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Portfolio selection theory and wildlife management

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Abstract

With a strong commercial incentive driving the increase in game ranching in Southern Africa the need has come for more advanced management tools. In this paper the potential of Portfolio Selection Theory to determine the optimal mix of species on game ranches is explored. Land, or the food it produces, is a resource available to invest. We consider species as investment choices. Each species has its own return and risk profile. The question arises as to what proportion of the resource available should be invested in each species. We show that if the objective is to minimise risk for a given return, then the problem is analogous to the Portfolio Selection Problem. The method is then implemented for a typical game ranch. We show that besides risk and return objectives, it is necessary to include an additional objective so as to ensure sufficient species to maintain the character of a game ranch. Some other points of difference from the classical Portfolio Selection problem are also highlighted and discussed.

Key words: Portfolio selection, multi-objective optimisation, game ranching, wildlife management.

1 Introduction

The trend towards transforming livestock production systems into game ranching has increased rapidly since the early 1990s. By the year 2000 it was estimated that there were approximately 5000 fenced game ranches and 4000 mixed game and livestock farms in South Africa covering more than 13% of the country's land area (ABSA Economic Research, 2003). In 2008 some 3000 additional livestock farms were in the process of conversion to integrated game and livestock production. Some concern about the economic sustainability of this activity and the lack of understanding of risk due to market and climatic variability has been expressed (Falkema and Van Hoven, 2000). Strategies to improve the economic returns from game ranches were formulated by Hearne *et al.* (1996), but this work did not deal with risk.

Theron and Van den Honert (2003) dealt with issues of risk and return in an agricultural context. They developed an agricultural investment model based on investment portfolio techniques first proposed by Markowitz (1952). Their objective was to optimise the

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proportion of land allocated to each of a number of agricultural products. The ideas of Theron and Van den Honert are followed in this paper. Their potential application to game ranches is explored by means of an illustrative case study.

2 The Problem

The portfolio selection problem is the bi-objective problem of choosing a portfolio of investments that minimises risk while maximising returns. As an acceptable trade-off between risk and return is usually required, an efficient frontier of Pareto optimal solutions is generated by repeatedly solving a single objective optimisation problem. Such a problem minimises risk for various given values of return.

Most modern Operations Research textbooks, such as Winston (2003) or Ragsdale (2004), include the formulation of a simple portfolio selection problem similar to the following formulation.

Suppose K is the total capital available to invest in n investment opportunities. Let p_i and r_i denote respectively the capital invested in and the expected return from investment opportunity i, and let $\boldsymbol{p} = (p_1, \ldots, p_n)^T$. Furthermore, suppose V is the portfolio variance and $\boldsymbol{C}$ is the covariance matrix of investment returns. Then the objective is to

$$\left.\begin{array}{rll} \text{minimise } V = \boldsymbol{p}^T \boldsymbol{C} \boldsymbol{p}, & & \\ \text{subject to } \displaystyle\sum_{i \in S} r_i p_i \geq R, & & \text{(acceptable revenue returned)}, \\ \displaystyle\sum_{i \in S} p_i = K, & & \text{(all capital invested)}, \\ p_i \geq 0, & & i \in \{1, \ldots, n\}. \end{array}\right\} \tag{1}$$

By repeatedly solving (1) with different specified values of R an efficient frontier of portfolio variances may be obtained.

Before pursuing the principles of (1) in a game ranch context some background information is necessary. The food requirements of large herbivores are often given in terms of animal units. An *animal unit* (au) is usually defined as the amount of food required to sustain a domestic cow of 455 kg. An impala, for example, only requires 0.16 animal units per head. Therefore six impala require $6 \times 0.16 = 0.96$ au of food resources which is still less than the food resources required by one domestic cow. The *carrying capacity* of a given area of land is defined as the number of animal units the land can sustain.

