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Mathematical preparedness for tertiary mathematics – a need for focused intervention in the first year?

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Abstract

Ongoing action research at the University of Pretoria investigates first-year students' preparedness for a study in calculus. In 2005 first-year engineering students completed a mathematics diagnostic survey at the beginning and end of the year. In this article the results of the 2005 survey are compared with the students' final school marks in mathematics, as well as with their performance in the calculus courses of the first year at university. The data of three different groups of students are considered to determine whether focused intervention in the first year can contribute to improve students' competence in introductory calculus. All three groups received calculus instruction through traditional lectures and practical sessions. Two of the groups received additional support through tutor sessions. One of these groups also received developmental support in mathematical and non-mathematical skills. Although results indicate an overall improvement in students' pre-calculus skills at the end of the year, they are not encouraging. Further statistical analysis of results indicates that students, whose entry level preparedness for mathematics was weak, and who had been exposed to additional developmental learning facilitation strategies, improved significantly compared with students who were better prepared, but were not purposefully exposed to a focused intervention strategy.

Keywords: Mathematical preparedness; tertiary mathematics; calculus learning; calculus pedagogy; mathematics diagnostic survey; first-year engineering students

Background

First-year students' mathematical skills have progressively become a focus in mathematics educational research, for example, the research by Ferrini-Mundy and Gaudard (1992); Frith, Frith and Conradie (2006) and Hooper (2006). With regard to first-year engineering students, recent reports also affirm that engineering students' entry level competence in calculus remains a growing concern (Bottomley, Hollebrands & Parry, 2006; Carpenter & Hanna, 2006; Jourdan & Cretchley, 2007 and Koch & Herrin, 2006).

At the University of Pretoria we have also increasingly become aware of freshmen engineering students' lack of understanding of fundamental mathematical concepts. In addition we have found that another consistent problem area in the background knowledge of freshmen engineering students is a lack of competence in communication skills, especially technical (including mathematical) communication skills. Similar observations were noted in the United States where a survey revealed that an increasing number of incoming students need remedial courses in mathematics and English (Graff & Leiffer, 2005).

The accreditation bodies for engineering programmes in the United States (ABET, 2000) and in South Africa (ECSA, 2000) list the ability to apply the knowledge of mathematics, science and engineering as a first exit-level outcome for engineering graduates. This ability presupposes not only a thorough knowledge of mathematics but includes all the so-called mathematical skills such as logical and procedural reasoning; analysis and synthesis; deductive and inductive reasoning and estimation skills based on sound principles. Educators of engineering students certainly have the task to equip students with these skills and to start doing so in the first year. A recent longitudinal study at Boise State University (Gardner *et al.*, 2007) indicates that success in first-semester mathematics is a most effective predictor of retention among engineering students.

The School of Engineering at the University of Pretoria offers a standard four year university engineering program, the Four Year Study Programme (4YSP) as regulated by the Engineering Council of South Africa (ECSA), as well as an extended study programme, the Five Year Study Programme (5YSP) with a duration of a minimum of five years. The purpose of the 5YSP is to create opportunities for students who have the potential to become engineers, but who do not meet the entrance requirements to enrol for the standard 4YSP and/or who are academically at risk of not coping with the high demands of engineering study. The 5YSP is structured in such a way that the courses of the first two years of the 4YSP are spread over the first three years of the 5YSP. Students on the 5YSP attend the same classes, have the same lectures, use the same textbooks and write the same tests and examination papers as the students on the 4YSP. Placement on the 5YSP is determined by a student's final school results as well as from the results of admission tests. These admission tests are currently in use at most South African tertiary institutions and were designed and compiled by the Centre for Higher Education Development at the University of Cape Town (AARP, 2007).

