoretical practices' included projects such as 'Social and economic relations in the world of a South African child' recontextualised from a similar Danish project (Skovsmose, 1994). However they also generated new ideas of their own, such as an electricity project on usage and wastage, and the forms of provisioning in different residential areas or a project investigating late-coming to school (Vithal *et al*., 1997). Each of these projects created spaces for developing mathematical ideas and concepts from learners' own life experiences and circumstances. Many of the project work ideas carried socio-political potential in making visible the injustices and inequalities of South African society and of schools. Within this imagined praxis the teacher education curriculum followed with possibilities for extension of "non-mathematical" and mathematical ideas in more traditional or typical ways, such as issues in student learning and difficulties.

The key point here is that issues of both content and pedagogy remain in the realm of the imagined and hypothetical. No matter how innovative, this teacher education praxis is confined largely to the lecture room. The student teacher is engaged in critique of research, theory and practice by studying cases, observing learning or teaching, by reading research, or developing lesson plans, projects or other types of material and activities. However it remains a hypothetical praxis because it draws on, and relies upon the student teachers' imagination shaped by their experiences as learners or as student teachers, and by the opportunities of having been in a variety of classrooms. The limitations and diversity of settings imagined by teacher educators are also hypothetical because they themselves have not usually undertaken substantial action and reflection in the pedagogy proposed. The mathematics likely to be developed, the difficulties learners and teachers are likely to experience, the resources needed to make the innovation or intervention work, can all be anticipated, but it is a "do as I say" teacher education pedagogy derived from each participant's personal histories, past experience and existing knowledge, skills, attitudes, values and ideology. This is a common teacher education praxis, but by itself it is often unlikely to be sufficient to effect the envisaged change.

Actual praxis

An actual praxis refers to the teacher educator deliberately changing his/her pedagogy to construe the student teachers as "learners" and him/herself as the "teacher" so that the prospective teachers can experience the innovation first-hand. It is a praxis that may be described as a real-life enactment of a new pedagogy intended to provide a different means for critique by

both teacher educators and student teachers. It allows for the anticipation of possible advantages and pitfalls should student teachers choose to emulate the innovation in their own practice. It may be regarded as a serious simulation of the innovation. The teacher education environment is transformed for the student teachers to participate as learners in a critical pedagogy and for the teacher educator to demonstrate and enact a praxis of emancipation and empowerment with reference to student teachers' own lives within the teacher education curriculum. For teacher educators who themselves may not have fully experienced or sustained the innovation they are advocating, it provides an opportunity to put theory into practice to deepen understanding of both through action-reflection. The inherent hierarchy between student and teacher educator is reduced as both grapple with what the innovation means in reality.

To engage an actual project work praxis, typically I set aside several weeks in a term, during which I become "the teacher as facilitator or supervisor" and student teachers become the "the learner participants" as we jointly engaged the problem-oriented and participant-directed features of this pedagogy. Student teachers are invited to consider or suggest "problems" that they might find interesting, important or relevant to their own lives. Such a discussion includes joint decision making about choosing different projects as individuals, in groups or as a whole class. In one year when the university closed down due to student protests against student exclusion for non-payment of fees, the entire class chose a project theme on "the economic relations in the life of a university student" inspired by a project described by Skovsmose (1994). In small groups they investigated academic and living expenses, as well as sources of income students rely on by engaging in a research-like process of conducting surveys and interviews. During data analysis students were provided with appropriate mathematical tools and technologies. The link to the school curriculum with respect to statistics education was maintained by giving students a variety of school and university statistics text book chapters. They delivered lessons to the class on different statistical topics, which served to develop their knowledge and skills in a new area of the new curriculum reforms and were used in the project research. The findings from the project were written up as a report and presented to the class and disseminated. Similarly, in another year the class chose a project on "crime on campus", following publicity in local newspapers about the university and its media image as an unsafe historically disadvantaged black university.

In some years not all students agreed to participate in the same project, and hence small groups or individuals chose different projects. In the year that HIV&AIDS became an important issue, one group undertook a project to investigate their fellow students' knowledge about the disease and safe sex practices, including their preparation for dealing with this problem as teachers in school. Another student chose to work individually in a project on levels of waste produced by staff and students and recycling in the university. In this a constant interplay in learning about mathematical content and pedagogy occurred. The student teachers learned the mathematical content on its own terms and acquired the content for teaching as they experienced the pedagogy. Actual project work experiences are usually powerful as student teachers come to see and experience first-hand how the teaching and learning of mathematics can be directly connected to their lives.