For a game ranch, a problem analogous to (1) is obtained if species represent investment opportunities and the carrying capacity of the land represents the capital available for investment. Let K be the number of animal units available (*i.e.* the carrying capacity) and denote the set of livestock species by S. Furthermore, suppose p_i animal units are allocated to species $i \in S$. Then the analogous problem is to

$$\left.\begin{array}{rll} \text{minimise } V = \boldsymbol{p}^T \boldsymbol{C} \boldsymbol{p}, & & \\ \text{subject to } \displaystyle\sum_{i \in S} r_i p_i \geq R, & & \text{(acceptable revenue returned)}, \\ \displaystyle\sum_{i \in S} p_i = K, & & \text{(utilizing carrying capacity)}, \\ p_i \geq 0, & & i \in S. \end{array}\right\} \quad (2)$$

A shortcoming of the above formulation is that the total food resources represented by the carrying capacity K are assumed to be homogeneous. The formulation may be improved by dividing the carrying capacity into three broad food classes: *bulk graze*, *concentrate graze* and *browse*. The actual utilisation of these food resources depends on both the number of animal units of each species and their respective diets. Let $F = \{\text{bulk graze, concentrate, browse}\}$, and suppose the proportion of food resource j in the diet of species i is denoted by α_{ij}. Then the additional constraint

$$\sum_{i \in S} p_i \alpha_{ij} \leq K_j, \quad j \in F \tag{3}$$

is required, where $\sum_{j \in F} K_j = K$ and $K_j \geq 0$ for all $j \in F$.

The expected returns generated in this model are more complex than those for the ordinary capital investment portfolio. Whilst the return on an investment in shares is mainly a function of changes in price over a certain period, wildlife returns comprise changes in both sales price and population numbers. For example, suppose that there are b buffalo on a ranch at time t, and suppose that the average market price of buffalo at this time is s_b. Then the market value of the buffalo population on the ranch at time t is $b s_b$. With an annual population growth rate of f_b a ranch owner may expect to own $(1 + f_b)\, b$ buffalo in year $t + 1$. Also, with an annual price growth rate of $\overline{\Delta s_b}$, the sales price of *buffalo* is expected to become $\left(1 + \overline{\Delta s_b}\right) s_b$ after one year. The value of the population after one year would therefore be $b s_b (1 + f_b) \left(1 + \overline{\Delta s_b}\right)$. From this value and the value at time t it is easily shown that the expected annual return on capital invested in the buffalo population is $\overline{\Delta s_b} + f_b + \overline{\Delta s_b} f_b$.

For species $i \in S$, the expected return on capital in livestock is therefore given by

$$R_i = \overline{\Delta s_i} + f_i + \overline{\Delta s_i} f_i, \tag{4}$$

where $\overline{\Delta s_i}$ denotes the average change in the sales price for species i over a certain time period. This is calculated as

$$\overline{\Delta s_i} = \frac{1}{T - 1} \sum_{t=1}^{T-1} \left(\frac{s_{i,t+1} - s_{it}}{s_{it}} \right), \quad i \in S \tag{5}$$

where s_{it} is the sales price of species i at time t, and T is the duration of the time under consideration.

The arithmetic mean is calculated in (5) above. This is the classical approach followed in most textbooks. However, there is a large body of literature with alternative formulations of the problem, including for example, the geometric approach suggested by Leippold *et al.* (2004). A thorough review of various methods for calculating $\overline{\Delta s_i}$ is given by Steinbach (2001).

3 Implementation

Consider a hypothetical but typical ranch in southern Africa. Suppose that twelve species are suitable for this ranch. Data relating to these species are given in Table 1. Typical

			Proportional Food Preference		
Species	**au/head**	**Growth Rate**	**Bulk Graze**	**Concentrate Graze**	**Browse**
White Rhino	2.45	7%	0.9	0.1	0.0
Blesbok	0.22	15%	1.0	0.0	0.0
Zebra	0.54	15%	0.7	0.3	0.0
Blue Wildebeest	0.49	16%	0.3	0.7	0.0
Reedbuck	0.19	15%	0.3	0.7	0.0
Red Hartebeest	0.37	15%	0.2	0.8	0.0
Nyala	0.26	20%	0.0	0.4	0.6
Eland	1.01	15%	0.4	0.2	0.4
Impala	0.16	25%	0.0	0.7	0.3
Giraffe	1.45	12%	0.0	0.0	1.0
Kudu	0.40	15%	0.0	0.1	0.9
Springbok	0.16	15%	0.25	0.25	0.5