First year calculus pedagogy at the University of Pretoria

The mathematics modules for all engineering students are presented by the Department of Mathematics and Applied Mathematics in the Faculty of Science. Without any recent changes, the calculus module in the first semester comprises four 50-minute lectures and one three-hour practical per week. Lectures are presented in a traditional style and class sizes vary between 90 and 160 students. A practical class usually has 80 students and is presented by a lecturer or a post-graduate student. The aim of a practical is to give students an opportunity to discuss the previous week's work. The last 20 minutes of a practical session are used for assessment which is either a closed book written test that is done individually or an open book test which is done in groups consisting of two to four students. The marks for the practical tests contribute 30% to a student's semester mark. Students write two formal semester tests (duration 90 minutes each) which are equally weighted and contribute 70% to the semester mark. The semester mark and the end of term examination (duration 90 minutes) each contributes 50% to the final mark for the calculus module of the first semester.

In addition to this strategy of four lectures and one practical per week, students on the 5YSP also have to attend two one-hour tutor sessions per week. The class sizes for these sessions vary from 10 to 20. The tutor sessions provide students with the opportunity to discuss the current week's work in mathematics. The main idea is that the tutor (a postgraduate or a senior undergraduate student) should act as facilitator to enhance work done in class. Tutors are provided with additional study material to aid their preparation. In 2005 the tutors (as a group) met once a week with an experienced lecturer in order to discuss the mathematical content and pedagogical strategy of the forthcoming week's tutor sessions. In these meetings a typical tutor session was simulated with the aim of training the tutors to act as facilitators and not as instructors. It was expected that a tutor revise the work to be covered in the tutor session before coming to the meeting.

The calculus module in the second semester comprises two 50-minute lectures and one 90-minute practical per week. The calculus module in the first semester has a credit weighting of 16 credits whereas the calculus module in the second semester has an eight credit weighting.

The strategy regarding the weekly practical sessions for all students and the additional tutor session for the 5YSP students as in the first semester is the same for the second semester. Assessment in the second semester is also similar to that of the first semester.

In the School of Engineering a proactive approach is followed to promote the academic development of students on the 5YSP. In addition to the mentioned admission tests, students, who are placed on the 5YSP, also complete a questionnaire. The purpose of the questionnaire is to gain information regarding a student's level of preparedness for tertiary study and to identify possible shortcomings in a student's educational background. The questionnaire serves as a tool for selection of students to enrol for a developmental course, Professional Orientation during the first year of study. The Professional Orientation course (POC) comprises two semester modules and is credit-bearing. Formal contact time in each of the POC modules is six hours per week. The POC (Steyn, 2003) consists of a conceptual mathematics component, the development of personal and academic skills, communication skills and skills in information technology. In the first semester the main focus in the POC is on mathematical and non-mathematical skills that are essential for understanding the fundamental concepts underpinning a study in calculus. The mathematics component of the POC is given in addition to, and separately from the mainstream mathematics modules that are presented by the Department of Mathematics. The following are viewed as core pedagogical principles in the POC (Steyn, 2003):

- Active learning that engages students in doing something 'hands-on' instead of only observing or listening to what can or should be done.
- Learning content in the POC is integrated with the learning process and the development of skills. Students are made aware of their own learning (meta-cognition) and that they should consciously plan, implement, monitor and evaluate their learning activities. A strong emphasis is placed on effective time management and time on task.
- Face-to-face interaction is the main mode of interaction between facilitator (lecturer or tutor) and the students. Most of the learning activities are done in a computer laboratory on campus. The layout of the laboratory promotes easy communication between the facilitators and students as well as amongst students themselves. A further advantage is that technology can be incorporated into the learning activities as necessary.
- Co-operative learning in and out of class is encouraged. Student-student interaction in class is frequent and done one-on-one or in informal small groups. For formal class activities in groups, group members are purposefully selected. The combination of students in a group is based on pedagogical principles and takes the didactical aim of the specific task into consideration.
- An approach of continual assessment and extensive feedback on performance is followed. A strong emphasis is placed on a high standard of work and students must make an effort to perform at their best. Academic progress is monitored continually for the purpose of proactive intervention. Students are referred for counselling or other assistance when necessary.