During the time that the projects run, the teacher educator takes on different roles, as resource person, facilitator, supervisor and teacher – making suggestions, reviewing data collection instruments such as interview schedules and questionnaires, teaching mathematics, interpreting mathematics in articles and information collected, making contacts with other disciplines or specialisations, as well as creating opportunities for using technology and engaging the school curriculum reforms. Since the point of departure in selecting the project is not learning some specific mathematics, but solving a real-life problem, opportunities for interdisciplinary engagement within teacher education are created.

On conclusion of an actual praxis experience, to facilitate understanding of the nature and scope of the innovative pedagogy, the process of evaluation and debriefing is essential. Through this substantial reflections take place on learning and teaching, and the responsibilities and roles of each participant. For the student teachers, reflecting on the actions of the teacher educator provides an opportunity to interrogate the role of the teacher in project work praxis from the perspective of being a learner. For the teacher educator, an actual praxis enactment provides opportunities for a critique of the administrative and educational viability of project work as a sustainable practice and its theoretical framing. A double analysis of praxis as both insider and outsider to the pedagogy is facilitated. In this there may be some overlap with an imagined praxis because this reflexivity is crucial to assist student teachers in their recontextualisation of the project for any later school setting.

An actual project work praxis allows for extensions of project work theory and practice, and mathematical content and pedagogy later, even if in more traditional formats. For example, in projects in which some aspects of the mathematics education were not dealt with, these followed later or specific pedagogical practice such as how to work more effectively with, and inside, groups. What is observed here is that an actual praxis of project work is not cut off from other mathematics teacher education practices and theories with which the student teacher is engaging. Instead, it often provides spin-offs for other areas of the teacher education curriculum. What an actual praxis does is change students' reasons for learning or investing in an innovative pedagogy through the power of the actual experience. It is a "do as I do" (as opposed to a "do as I say") praxis. However, depending on the quality of the action and reflection in the project work experience, student teachers will either take up or limit the possibilities for its use in the future as teachers in school.

Arranged praxis

In an arranged praxis student teachers enact an intervention or innovation in real time with real learners in their role as learner-teachers. It is an arranged praxis because it takes place within a setting which is deliberately created for prospective teachers to implement a particular praxis or pedagogy in a real mathematics teaching-learning situation. Mellin Olsen (1989, 184) described how he "provokes them (student teachers) by enforcing them to do a project with their classes, which is based on the students' knowledge, thus supporting their interests". An arranged praxis may be constituted in a number of ways. For instance as a "micro-teaching" environment in a higher education context, with learners being brought in or within the school context during what is typically referred to as "teaching practice" or "internship". The arranged nature of this praxis is also emphasised by the co-operation needed between teacher educators and other practitioners, such as teachers and school leaders to reflect jointly on the possibilities and pitfalls of a new pedagogy. The uncertainty of the outcomes of an innovative pedagogy that is not widely available in the education system has implications for the triad of student teacher, teacher educator and school teacher, as each recognises their different knowledge, skills, attitudes and values in relation to the teaching-learning setting. The theories and practice experiences of each participant may complement each other, or there may be conflicts, but the arranged praxis makes it possible to engage the differences in a real setting and in real time. Action-reflection is ongoing as the practice and its underlying theory come under critique and develop.

An arranged praxis is generative of a range of interpretations of the innovation. In collaboration with their learners, student teachers undertook projects such as building a school fence for security, redesigning the agricultural science school garden, developing a school vegetable garden to generate school funds, developing a school mathematics newsletter, and creating a dream fantasy school ground (Vithal, 2003; Vithal *et al*., 1997). The intensity, depth and scope of these varied. For example, two students who worked on the project on "economic

relations in the life of a child" led their classes in an activity of calculating "pocket money" over only two or three units, while the fence building project ran over almost the entire six-week period set aside for teaching practice and included measuring, calculating area and perimeter, costing the fence, fund raising, meeting the school principal, and sending a letter to the department of education for financial assistance (Paras, 1998). Furthermore such practices were presented and written up by student teachers for wider dissemination and discussion (Vithal *et al.*, 1997; Paras, 1998), which may be more readily taken up by teachers and continued later by student teachers because they speak more directly to challenges of practice and context. In this regard an arranged praxis provides opportunities for developing a professional praxis. As a result of having done it by themselves, and having seen it and heard the learners themselves, the power of an arranged praxis is considerable in leaving lasting impressions on student teachers. This may include negative and positive experiences and reflections with different consequences for the survival of the innovation later when they become teachers.

