Durability and Strength of Reinforced Concrete Bridges subject to Corrosion: Fuzzy Random and Probabilistic Analysis

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ABSTRACT: This paper established a robust theoretical framework and applied fuzzy random and probabilistic theories to evaluate the durability and strength of reinforced concrete bridge structures subject to corrosion. The study systematically predicts moment capacity degradation, failure probability, and service life, while accounting for uncertainties in input parameters through Monte Carlo simulation. The MATLAB programming tool was used to calculate and analyze the bridge structures considering the uncertainty of input parameters and Monter Carlo simulation. A targeted application focused on estimating the service life of an aging bridge affected by chloride intrusion yielded noteworthy findings. Notably, the predicted initial corrosion times from both fuzzy and non-fuzzy analyses were found to be comparable, at approximately 29.96 years and 29.94 years, respectively. Additionally, the analysis indicated that the probability of failure for reinforced concrete bridge structures effectively doubles after 25 years. These results underscore the robustness of the proposed model, emphasising that the incorporation of comprehensive empirical data on the input parameters used to calculate durability and resistance, particularly from field experiments, significantly enhances the reliability and accuracy of lifespan and resistance predictions for reinforced concrete structures in practical applications.

KEYWORDS: Steel corrosion, Probabilistic analysis, Randomness, Fuzzy random, Reinforced concrete bridge, Service life

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I. INTRODUCTION

The corrosion of reinforcement in concrete structures is a prolonged and intricate process influenced by various factors such as environmental conditions, loading conditions, and construction materials (Poursaee, 2016, Nogueira et al., 2023, Rodrigues et al., 2021, Zhao and Fu, 2018). This degradation of steel reinforcement and concrete significantly reduces the lifespan of these structures, leading to increased repair and maintenance costs. Numerous authors have highlighted the diminished serviceability and longevity of reinforced concrete (RC) structures caused by chloride intrusion and carbonation, which in turn cause corrosion of steel reinforcement and concrete (Nguyen et al., 2018, Li et al., 2019, Alexander and Beushausen, 2019, Li et al., 2024). In general, steel corrosion caused by carbonation is less aggressive and destructive compared to corrosion induced by chlorides (Poursaee, 2016, Rodrigues et al., 2021). Accurate prediction of the corrosion process of steel reinforcement in reinforced concrete structures will provide more accurate service life estimation. However, accurately predicting the corrosion process of steel reinforcement in RC structures remains a significant challenge. Many structures lack regular monitoring, and historical corrosion data is often unavailable, resulting in insufficient necessary data.

Furthermore, environmental factors such as humidity, temperature, and airborne corrosive concentrations can fluctuate over time and across locations, complicating the collection of accurate and continuous data. Current measurement methods and equipment may lack the precision required or may not be designed to function optimally in all environmental conditions, thereby reducing the reliability of the data collected. In developed countries, the collection and processing of detailed and continuous data necessitate substantial financial and human resources, which not all projects can afford. The corrosion of steel reinforcement is a complex process influenced by various factors, including initial material quality, usage and maintenance conditions, complicating modelling and prediction efforts (Vachtsevanos, 2020). Additionally, the absence of clear standards and norms for data collection and analysis can lead to inconsistencies and difficulties in comparing and synthesising data from diverse sources. These factors constitute a significant research gap, introducing major uncertainties into the inputs for predicting the service life of structures. This shortfall undermines the reliability of service life predictions and underscores the necessity for more robust analytical methods to identify regularities in data distribution.

The issue of determining the service life of structures in corrosive environments is a pressing concern in current research. Existing studies have proposed various definitions

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and models for the service life of RC structures. According to ACI-365 (2000), the service life of RC structure is the period from installation until the properties of the structure no longer meet the required performance, even with routine maintenance. Chai et al. (2011) defined service life as the period from the initial utilisation to steel depassivation. Zhang and Ba (2011) gave the definition of service life as the time before chloride concentration reaches a threshold leading to corrosion of reinforcement. Parameswaran et al. (2008) developed a degradation model to assess the impact of carbonation on concrete service life, considering the time required for the concrete covered corrosion to reach the reinforcement surface. In their study, parameters with specific values without taking into account randomness were used for analysis. The time period required for the concrete covered corrosion to reach the surface of the reinforcement due to carbonisation was considered. The service life of RC structures is ended when generalised cracks caused by rust appear around the reinforcement. The model primarily focuses on carbonation without considering other factors such as chloride ion penetration, abrasion, or the effects of cyclic loading. In many cases, the service life of concrete is influenced by a combination of multiple degradation factors, and this model may not fully account for them.