Problem statement

In all engineering disciplines at the University of Pretoria, both the 4YSP and the 5YSP have the same compulsory mathematics modules in the first two years of study. Continuation to the final year of engineering study is not possible if a student has not passed the mathematics modules of the first two years. Since the mid-1990s, experienced lecturers in the Department of Mathematics have expressed concern that students in the second-year modules make algebraic mistakes that would not be expected from a student in senior high school. Similar concerns regarding the level of mathematical skills and knowledge of first-year students are continually expressed by the Module Committee: Mathematics of the School of Engineering (2004-2007). Numerous research studies, for example, by Budny, Le Bold and Bjedov (1998) and by Koch and Herrin (2006), indicate that a strong correlation exists between performance in calculus and the eventual success of students graduating in engineering. Similarly, one of the most accurate predictors for retention in engineering study is academic performance and success in the first year (Chalton, Yeld & Visser, 2001; Hampikian *et al.*, 2006; Gardner *et al.*, 2007). Our experience and research results concur with these findings (Du Plessis, 2004; Du Preez, 2004; Steyn & Du Plessis, 2007). Therefore success in the first year and success in mastering the mathematics modules of the first two years are crucial for both the 4YSP and the 5YSP engineering students. We believe that in order to facilitate students' competency in mastering mathematics, lecturers must know when and where to start the mathematical intervention with students and how to devise the most suitable intervention strategy. Davis (2000, 191) points out that lecturers (educators) of mathematics need to know what a student's "useable knowledge" is. Du Preez (2005, 10) refers to mathematical "must knows" as those knowledge components that are "vital for some [mathematical] activity and become so ingrained that they can be recalled effortlessly". Engineering students and engineering graduates must have a sound base of mathematical 'useable knowledge' and 'must knows'. The question arises, how to assess if freshman engineering students have the mathematical knowledge base to effectively undertake their first-year mathematics courses; and how to assess if they retain the mathematical 'must knows' that are crucial to complete their engineering study and practice as engineers.

Inventory

In response to the concerns expressed by lecturers in the School of Engineering (Module Committee: Mathematics, 2004) as well as by lecturers in the Department of Mathematics, an inventory, the Mathematics Diagnostic Survey (Du Preez, 2005), was compiled. The Mathematics Diagnostic Survey (MDS) is not regarded as a test. The aim in setting up the inventory was to:

- determine entry level knowledge, 'must knows', in mathematics of first-year students;
- gain insight into students' information-processing ability;
- gain insight into students' problem-solving behaviour; and
- establish the students' confidence in their mathematical ability.

Information processing in mathematics refers to reading strategies, critical thinking and understanding strategies (Maree & Steyn, 2003). Problem-solving behaviour in mathematics includes planning, self-monitoring, self-evaluation and decision making during the process of problem solving in mathematics (Maree & Steyn, 2003).

The format of the 2005 survey was paper-based and questions were answered on forms that could be marked by an optical reader. Questions are grouped in seven content domains, with each content domain focusing on a specific topic (see Table 1). Each content domain comprises two parts. The first contains the questions and possible answers and in the second the student has to indicate how sure he/she is that the answer is correct. The format of the answers in part one (mathematics) is either multiple choice or match. The given answers apply to all the questions in a specific content domain. The number of answer options per content domain varies between six and ten. The aim in listing answers that apply to all questions in a specific content domain, is to discourage guessing but rather encourage reasoning and computing when determining an answer. At each answer, space is provided for computations and students are required to use it, showing the steps of their reasoning. In setting up the survey, the choice of distractor answers was based on our experience of the mistakes that students make. The format of the answers in part two (confidence) of each content domain is a choice of one option out of four. Each of the four possible options indicates a level of confidence with which the given answer has been determined (selected). The confidence level options are: completely sure; reasonably sure; unsure; I am guessing.