Table 1: *List of species, animal units per head, growth rates, and the proportions of each food type in their preferred diet.*

carrying capacities available on such a ranch would be 250 au of bulk graze and 200 au for each of concentrate graze and browse. Previous annual sales prices over the last fifteen years for each species are given in Table 2 and these prices are used to estimate the rate of price change and the covariance matrix required. The model was implemented using the built–in solver of Microsoft® Excel [2].

The efficient frontier for this problem is shown in Figure 1. In the absence of risk considerations, a return of nearly 31.28% can be obtained. This drops to 26.31% when risk is minimised without any consideration for returns. Normally a decision-maker can use such a graph to choose the preferred trade-off between risk and return. There are other considerations, however, for decision-makers in this problem.

For a quality hunting experience the ranch needs to have a good spread of species. In Figure 2 the populations of each species are shown for the two extremes of the efficient frontier. In the case where "Return" is maximised it may be seen that only three species are maintained at non-zero population levels. In the case where "Risk" is minimised with no constraint on the required return only five species have non-zero populations.

In terms of a "quality wildlife experience" both the solutions shown in Figure 2 would probably be considered undesirable. It is reasonable to argue that a third objective is required, namely to maximise the minimum proportion of the carrying capacity allocated

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
W/Rhino	50 172	43 800	26 450	27 400	32 767	48 063	43 812	82 051	107 500	117 949	211 429	176 785	237 500	138 325	142 081
Blesbok	250	375	240	281	370	289	425	545	650	491	650	604	580	711	771
Zebra	2 595	2 175	1 320	1 881	1 675	1 457	1 441	1 715	2 300	1 670	2 336	3 093	5 260	4 385	4 404
B/Wildebeest	682	700	443	1 285	1 658	1 322	1 449	1 796	1 900	1 564	2 400	2 503	3 316	1 350	1 333
Reedbuck	1 283	1 450	1 800	800	2 365	1 400	1 886	2 075	2 500	2 338	3 611	4 562	4 088	4 656	3 806
R/Hartebeest	1 000	2 363	979	1 084	1 900	1 560	1 665	2 600	2 850	2 250	3 013	3 247	2 946	3 720	3 906
Nyala	1 487	1 958	1 407	1 224	1 920	1 970	2 348	2 664	4 450	2 726	5 914	7 362	7 538	5 648	5 588
Eland	2 653	2 641	2 550	4 058	4 308	3 136	4 502	4 195	6 750	4 487	6 114	3 904	4 800	4 945	6 802
Impala	288	375	234	247	320	286	416	480	487	421	480	634	590	469	486
Giraffe	9 750	9 000	6 880	8 800	6 725	7 909	6 150	9 769	11 750	10 141	12 333	12 100	13 350	10 931	11 619
Kudu	1 400	1 600	827	1 175	1 463	1 074	1 054	1 866	3 200	1 747	1 933	2 960	2 050	1 900	2 591
Springbok	550	1 000	525	524	500	842	387	476	650	378	587	718	600	514	602

Table 2: *Annual sale prices (in South African Rands) over fifteen years. The data were gathered by the third author over a period of 15 years from the annual KwaZulu-Natal wildlife game auction, South Africa. Some of the missing data (which accounts for less than 10% of the whole data set) were estimated.*