Tuutti (1982) proposed a formula for estimating the service life of RC structures based on CO_2 penetration and protective concrete layer thickness. Subsequent analytical models aimed at predicting the service life of RC structures under corrosive environmental conditions have been developed (Saetta et al., 1993, Liang et al., 1999, Sagüés, 2003). Recently, innovative research has focused on using artificial neural networks, machine learning, and grey system theory for service life prediction (Li et al., 2017, Taffese and Sistonen, 2017, Dey et al., 2019).

Despite these advancements, a critical gap exists in the application of these new methods to real-world conditions, where data limitations and environmental variability present significant hurdles. Moreover, the current models often fail to fully account for the inherent randomness and uncertainty of key factors influencing the corrosion process, such as chloride concentration, diffusion coefficient, and environmental conditions.

The use of probability theory has been extensively used to estimate the service life of reinforcement concrete structures. Oslakovic et al. (2010) employed two deterministic models, including the Life-365 model (Thomas and Bentz, 2001) and the model utilised in Croatia, to estimate the service life of the concrete structures under exposure to the marine environment, comparing with the DuraCrete model (2000). Vorechovska et al. (2010) introduced an analytical model based on the probabilistic theory to predict the service life of concrete structures under steel corrosion conditions. Both the initiation and propagation stages of steel corrosion were considered in their study. Ma et al. (2020) developed a probabilistic approach for estimating the service life of RC structures exposed to seasonal corrosion environments. The conclusions of these studies indicate that no model is perfect; each model possesses its strengths and weaknesses. However, the combination of deterministic and probabilistic approaches may provide a more comprehensive understanding of the service life of concrete structures in marine environments.

Recently, fuzzy random theory has emerged as a promising tool for estimating the service life of RC structures. Anoop et al. (2012) proposed a novel approach by considering the fuzziness and randomness of the relevant uncertain parameters for predicting the service life of structures. In their research, the influence of cracks, a significant factor affecting the corrosion rate of steel, on the life of reinforced concrete structures was not considered. Wang et al. (2013) predicted the corrosion of steel reinforcement in RC structures using probability and fuzzy random theory. However, the service life of the RC structure was not studied in their research. Besides, fuzzy theory was also used in rating bridges (Sasmal et al., 2006), seismic resilience of bridges (Andrić and Lu, 2017), and time-dependent failure possibility analysis (Fan et al., 2019). Nevertheless, the application of fuzzy random theory in predicting service life has not yet been extensively explored or implemented, particularly in the context of assessing the condition and longevity of ageing infrastructure, such as an old bridge in Vietnam. This represents a notable research gap that merits further scholarly attention.

The key factors influencing the corrosion process include chloride concentration, diffusion coefficient, protective concrete layer thickness, critical chloride ion concentration at the reinforcement surface, humidity, and temperature (Rodrigues et al., 2021). Despite typically being derived from empirical laboratory and field test results, data for these parameters are often difficult to obtain and insufficient. Recognising the randomness and uncertainty of these factors is essential for accurate and reliable service life prediction of RC structures under corrosive conditions.

This study establishes a theoretical framework and applies fuzzy random and probabilistic theory in designing the load-bearing capacity of bridge structures based on durability, evaluating failure probability, and predicting structural service life. The research considers the uncertainty of input parameters and employs Monte Carlo simulation in the prediction process. The focus of this study is the service life of old RC bridge structures, particularly concerning reinforcement corrosion.

II. PROBABILISTIC MODEL FOR STRUCTURE ANALYSIS

In structural design, the limit state function including resistance of structures R and load S should be satisfied, i.e.:

$$g = R - S \geq 0 \quad (1)$$

where g is the limit state function. Nonetheless, R and S are considered time-dependent parameters. The time-dependent limit state function can be written as:

$$R(t) - S(t) \geq 0 \quad (2)$$

where *R(t)* is the resistance of structure remaining unchanged or degrading with time, *S(t)* is the load effect remaining unchanged or increasing with time. When R<S, the failure of structure will occur. Thus, the failure probability failure probability of structure within intended service period P_f can be expressed as:

$$P_f = P[R(t) - S(t) < 0] \quad (3)$$

Therefore, the probability of failure increase with time. Figure 1 illustrates the probabilistic of service life taken from Siemes and Rostam (1996). As the operating time of the structure increases, the two curves of the distributions *R(t)* and *S(t)* tend to move close and overlap. The larger the overlapping area, the greater the probability of the incident occurring. Figure 2 shows the intended service period design and the service life of the structure. In the intended service period design, the limit state may not be attained within this period with a certain reliability. In service life design, structural reliability is related to the probability of exceeding the service life design. At the time the limit state is exceeded, the service life of structure ends. To determine the intended service period, the failure probability should not exceed the target failure probability. Hence, the failure probability of structure within intended service period P_f can be expressed as follows:

$$P_f = P[R(t) - S(t) < 0] \leq P_{target} = \Phi(-\beta) \tag{4}$$

where P_{target} is the maximum acceptable probability of failure. Φ and β are the standard normal distribution function and reliability index, respectively.