The total score for the MDS is 21 with one mark per correct answer. Of the 21 questions, 19 are regarded as pre-calculus 'must knows' for calculus study and a score $\geq 80\%$ can be regarded as indicative of a satisfactory entry level for calculus study.

The only type of validity that is relevant for the MDS is content validity (Anastasi, 1961). In this case, content validity was established by including only selected questions that are significant in a specific content domain.

Table 1: Summary of the content domains of the Mathematics Diagnostic Survey (MDS)

MDS Content domain	Topic	Some examples where topic is applicable in calculus in the first study year	Number of possible answers		'Must knows'	Information Processing	Problem solving behaviour
Content domain 1	Solving equations	Intercepts	10	Q7-Q9	✓	✓	
		Domain		Q10-Q11	✓		
Content domain 2	Solving inequalities	Increasing/Decreasing	10	Q12-Q13	✓	✓	
	Concavity			Q14	✓	✓	✓
Content domain 3	Trigonometric identities	Integration	10	Q15	✓		
		Solving equations		Q16	✓	✓	
		Solving inequalities		Q17		✓	✓
Content domain 4	Absolute value graphs	Solving equations	8	Q18	✓		✓
		Solving inequalities		Q19	✓		
		Numerical integration					
Content domain 5	Radian measure	Inverse trig functions	6	Q20		✓	✓
	Completing square	Extreme values		Q21	✓	✓	✓
Content domain 6	Fractions	Limits, Asymptotes Differentiation Partial fractions Integration	10	Q22-Q24	✓		
Content domain 7	Translations of sine and cosine graphs	Integration	9	Q25-Q26	✓		
		Parametric representations		Q27	✓	✓	

Research project

In 2005 the MDS was used to ascertain first-year students' mathematical preparedness for study in calculus and to investigate the possible effect that a focused intervention could have on students' learning and performance in the first year modules in calculus. Students completed the MDS(Pre) in a formal lecture period within the first week of the academic year, and the MDS(Post) at the end of the academic year. These action research activities thus form part of course activities and the students are never regarded as merely 'research objects'. The data in this research project, and the results presented here are for groups of students and comply with the requirements for research ethics and research integrity of the School of Engineering at the University of Pretoria.

In the research reported here we consider only first entrant engineering students who enrolled at the University of Pretoria in 2005. First entrant engineering students exclude students who repeat their first year of study as well as students who had previously taken any tertiary course. The research involves 782 students and three groups are considered:

- The 4YSP group (N=662) comprises those students who had qualified for entry into the 4YSP in engineering;
- the 5YSP non-POC group (N=78) are students who did not meet all the criteria for entry into the 4YSP and were placed on the 5YSP; and
- the 5YSP POC group (N=42) are students on the 5YSP who were also enrolled for the Professional Orientation Course (POC) in 2005.

Students in all three of the groups were distributed between various mathematics classes (and various lecturers) according to their field of engineering study. This implied that none of the groups received their calculus instruction as a separate group from a specific lecturer. Shortcomings in the students' pre-calculus knowledge, based on the results of the MDS(Pre), were not purposefully incorporated into the learning content.

Although our ongoing action research activities are locally focused and may have limited generalisation value, we are of the opinion that our findings can contribute to other educators' continuing efforts to improve tertiary students' experiences in mathematics.

Results

Admission to the School of Engineering at the University of Pretoria requires a minimum (so-called) M-score (which is based on final school results) of 18 out of a possible 30 points as well as a minimum of 60% in both Mathematics and Physical Science Higher Grade (HG). Prospective students with at least 50-59% in Mathematics (HG) and/or Physical Science (HG), or an M-score between 12 and 17 are required to write additional admission tests for possible placement on the 5YSP (Regulations and Syllabi, 2005).