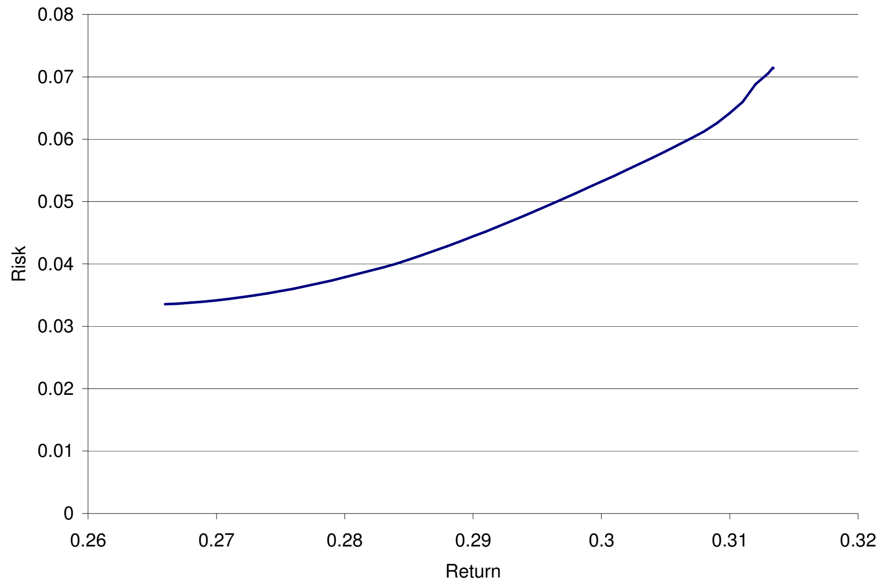

Figure 1: *Efficient frontier of (risk, return) values as solution to (2)–(3).*

to a given species. With three objectives it is best to re-formulate the problem as a multiple objective optimisation problem. "Best" solutions or goals have already been determined for "Returns" and "Risk". Let Q denote the smallest proportion of the carrying capacity allocated to a single species. The following maximin problem determines a goal for Q:

$$\begin{aligned}
&\text{Maximise} && Q \\
&\text{subject to} && \sum_{i\in S} p_i = K, && \text{(utilizing carrying capacity)}, \\
& && p_i \geq Q, && i \in S, \\
& && Q \geq 0.
\end{aligned}$$

The solution to the above problem gives Q as 5.39% of the carrying capacity. This means that each species is allocated *at least* this proportion of the carrying capacity. In terms of individuals this allocates resources sufficient to sustain 35 Eland and greater numbers for other species. Note that due to the constraints relating to the three different types of food resources making up the carrying capacity not all species are allocated equal proportions. So, for example, giraffe are allocated nearly 16% and white rhino just over 30%.

4 Multiple objective optimisation

Having determined goals or best values for the three objectives the multiple objective optimisation problem can now be formulated. Let g_1, g_2 and g_3 be the best values obtained for return, risk, and Q, respectively. Furthermore, let w_1, w_2 and w_3 denote the weights allocated to the objectives of return, risk and Q respectively. Then the objective is to

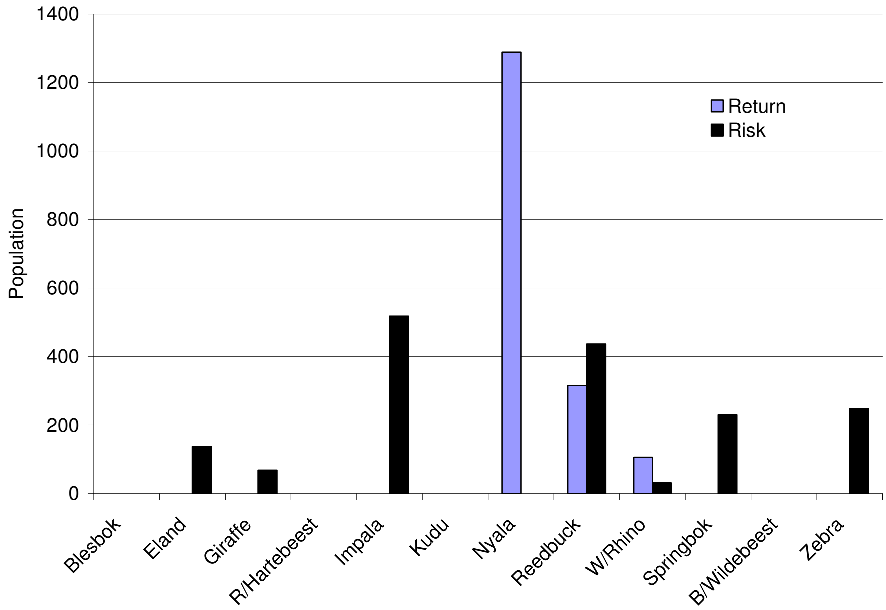

Figure 2: *Populations for the two extreme cases where 'Return' is maximised and where 'Risk' is minimised.*