Since an arranged praxis is unlikely to be the only approach to introducing student teachers to an innovation, the impact of the extent and quality of each of the imagined and actual praxis must be considered for understanding what happens in an arranged praxis. How elements of what has been described as imagined or actual praxis precede or are offered concurrently with any arranged praxis will shape the learning that student teachers will undergo and the potential for future engagement with the innovation. The arranged praxis offers opportunities for critical engagement by student teachers with a new theory and practice they may have been asked to imagine or tried out as a learner, but from the perspective of a teacher. Gaps in the theories and practices become visible when the pedagogy has to be woven into the fabric of everyday school life. For example once in the mathematics classroom, issues of assessment, which were not dealt with as imaged or actual praxis, became important for the student teachers and the school teachers.

In an arranged praxis the limits of the innovation for real classrooms come into sharper relief. From the perspective of student teachers, this is a praxis based not in "doing what the teacher educator says or does", but rather "doing it for themselves for real". Opportunities for learners to engage in the outcomes of project findings, an important aspect of a critical approach, could seldom be fully realised for a number of reasons, such as constraints of time or resources, resistance in the school, and lack of agreement or support for the action to be taken. Nevertheless, that student teachers with their learners tried to do something differently meant that they took action and learned about the limits or possibilities for action.

Praxis: Connected and transforming

Although a teacher education pedagogy may be analysed with respect to these different praxes, they do not exist as discrete entities. Typically, various combinations of these praxes occur within any teacher education curriculum, though a particular programme may be driven more by one than the other in terms of how a particular praxis is valued. These praxes are connected to one another and provide different opportunities for learning since each has different strengths and weaknesses. This means that some kind of movement from one praxis to another has to be effected, as each praxis also changes, taking on new forms. For such shifts to take place, several qualities connecting and transforming these praxes may be identified. These are again derived from critical research, theory and practice in mathematics education (Vithal, 2003; Skovsmose, 2004), and re-interpreted for a teacher education analytic framework that seeks to introduce new teachers to that same praxis.

First, *pedagogical imagination* is required, which refers to the creative process of continually conceiving alternatives to the dominant ideas and practices to think or do something differently. This quality emphasizes that imagination and creativity are needed in each of the

praxes described. Pedagogical imagination, which Skovsmose and Borba (2004) derive from "sociological imagination" in the work of Mills and Giddens is also relevant here because any innovation has to recognise a *historical* sensitivity to what led to the existing pedagogy, an *anthropological* sensitivity about what has been done before and across contexts, and a *critical* sensitivity that makes it possible to break out of the status quo. In moving from any one praxis to another, the teacher educator not only recontextualises theories and their associated practices, but brings new intentions and aspirations to bear on the construction of the praxis. The teacher educator and student teachers modified practices observed elsewhere, but also developed new ideas of their own. Pedagogical imagination is both an individual, but also a collective engagement which requires co-operation among those involved.

This brings into play a second quality for the movement within and between praxes – *pedagogical action*. All participants' active involvement in, and consideration of the practical organisation of the praxis is needed for any innovation to take root. Pedagogical action mobilises pedagogical imagination as it works with the constraints and the potential of the educational setting to change. Teacher educators and student teachers had to negotiate different resource and administrative requirements such as modifying timetable arrangements, finding space for consultation, and seeking the co-operation and participation of a broad range of people, both inside and outside institutions, to operationalise the project. The extent and quality of the practical organisation required is shaped by how far the innovation is from the culture and ethos of the institution and its receptiveness to change.

The initiation of an intervention or innovation and its continual analysis and evaluation, as it is conceptualised through pedagogical imagination and enacted through pedagogical action, requires a third quality: that of *critical pedagogical reasoning*. Privileging action-reflection gives pedagogical reasoning an important place in the movement from one praxis to another. For example although an imagined praxis may offer strong preparation for an arranged praxis, many aspects of theory and practice may need to be modified and adapted for the arranged praxis to be realised in a diversity of contexts. This quality is important for valuing and legitimising the role and function of critique in the process of innovating, given the diverse knowledge and experience domains of the participants, as well as the unequal relations of power as a form of praxis is realised.