In all of the following results the maximum available data (N) are used. Only the results relating to the mathematics questions (part one of the content domains) of the MDS are discussed in this article. Item analysis was done on all the questions in the MDS for the groups combined. The Cronbach Alpha reliability coefficient is 0.78. The difficulty index for all the questions was between 20% and 80% and the item-total correlations ranged between 0.20 and 0.46.

The data in Table 2 summarises the detail of the research participants and shows their mean M-scores.

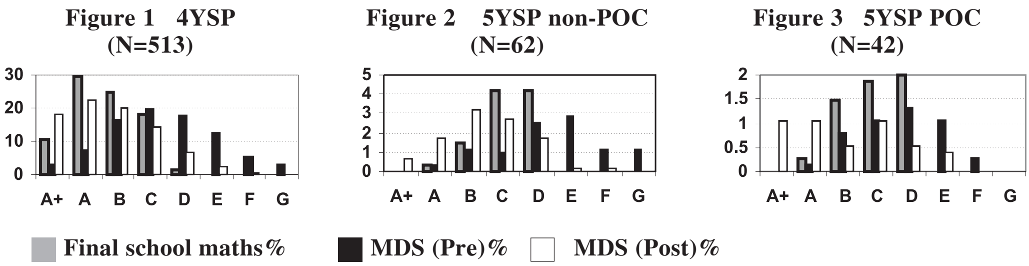
Table 2: M-scores of research participants

Group	N	M-score
4YSP	662	25
5YSP non-POC	78	19
5YSP POC	42	19

Although the mean M-scores of both the 5YSP non-POC and 5YSP POC groups in 2005 are above the required minimum of 18 for admittance to engineering study, they are significantly lower than those of the 4YSP group. A low M-score is an indication of shortcomings in schooling and for engineering students with a low M-score, mathematics is the main area of concern. This concern applies to all students on the 5YSP.

Figure 1 illustrates the distribution of the percentage of students in the 4YSP group according to achievement per score interval for the final school mathematics mark, the MDS(Pre) and the MDS(Post) mark. Figure 2 illustrates the same data for the 5YSP non-POC group and the data for the 5YSP POC group are shown in Figure 3.

The labels on the horizontal axis indicate the score intervals where A+>90%; A=80-89%; B=70-79%; C=60-69%; D=50-59%; E=40-49%; F=30-39%; G and H<30%. The values on the vertical axis indicate the percentage of students per score interval. The percentages per score interval are calculated as a percentage of the total number (N=610) of participants with data available for the final school mathematics %, the MDS(Pre) as well as the MDS(Post). The maximum value on the vertical axis in Figure 2 is 5% and in Figure 3 it is 2% to give a better visual display of the distribution of the data.



Figures 1, 2 and 3: Distribution of 4YSP, 5YSP non-POC and 5YSP POC students according to scores in the final school mathematics, MDS(Pre) and MDS(Post)

A distribution that is skewed to the left is expected of marks for the final school mathematics, the MDS(Pre) and the MDS(Post) for all first-year engineering students. The diagrams in Figures 1, 2 and 3 show that for all the groups the distribution of marks for the final school mathematics and the MDS(Pre) are not in par. However, for the 4YSP students (Figure 1) as well as for 5YSP POC students (Figure 3) the distribution of marks for the MDS(Post) tends to a distribution that is skewed to the left. Although the distribution of marks for the MDS(Post) for all the groups is better than the distribution of the MDS(Pre) after one year of tertiary calculus, their performances in the MDS(Pre) as well as in the MDS(Post) are not encouraging. Of the participants, only 10% achieved the expected minimum score of 80% in the MDS(Pre) and 45% achieved this minimum in the MDS(Post).

The data in Table 3 show the mean scores (as percentages) for the different content domains of the MDS(Pre) and MDS(Post) for the different groups as well as for the groups combined.