$$\left.\begin{array}{lll} \text{minimise} & w_1 \dfrac{g_1 - \sum\limits_{i \in S} r_i p_i}{g_1} + w_2 \dfrac{p^T C p - g_2}{g_2} + (1 - w_1 - w_2) \dfrac{g_3 - Q}{Q} & \\ \text{subject to} & \displaystyle\sum_{i \in S} p_i \alpha_{ij} \le K_j, \qquad j \in F, \ \text{(enforcing species diversity)}, & \\ & \displaystyle\sum_{i \in S} p_i = K, \qquad \text{(utilising carrying capacity)}, & \\ & p_i \ge Q, \qquad i \in S. & \end{array}\right\} \quad (6)$$

Solving this problem with $w_2 = 0$ and w_1 varying from 0 to 1 the results shown in Figure 3 are obtained. It may be seen that placing more weight on returns reduces the minimum allocation received by a species. Similarly, omitting returns from the objective and varying weights between risk and Q yields the results shown in Figure 4. It is seen that higher risks have to be incurred as Q is increased. It is clear from this analysis that ensuring a "good wildlife experience" comes at the cost of reduced returns and increased risk.

5 Land as capital

We have been dealing with problems that allocate food resources (animal units) rather than capital. Nevertheless, like in the capital investment problem one of the objectives is to maximise the return on capital. Food resources are directly related to the area of land available. In calculating returns on investment, therefore, it would be reasonable that the capital value of the land be taken into account. In §2 we considered the returns that would

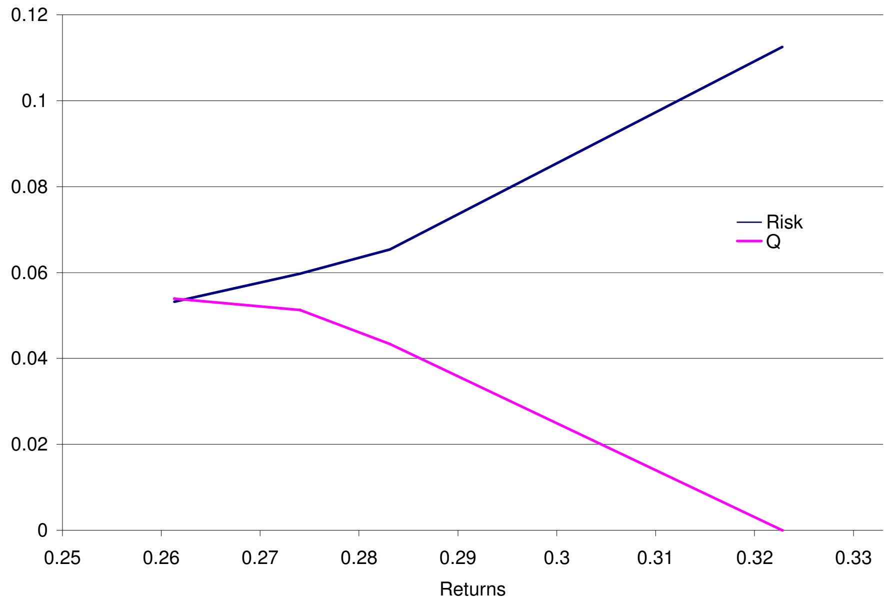

Figure 3: *Solutions to the multiple objective problem (6) with $w_2 = 0$. Here Q is the minimum proportion of carrying capacity (food resources) allocated to any species. The risk associated with each solution is given, but risk was omitted from the objective function.*

be achieved from an initial investment in b buffalo. If L is the value of land utilised by a single buffalo then the return on investment is given by

$$\frac{bs_b\left((1+f_b)\left(1+\overline{\Delta s_b}\right)\right)+bL-(bs_b+bL)}{(bs_b+bL)}.$$