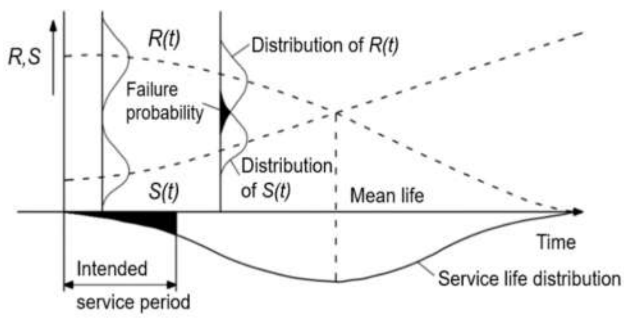


Figure 1: Probabilistic of service life (Siemes and Rostam, 1996)

III. CORROSION INITIATION TIME

A. *Chloride corrosion model*

A diffusion model based on Fick's second law, considering the effect of cracked, was proposed by Crank (1975) to predict the chloride ingress process as follows:

$$C(x,t) = C_s\left(1 - erf\left(\frac{x}{\sqrt{4D_{cc}t}}\right)\right) \tag{5}$$

where *C(x,t)* is the chloride concentration at depth *x* in the concrete protective layer at time *t*, (kg/m^3); C_s is the surface chloride concentration (kg/m^3); D_{cc} is the chloride diffusion coefficient for cracked concrete (m^2/s). *erf* is the Gaussian error function. A power function is employed to describe the chloride diffusion coefficient for cracked concrete D_{cc} as follows (Gérard and Marchand, 2000, Djerbi et al., 2008):

$$D_{cc} = \frac{AD_a + A_{cr}D_{cr}}{A + A_{cr}} = \frac{A(k_e k_t k_c D_o) + A_{cr}D_{cr}}{A + A_{cr}} \tag{6}$$

where D_a is the apparent diffusion coefficient (m^2/s); D_o is the empirical diffusion coefficient (m^2/s) ; k_e, k_t, k_c are the coefficients that take into account the influence of the environment, the test method and the curing, respectively (DuraCrete, 2000); A is the area of uncracked concrete at the soffit of the beam; A_{cr} is area of the crack at the soffit of the beam; D_{cr} is chloride diffusion coefficient inside the crack.

D_o (m^2/s) can be calculated according to function of Swatekitiham (2001) as follow:

$$D_0 = 10^{4.5(w/c)^2 + 0.14(w/c) - 8.47} \times 10^{-4} \tag{7}$$

and *w/c* is the water-to-cement ratio, determined by functions according to the standard ACI 211.1-91 (ACI211.1-91, 2009). D_{cr} is provided by Djerbi et al. (2008), effected bay width of crack as the follow:

$$D_{cr} = \begin{cases} D_{cc}, w_k > 30\mu m \\ (0.16w - 3) \times 10^{-10}, 30\mu m \leq w_k \leq 100\mu m \\ 13 \times 10^{-10}, w_k > 100\mu m \end{cases} \tag{8}$$

Where w is the crack width; Parameters of crack (w_k) is calculated according to the standard Eurocode 2 (2004), expressed as follows:

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \tag{9}$$

Where ε_{sm} is the mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and considering the effects of tension stiffening. Only the additional tensile strain beyond the state of zero strain of the concrete at the same level is considered; $\varepsilon_{sm}$is the mean strain in the concrete between cracks; $s_{r,max}$ is the maximum crack spacing expressed as the follow:

$$s_{r,max} = 3.4c + 0.425(k_1 k_2 \phi/\rho_{p,eff}) \tag{10}$$

Where c is the cover to the longitudinal reinforcement; ϕ is the diameter of the bar; k_1 is a coefficient which takes account of the bond properties of the bonded reinforcement, $k_1 = 0.8$ for high bond bars, $k_1 = 1.6$ for bars with an effectively plain surface (e.g. prestressing tendons); k_2 is a coefficient which takes account of the distribution of strain, $k_2 = 0.5$ for bending, $k_2 = 1.0$ for pure tension and $\rho_{p,\text{eff}}$ is the ratio of the reinforcement and prestressing tendons area within the tensile concrete zone to the tensile concrete area. The $\rho_{p,\text{eff}}$ can be expressed as follows:

$$\rho_{p,\text{eff}} = \frac{A_s + \xi_1 A'_p}{A_{c,\text{eff}}} \tag{11}$$

With $A_{c,eff}$ is the effective area of concrete in tension surrounding the reinforcement or prestressing tendons of depth; $A'p$ is the area of pre or post-tensioned tendons within $A_{c,eff}$; $\xi_1$is the adjusted ratio of bond strength taking into account the different diameters of prestressing and reinforcement; A_s is the cross-sectional area of reinforcement.

B. *Prediction of corrosion initiation time*

The process of corrosion of reinforcement begins when the chloride concentration at the steel surface reaches the critical chloride concentration ($C_{(x,t)} = C_{cr}$), then $x = h$ (thickness of the concrete cover):

$$C_{cr} = C_s\left(1 - erf\left(\frac{h}{\sqrt{4D_{cc}T_{in}}}\right)\right) \tag{12}$$

The corrosion initiation time (T_{in}) of reinforcement using probabilistic analysis takes into account the uncertainty in the critical chloride concentration and the surface chloride concentration is determined by transforming Equation 12 expressed as follows:

$$T_{in} = X_i\left[\frac{h^2}{4D_{cc}}\left(erf^{-1}\left(\frac{C_s - C_{cr}}{C_s}\right)\right)^{-2}\right]^{1/(1-n)} \tag{13}$$

Where T_{in} is the corrosion-initiation time (years); h is the thickness of concrete cover (mm); C_{cr} is the critical chloride concentration (% weight of concrete); C_s is the surface chloride concentration (% weight of concrete); D_{cc} is the chloride diffusion coefficient for cracked concrete (mm^2/year); X_i is the model uncertainty coefficient to account for the idealisation implied by Fick's second law (1975); and n is the factor of time life.

IV. FUZZY MODEL FOR THE STRUCTURE ANALYSIS

The onset time of steel reinforcement corrosion in RC structures depends on four factors: chloride diffusion coefficient, equilibrium chloride concentration at the concrete surface, critical chloride concentration, and concrete cover. Monte Carlo simulation was used to determine the means, SDs, and corrosion onset time distributions with specific experimental parameters. These four factors are assumed to have different distributions including fuzzy distribution or logarithmic distribution.

For RC structures, typically bridge works do not have a quality management system, so it is not possible to collect the exact statistical parameters of the four variables in the previous periods. These statistical parameters can be estimated based on design information and data for similar structures or as term variables from expert opinion. In these cases, the statistical parameters are considered to lie in a fuzzy range. The fuzzy random theory is used in this study to consider both types of uncertainty of the four variables (Wang et al., 2013). Therefore, four variables are assumed to be fuzzy random variables in the present study. The time of onset of corrosion for chloride ions is rewritten as:

$$\widetilde{T_{in}} = \cup_{\alpha\in[0,1]} \alpha[T_{in\alpha}^{-}, T_{in\alpha}^{+}] = \frac{h^2}{4\tilde{D}_{cc}}\left[erf^{-1}\left(\frac{\tilde{C}_s-\tilde{C}_{cr}}{\tilde{C}_s}\right)\right]^{-2} \quad (14)$$

$$T_{in\alpha}^{-} = \frac{h^2}{4D_{cc\alpha}^{+}}\left[erf^{-1}\left(1-\frac{C_{cr\alpha}^{-}}{C_{s\alpha}^{+}}\right)\right]^{-2} \quad (15)$$

$$T_{in\alpha}^{+} = \frac{h^2}{4D_{cc\alpha}^{-}}\left[erf^{-1}\left(1-\frac{C_{cr\alpha}^{+}}{C_{s\alpha}^{-}}\right)\right]^{-2} \quad (16)$$

Where $[]_{\alpha}^{-}$ and $[]_{\alpha}^{+}$ are infimum and supremum of variables, respectively; [] denotes four variables influencing the time to corrosion initiation (i.e., chloride diffusion coefficient, equilibrium chloride concentration at the concrete surface, critical chloride concentration, and concrete cover).