Table 3 shows that overall the students performed better in the MDS(Post) compared with the MDS(Pre). Detailed analyses and comparisons of performances in the MDS(Pre) and MDS(Post) with regard to the different content domains of the MDS are beyond the scope of this article. For all the groups the highest and the lowest means (respectively) for the MDS(Pre) occurred in the same content domains. Students performed best in Content domain 1 (Solving equations) and worst in Content domain 4 (Absolute value graphs). The 4YSP and 5YSP non-POC showed the highest gain in performance in this content domain when comparing means of the MDS(Post) with means of the MDS(Pre).

Table 3: Mean scores for the content domains of the MDS(Pre) and MDS(Post)

Content domain	Topic	Mean score %							
		4YSP N=513		5YSP non-POC N=62		5YSP POC N=35		All N=610	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post
Content domain 1	Solving equations	77.7	84.0	73.4	78.7	78.0	84.0	77.1	83.5
Content domain 2	Solving inequalities	67.4	68.3	51.4	62.8	74.4	67.8	66.1	67.7
Content domain 3	Trigonometric identities	47.3	65.9	36.1	55.2	41.1	63.3	45.7	64.6
Content domain 4	Absolute value graphs	32.9	71.7	18.9	65.6	38.3	70.0	31.8	71.0
Content domain 5	Radian measure. Completing the square	61.0	82.6	55.7	80.3	51.7	76.7	60.0	82.1
Content domain 6	Fractions	52.0	78.8	40.4	67.8	41.1	80.0	50.1	77.8
Content domain 7	Translations of sine and cosine graphs	63.4	78.7	52.5	73.8	53.3	84.0	61.7	77.8

Content domain 3 (Trigonometric identities) shows the lowest mean value for all the groups in the MDS(Post). The level of students' trigonometry knowledge with which they leave school, and the retention of this knowledge are the issues at stake in this case. Basic trigonometry (including identities) is a crucial 'must know' for calculus and for engineering study. In the MDS(Post) the 5YSP POC group outperformed the other groups in Content domain 7 (Translation of sine and cosine graphs). It seems as if the focused intervention in the POC, where students explore 2-D graphs with use of a computer graphing tool has a positive effect.

The data in Table 4 indicate the retention of students in the first-year mathematics modules. The data in Table 4 are given as percentages of the initial enrolment of first entrant engineering students in 2005.

Table 4: Retention of students in the first-year calculus modules in 2005

Group	N	Pass % first semester calculus	Pass % second semester calculus
4YSP	662	84.7	67.2
5YSP non-POC	78	59.0	44.9
5YSP POC	42	88.1	66.7
All	782	82.4	65.0

Table 4 shows that the 5YSP POC group has the highest pass rate with the first attempt in the first-year calculus modules and the 5YSP non-POC group has the lowest pass rate. It should be pointed out that the data in Table 4 indicate the pass rates for the first attempt in 2005. Longitudinal data of the 2005 first entrant engineering students reflecting on retention in the calculus modules of the first year (pass with a second or third attempt) will only be available in 2008.

Further data regarding the performance in the calculus modules are given in Table 5, namely, the means and standard deviations for marks in the first semester and second semester calculus modules. The calculus marks are for the first attempt in 2005.

Table 5: Means (as percentages) and standard deviations for marks in the first semester calculus and second semester calculus modules

	First semester calculus			Second semester calculus		
Group	**N**	**Mean**	**Standard deviation**	**N**	**Mean**	**Standard deviation**
4YSP	662	60.03	13.84	534	58.87	13.44
5YSP non-POC	65	49.27	11.06	43	53.59	8.33
5YSP POC	42	57.83	10.55	33	57.94	11.54

The data in Table 5 indicate that the 4YSP group has the highest mean score for both the first-year calculus modules. This is expected as the 4YSP group has the highest M-score of the three groups (see Table 2). The performance of the 5YSP POC group, in both the first-year calculus modules, compares favourably with that of the 4YSP group. As the standard deviations of both the 4YSP and 5YSP POC groups are of the same order of magnitude for the calculus modules, the distribution of these marks is similar for the two groups.