The assumption of connection and movement between praxes implies that they are not static. Each of these praxes is *dynamic*, changing and shaping each other as they are realised and as translation occurs from one to the other. Each praxis itself, furthermore, can never be identical to any previous enactment. That is, there are possibilities for transformation and improvement of each imagined, actual and arranged praxis, in each cycle of innovation as teacher educators gain deeper insights through successive attempts in working with and through a new pedagogy. It is important to recognise that different groups of student teachers bring different knowledge, attitudes, values, mathematical life histories and interests to their praxis. The actual direction of movement from one praxis to another, as well as how they are combined, will also impact the dynamism of any particular praxis.

While each praxis makes visible and possible a certain actuality that may be realised through pedagogical action, each also carries a large potential of thoughts and ideas that may remain unexpressed as pedagogical imagination, but may be carried into the future. *Potentiality*, a fifth quality allows teacher educators to understand what student teachers do and say, not as failure, but as alternate interpretations and unexpressed actions, and refers to what student teachers might deem possible in some future teaching and learning setting. This may be influenced, but not controlled, by teacher educators. Whatever the take-up of new praxes, the coverage of these in university teaching sessions introduces prospective teachers to the existence of innovative praxis and makes it available to them.

Reflections and concluding remarks

A major reason for offering this analytic framework is to seek a link between particular praxis that might dominate a teacher education curriculum, and student teachers' learning and action. Over-reliance on any one praxis has consequences for what student teachers think and do, and how they interpret and recontextualise a particular praxis for a variety of settings.

It may be posited that all teacher education curricula engage an imagined praxis, though to different degrees. However, if an imagined praxis is the only or dominant praxis within a teacher education curriculum, student teachers' learning tends to remain in the realm of theory, which seldom sees a substantial transformation into practice once they are in a classroom. Even though they may have been involved in debates and discussions on a particular pedagogy, it still remains as a kind of "theoretical practice". Not having actually experienced or tried the innovation, it may seem beyond the range of possibility. Hence it requires a strong deliberate negotiation on the part of the teacher educator for it to be brought into an actual or arranged praxis. An imagined praxis by itself is also unlikely to provide the impetus and basis for student teachers to consider trying out the innovation when they become teachers, saying "nice to know but too hard to do".

The sharper focus and dominance of practice considerations, often a characteristic of an arranged praxis, can have a different consequence if overemphasised. It could undermine the development of a theoretical understanding of the pedagogy because of the strong concern to try to make the innovation work and the limited capacity and time to analyse, understand and deal with problems that arise, as any innovation inevitably will throw up. Depending on the quality of the support, experience and reflection in the arranged praxis, student teachers may come out with a positive orientation to the innovation with a commitment to do project work in the future, or with a negative view that may discourage any further experimentation.

While an imagined praxis risks an innovation remaining in theory and an arranged praxis risks stagnating in practice, an over-reliance on the experience of an actual praxis can leave prospective teachers struggling to acquire both theoretical understanding and practice experiences for teaching. This is because they can become engrossed in the project work experience itself and the content of the project problem. However, having experienced a new pedagogy first-hand as a learner, they may be more interested and motivated to make the additional effort to try it out in a classroom. What has been observed if they do take up project work is that they tend to implement versions and variations of the specific projects they themselves had participated in as learners in their actual praxis.

Ideally, a teacher education programme should attempt to integrate all three praxes to give students varied and holistic opportunity to learn about, and through innovative pedagogy. However what may be desirable is seldom what is possible in the real world of higher education. In my teacher education curricula, I have experimented with various combinations of the praxes described, with different project problems and different school conditions. This framework offers a first step towards finding a way of relating what we do in our praxis as teacher educators to the experience and development of the teachers we train and attempts to do so in a way that tries to retain an intellectual integrity to the theories and practices we advocate. Fuller analysis is needed of teacher educators' own curricula and pedagogy, and a much deeper critical research of their own theories and practices, especially those that they themselves may have had limited classroom experience with, to develop stronger reflexivity. What must be recognised is our responsibility as teacher educators in what teachers who come through our programmes think, say, do and take away as learning.

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