After some simplification the return on investment is given by

$$R_b = \frac{\overline{\Delta s_b}+f_b+\overline{\Delta s_b}f_b}{1+\rho_b},$$

where $\rho_b = \frac{L}{s_b}$ and $L = u_b s_r \pi$. Here u_b denotes the animal unit equivalent for one buffalo (au), s_r denotes the stocking rate (ha.au^{-1}), and π denotes the price per hectare of land (Rand.ha^{-1}). Note that when land value is included, the original return on investment is simply divided by $1 + \rho_i$ for species i.

As an example, using the animal unit equivalent from the second column of Table 1, a stocking rate of 6 hectares per animal unit, and a nominal price of land at R4000 per hectare, the values of ρ can be obtained for each species. For impala and white rhino the calculations yield values of 7.87 and 0.41 respectively. The effect of land price on the returns from these two species may be seen by multiplying the land price by a multiplier. Figure 5 shows the results for land prices from zero through to 1.5 times the nominal land price.

There is an important conclusion to be drawn from Figure 5. Although not true, suppose that impala and white rhino had identical food preferences. In the absence of land costs it would be preferable to stock a ranch with as many impala as possible. As the value

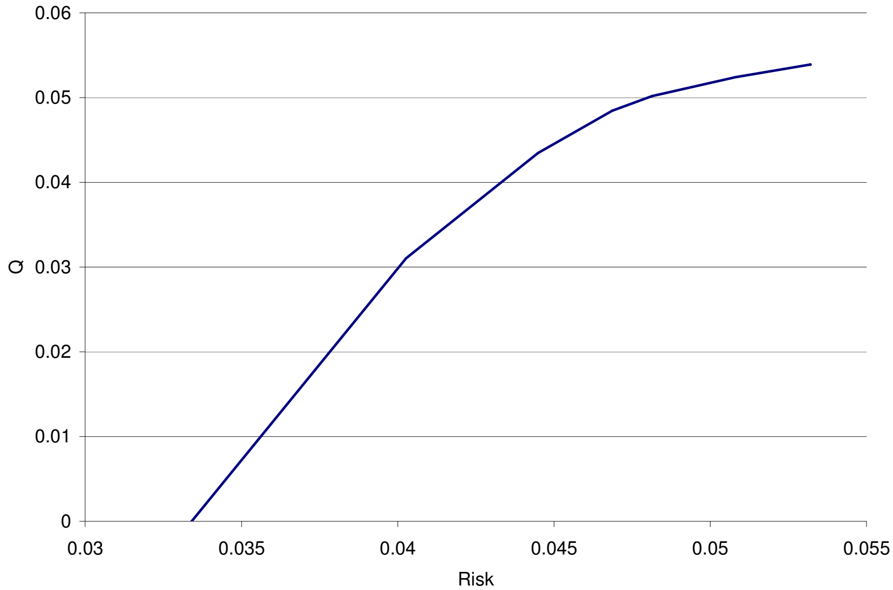

Figure 4: *The relationship between Q and risk in solutions to the multiple objective problem (6) with $w_1 = 0$. Here Q is the minimum proportion of carrying capacity (food resources) allocated to any species.*