The data in Table 6 give a comparison of the difference between the groups regarding the arithmetic means of the final school mathematics mark, the score in the MDS(Pre) and the score in the MDS(Post). The data are given for all participants who did both the MDS(Pre) and the MDS(Post). As the data in this research are not normally distributed, the Kruskal-Wallis Test is used to determine the statistical significance of the results.

Table 6: Results of Kruskal-Wallis comparing the arithmetic means (as percentages) of the final school mathematics, MDS(Pre) and MDS(Post) marks between the groups

Group	N	Final school	MDS(Pre)	MDS (Post)	MDS (Post-Pre)
4YSP	513	77.05^{a}	58.76^{a}	75.62^{a}	16.86^{a}
5YSP non-POC	62	62.18^{b}	49.38^{b}	69.76^{b}	20.38^{ab}
5YSP POC	35	65.97^{b}	44.48^{b}	73.19^{ab}	28.71^{b}
Overall p-values		<0.0001	<0.0001	0.0020	0.0426

^{ab} Means with different superscripts differ at $p<0.05$

Both 5YSP groups have the same M-score (see Table 2) and the data in Table 6 show no significant difference between the means of these groups in the final school mathematics. It is thus likely that the two 5YSP groups should perform similarly in the calculus modules (see Table 5). However, the 5YSP POC group outperformed the 5YSP non-POC group in the calculus modules.

Analysis of the results in Table 6 shows a statistically significant difference between the 4YSP and both the 5YSP groups (respectively) with regard to the final school mark in mathematics as well as the MDS(Pre) mark. Analysis of the scores for the MDS(Post) shows that there is no statistically significant difference between the 4YSP and the 5YSP POC groups. The Kruskal-Wallis Test confirms that the 5YSP POC performed as well as the 4YSP after two semesters of calculus instruction, irrespective of the fact that the 5YSP POC embarked on the calculus modules with a lower final school mark in mathematics (see Table 5) and a lower M-score (see Table 2). However, there is a statistically significant difference between the 4YSP group and the

5YSP non-POC group. The 5YSP non-POC group did not perform as well as the other groups even though they had some intervention through tutoring.

The data in Table 6 further shows that the 5YSP POC group has the highest gain when the difference MDS(Post-Pre) between the MDS(Post) and MDS(Pre) is considered. There is a statistically significant difference (regarding gain) between the 5YSP POC and the 4YSP group when the difference MDS(Post-Pre) is considered. However, there is no statistical difference (regarding gain) between the 4YSP and the 5YSP non-POC groups. Table 6 confirms that the pedagogical intervention and didactical strategy followed in POC have a beneficial effect on the students' mathematical skills.

Discussion

The results of this action research study reveal two definite areas of concern regarding mathematics teaching and learning in a South African context: the pre-calculus mathematical knowledge of first-year engineering students is not at the expected level necessary for a study in calculus; and the teaching of calculus at first-year tertiary level needs to equip engineering students with the fundamental mathematical skills required in engineering study.

Continuing speculation about possible reasons for the students' low level of preparedness such as inadequate schooling; poorly qualified school teachers; coaching for the final school examination and inadequate content of the school syllabus, will not solve the problem. However, ignoring data on the results of the final school examination in mathematics will also not serve the case of mathematics education. It can be assumed that the majority of participants in this research wrote their final school examination in 2004. The data for the 2004 final school results in mathematics can thus serve as an example of the status of the mathematical knowledge first-year engineering students should have. Of the 469 056 candidates who wrote the 2004 final school examinations in South Africa, 330 014 passed. Of the 28 448 candidates who wrote mathematics at Higher Grade, 73.7% obtained a final score of ≥ 50% (Summit information, 2005). A pass rate of 73.7% is positively high and further analysis of the 2004 data regarding final school mathematics at Higher Grade should also be encouraging: 17.1% of the candidates achieved a score ≥ 80%; 13.1% achieved a score of 70-79%; 19.8% scored 60-69% and 23.6% obtained 50-59%. Unfortunately the seemingly positive data is not supported by the research results reported in this article. Most recently it was reported (Rademeyer, 2007) that the marks in the final school examination were adjusted by up to 10% and by implication, this adjustment also applied to mathematics. The use of an inventory, such as the MDS, can enable lecturers to determine students' entry level preparedness in mathematics and can then focus on specific shortcomings that first-year students may have.