Corrosion of reinforcement leads to a reduction in the diameter and cross-sectional area of the reinforcement with the time-dependent corrosion rate ($i_{corr}(t)$), provided by Vu and Stewart (2000):

$$i_{corr}(t) = 0.85 i_{corr,o}(t-T_{in})^{-0{,}29}, \quad t \geq T_{in} \quad (17)$$

$$i_{corr,o} = \frac{37.8(1-w/c)^{-1.64}}{h} \quad (18)$$

Where $i_{corr,o}$ is the corrosion rate at the start of corrosion propagation; w/c is the water-to-cement ratio; h is thickness of the concrete cover.

The reduction in the diameter over time ($d_b(t|T_{in})$) is determined according to the function of Choe et al. (2008) as follows:

$$d_b(t|T_{in}) = f(x) = \begin{cases} d_{b0} & , \ t < T_{in} \\ d_{b0} - \frac{1.0508(1-w/c)}{h}(t-T_{in})^{0.71} & , \ T_{in} < t \leq T_f \\ 0 & , \ t > T_f \end{cases} \quad (19)$$

Where d_{b0} is the diameter of the reinforcement at the time t=0, T_{in} is the corrosion initiation time of reinforcement, and T_f is the time when diameter of reinforcement reaches value 0 in theory, that is:

$$T_f = T_{in} + d_{bi}\{d/[1.0508(1-w/c)^{-1.64}]\}^{1/0.71} \quad (20)$$

Using the previously derived model and fuzzy random variable, the corrosion process according to the fuzzy variable and the random variable can be predicted. Experimental data are required to correct the input variables from which to calculate the life predictions of the structure.

V. FAILURE PROBABILISTIC ANALYSIS OF RC BRIDGE

The limit state equation for flexure of the RC slab at the support location can be written in terms of the negative flexural capacity of the section M_n^{-}, the dead load of structural components and nonstructural attachments moment M_{DC}, the dead load of wearing the surface and utility moment M_{DW}, the moment due to live load from traffic including impact load moment M_{TR+IM}, and lane load moment M_{LL}, as follows:

$$g_{slab} = M_n^{-} - (M_{DC} + M_{DW} + M_{TR+IM} + M_{LL}) \quad (21)$$

The nominal flexural resistance of the RC slab is calculated as follows:

$$M_n^{-} = A_s^{top} f_y\left(d_s - \frac{a}{2}\right) \quad (22)$$

Where A_s^{top} is the top reinforcement area in the section over the support; f_y is the yield strength of reinforcement; d_s is the distance from extreme compression fiber to the centroid of non-prestressed tensile reinforcement; and a is the corresponding depth of equivalent stress block, which is taken as:

$$a = c\beta_1 \quad (23)$$

$$c = \frac{A_s^{top} f_y}{0.85\beta_1 f_c' b} \quad (24)$$

Where β_1 is the stress block factor taken from AASHTO LRFD Bridge Design Specifications (2020); b is the unit width of design strip, which may be taken as 1 m.

The probability of failure of the RC slab bridge at time t can be defined mathematically as follows:

$$P_f = P[g_{slab}] \geq 0 = P[M_n^{-} \leq (M_{DC} + M_{DW} + M_{TR+IM} + M_{LL})] \quad (25)$$

The general methodology flowchart of the model for predicting the service life of RC bridges in this study is illustrated in Figure 2. Reliability is expressed as a reliability index, $\beta = \Phi^{-1}(1-P_f)$, which represents the failure probability P_f