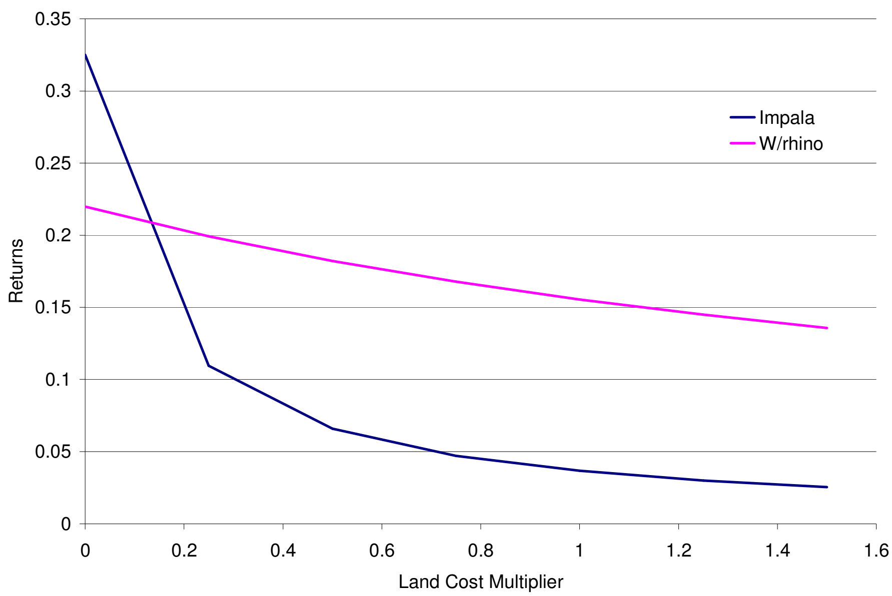

Figure 5: *The effect of land costs on the returns from impala and white rhino.*

of land increases, eventually better returns on investment are obtained from white rhino rather than from impala. It is therefore to be expected that the optimal population of each species will be affected by the value of land.

The effect on return on investment when the cost of land is included in the capital is now further explored. Equal weights were assigned to each of three objectives (returns, risk and

Land cost multiplier	**Blesbok**	**Eland**	**Giraffe**	**R/Hartebeest**	**Impala**	**Kudu**
0	142	146	46	85	194	78
1	139	79	66	84	191	76
2	134	29	81	81	184	73
	Nyala	**Reedbuck**	**W/Rhino**	**Springbok**	**B/Wildebeest**	**Zebra**
0	121	169	63	194	64	58
1	119	273	73	191	63	57
2	115	374	80	184	61	55

Table 3: *The effect of the cost of land on the optimal population numbers of each species. The nominal cost of land is multiplied by 0, 1 and 2 as indicated. For each case the three objectives (returns, risk and Q) in (6) are equally weighted.*

Q). The multiple objective optimisation problem (6) is solved again with three different land costs. This was achieved by multiplying the nominal land costs by 0, 1, and 2. The effect on the optimal populations is shown in Table 3. The two rows commencing with '0' represent the case where land costs are not considered in the calculations. The two rows commencing with '1' use recent or 'nominal' land costs, while the rows commencing with '2' represent the case where land costs are double the nominal value. For each case the three objectives (return, risk and Q) are equally weighted. It can be seen that as land costs increase the optimal balance of species changes: Giraffe, reedbuck and white rhino are allocated a greater proportion of the resources while the population of all other species are decreased. Optimal numbers of Eland, for example, decrease from 146 with no land costs to 79 with nominal land costs.

6 Discussion

The problem of determining population levels for each species on a game ranch so as to maximise returns while minimising risk is essentially analogous to the portfolio selection problem. A difference is that growth in investment value occurs through both natural growth and price change. In our illustration, natural growth was fixed. In practice, however, there will also be some fluctuations in growth rates. It is possible also that changes in price and growth are not independent random variables. There is insufficient data available at present to explore this question further.

A static problem formulation has been used here for illustration purposes. However, these ideas are easily extended to multiperiod problems. In such a case another difference from the standard multiperiod portfolio selection problem emerges. The game ranch problem would not necessarily incur the commission or transaction costs involved in buying and selling shares. Species offering improved returns may simply be allowed to grow to a new level. Of course, this might not always offer an optimal transition path from one 'portfolio' to another.

The purpose of this paper has been to show the connection between portfolio selection problems and the game ranching problem discussed. There have been many advances in Portfolio Selection Theory since the original work by Markowitz (1952). Much of this work can be applied to the game ranch problem in a similar way. The main difficulty is that lack of awareness of this type of approach has meant that the appropriate data has never been collected.

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