Although the results in the study show that students' mathematical 'must knows' improved after two semesters of calculus, only 45% of the students scored above the expected 80% in the MDS(Post). If a student's pre-calculus knowledge is poor it has consequences for calculus study (see Table 5). The results reported in this study (see Table 4) affirm that "an inadequate grounding in pre-calculus can be a barrier in the study of calculus" (Schattschneider, 2006, 285).

Mathematics educators cannot start a calculus course at a knowledge level where they assume the students should be. They need to meet the students where they are (Davis, 2000) and get them actively involved (Kuh, Pace & Vesper, 1997). It is thus essential that thoughtful planning must go into deciding what, when and how to incorporate the review of pre-calculus 'must knows' to support calculus topics so that students have a solid grounding in calculus.

Results of the MDS and the performance in calculus (see Table 6) indicate that 5YSP students benefit from the developmental approach given in the POC. The integration of learning content with the learning process, combining mathematical and non-mathematical skills, has a broader focus compared to merely reinforcement of mathematics class-work through tutorial sessions which was the only additional intervention strategy followed with the 5YSP non-POC group. It is difficult to determine the relative contribution that non-mathematical skills may have on mathematical competence and to measure intrinsic academic gain as a result of an educational strategy. However, past POC students specifically mention time management, communication skills (including speaking, reading and writing mathematics), study skills, thinking skills and group work as contributors to their continual academic success. Literature conclusively demonstrates the positive effect that the incorporation of these skills has on student success (Nolting, 1997; Weinstein, 1999; Springer, Stanne & Donovan, 1999; Anthony, 2000; Hagedorn, Sagher & Vali, 2000; Taylor & Mander, 2003; Felder & Brent, 2004). Taylor and Mander (2003, 224) remark with regard to mathematics instruction that "strategies that were once thought to be acquired by osmosis" should become an "explicit component of university study".

Although the results reported here only reflect on performance and retention in the first-year mathematics modules, longitudinal data of the 2000 and 2001 enrolment of first entrant engineering students at the University of Pretoria indicates that the developmental intervention as done in the POC seemingly has a long-term effect and that learnt skills are transferred to subsequent (mathematics) modules (Steyn & Du Plessis, 2007).

Teachers of calculus at first-year tertiary level face the challenge to equip engineering students with the mathematical skills they need because "without a solid foundation in mathematics at the calculus level, an engineering student will find difficulty in understanding and applying the knowledge involved in upper level engineering classes" (Silverstein & Baker, 2003). This necessitates the rethinking of the learning content and the learning facilitation strategies and to purposefully refocus the (mathematical) interventions so that the process of learning is promoted and encouraged. In these endeavours it should be kept in mind that "no style of teaching mathematics can substitute for insisting that students pick up their share of the work, unless one is willing to compromise standards" (Zucker, 2000, 277).

Conclusion

As the need for more and well trained engineers increases, it is essential that (mathematical) educators of engineering students seriously take cognisance of research results that stress the importance of the mathematical intervention in the first year of tertiary study. Research results undoubtedly show that the mathematical experience in the first year has a definite effect on students' perseverance in engineering study. It is the social and educational responsibility of the tertiary institution and its teaching staff to make the most of the educational opportunity represented by a population of students who have the potential of succeeding in engineering.

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