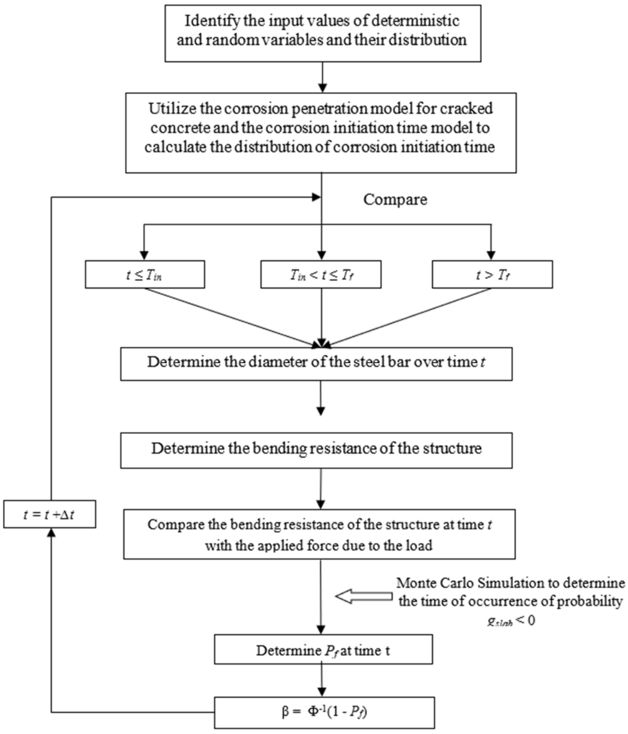


Figure 2: The flowchart of the general methodology

VI. APPLICATION TO PREDICT DURABILITY ACCORDING TO THE CHLORIDE INTRUSION MODEL

A. *Initial parameters*

The problem is considered for Trung Chinh bridge located in Thanh Hoa city, Vietnam. This is an old RC beam bridge consisting of 3 units of which each unit has 4 continuous spans with each span 12 m long. The bridge width is 17.5 m with four designed truck lanes. The design live load is HL-93 according to Highway Bridge Design Specification, TCVN 11823-2017 (2017). The problem of analysing the loading capacity of beams assumes corrosion of steel reinforcement in concrete. This study considers two continuous spans of 2×12 m from pier P6 to P8 as shown in Figure 3. The diagram of a two-span continuous girder bridge with hinge support in the middle and roller supports at both ends is illustrated in Figure 4. The considered bridge was subjected to design loads modelled as uniformly distributed and concentrated loads.

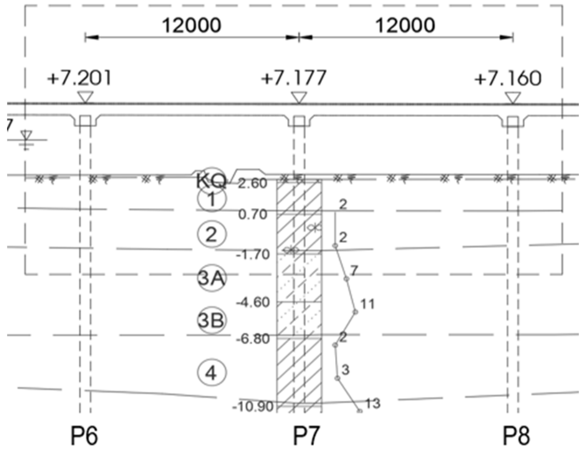


Figure 3: Considered span structure of Trung Chinh Bridge

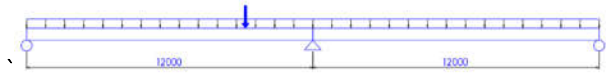
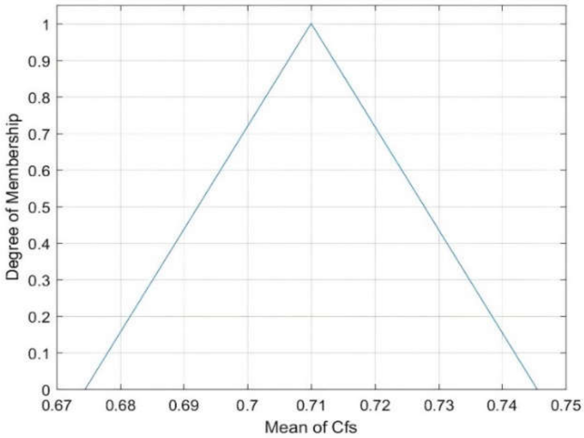
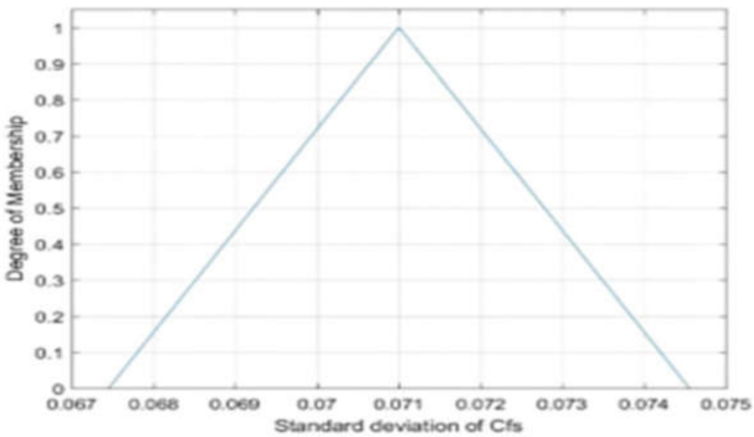


Figure 4: Diagram of a two-span continuous plate girder bridge

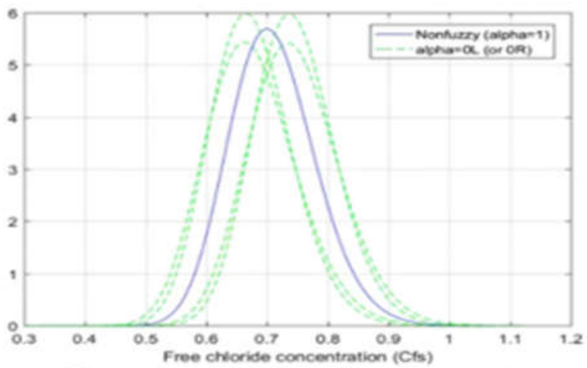
Similarly, material characteristics, environmental parameters, and design loads are also uncertain values with their own distribution rules. Particularly, the two parameters chloride content on the surface and critical chloride content are distributed with characteristic parameters being fuzzy variables, specifically according to the fuzzy-lognormal distribution rule. The problem of calculating the bearing capacity of a bridge according to the section of the top of the pier (the most unfavourable moment position) through the value of the probability of failure according to the construction's operating time. Besides, the problem takes into account the influence of crack development in normal RC structures, the crack size is determined according to Eurocode 2 (2004); as well as considering the decrease in strength of reinforcement over time during the development stage of reinforcement corrosion. Figures 5(a), (b), and (c) show the mean value, standard deviation, and distribution of chloride content on the concrete surface, respectively. Input data are shown in Table 1.



(a) Mean of *C*



(b) Standard deviation of C_{fs}



(c) Distribution of C_{fs} (Fuzzy –Lognormal distribution)

Figure 5: Fuzzy distribution of chloride content on the concrete surface

Table 1. Input parameters

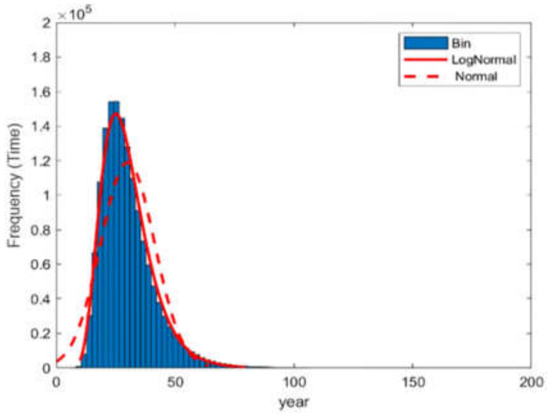
No.	Deterministic/random variable	Symbol	Unit	Determined value/expected value (m)	Standard deviation (s)	Distribution
General parameters of algorithm analysis						
1	Number of times	*N*	time	1500000		
2	Number of years	*N*_year	year(s)	25		
3	Analysis time step	Δt	year(s)	2.5		
Bridge specifications						
1	Span length (2 continuous spans)	*L*	m	12	0.005	Normal*
2	Bridge width	*B*	m	12	0.005	Normal*
3	Beam height	*H*	m	0.65	0.003	Normal*
4	Bridge deck coating thickness	T_{ws}	m	0.07	0.002	Normal*
Material specifications						
1	Concrete strength	*f'c*	Mpa	35	5.25	Lognormal*
2	Concrete specific gravity	G_{con}	kN/m^3	24.5	0.49	Lognormal*
3	Yield strength of steel reinforcement	f_{y0}	Mpa	400	20	Lognormal*
4	Experimental coefficient of steel reinforcement strength loss over time	α_y		0.005		
5	Modulus of elasticity of steel reinforcement	*Es*	Mpa	200000	2000	Lognormal*
6	Specific gravity of bridge deck coating	*G*_ws	kN/m^3	22.5	0.45	Lognormal*
Design parameters for bearing top pier cross-section (negative moment)						
1	Diameter of steel reinforcement	*D*_bar_top	mm	20	1	Lognormal**
2	Number of steel reinforcements	*N*_bar_top	bar	158		
3	Concrete cover thickness	*d*cover_top	mm	50	2	Normal**
Design parameters for mid-span cross-sectional load bearing (positive moment)						
1	Diameter of steel reinforcement	*D*_bar_bot	mm	20	1	Lognormal**
2	Number of steel reinforcements	*N*_bar_bot	bar	158		
3	Thickness of protective concrete layer	*d*cover_bot	mm	40	2	Normal**
Environmental parameters and adjustment factors						
1.	Chloride content on the surface					
	non-fuzzy	*C_s_f*	% cement	0.71	0.071	Lognormal**
	fuzzy-lower bound	*C_s_f_inf*	% cement	0.6745	0.06745	Fuzzy-Lognormal**

No.	Deterministic/random variable	Symbol	Unit	Determined value/expected value (m)	Standard deviation (s)	Distribution
	fuzzy-upper bound	*C_s_f_sup*	% cement	0.7455	0.07455	
2.	Critical chloride content					
	non-fuzzy	*C_cr*	% cement	0.15	0.02	Lognormal**
	fuzzy-lower bound	*C_cr_inf*	% cement	0.1425	0.019	Fuzzy-Lognormal*
	fuzzy-upper bound	*C_cr_sup*	% cement	0.1575	0.021	
3	Environmental coefficient	k_e		0.924	0.155	Gamma*
4	The coefficient influenced by the experimental method determines the dispersion coefficient D0	k_t		0.832	0.024	Normal*
5	Impact coefficient of maintenance work	k_c		1		
6	Age coefficient	n		0.23	0.04	Beta*
7	Idealized model adjustment coefficient when applying Fick's 2nd law	X_i		1	0.05	Lognormal**
8	Characteristic adhesion coefficient of reinforcement	k_1		0.8		
9	Improvisation distribution coefficient	k_2		0.5		
10	Load time coefficient	k_{tl}		0.6		
11	Tensile stress of reinforcement if the section is cracked	σ_S	Mpa	160	1.6	Lognormal*
Live load parameters						
1	Truck	Truck_P1	kN	35	7	Lognormal*
		Truck_P2	kN	145	30	Lognormal*
		Truck_P3	kN	145	30	Lognormal*
2	Tandem	Tandem_P1	kN	110	22	Lognormal*
		Tandem_P2	kN	110	22	Lognormal*
3	Impact coefficient	IM	%	33		
4	Lane load	Lane	kN/m	9.3	2	Lognormal*
5	Number of lanes	n_lane	lane	3		

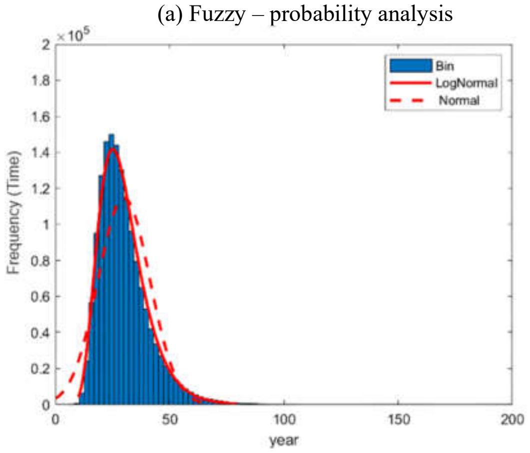
Note: (*) – assumption value; (**) – reference value

B. Results and Discussion

The effect of corrosion on the performance of RC plate beam structures, taking into account the uncertainty of input parameters and Monter-Carlo simulation, was calculated and analysed using the MATLAB programming tool. The obtained results include the corrosion initiation time of steel, the change in bending resistance of the plate beam with respect to the time, the change in probability of failure, and the corresponding reliability index over time, through determined and uncertain parameters. In this study, cracks appearing in concrete are assumed to be affected by load bearing, and then the reinforcement stress has a lognormal distribution with a mean value of 160 MPa. The calculated results using the proposed model were applied to a specific bridge case in the context of Vietnam, making it difficult to compare with other results in the literature, as the boundary conditions are unique to this calculation scenario. The computational model and the results obtained can be applied to the analysis of other ageing bridge structures, provided that the input data includes comprehensive statistical information. Figures 6(a) and 6(b) present the results of probabilistic analysis for the initiation time of steel corrosion with and without considering Fuzzy theory, respectively.



(a) Fuzzy – probability analysis



(b) Non-fuzzy – probability analysis

Figure 6: Initiation time of steel corrosion

Based on these figures, it is found that the initiation time of steel corrosion has a distribution law close to the normal distribution law - logarithm. The typical values are shown in Table 2. The prediction results using Fuzzy and Non-Fuzzy probability analysis for the initial time of corrosion are similar with values of 29.96 and 29.94 years, respectively.

Table 2. Typical values for the sample at the initial time of corrosion

Fuzzy – probability analysis			Non-fuzzy – probability analysis		
Mean (year)	Variance (year^2)	Standard deviation (year)	Mean (year)	Variance (year^2)	Standard deviation (year)
29.96	129.3	11.37	29.94	127.9	11.31

Figure 7 shows the degradation of the flexural resistance (Mn) of the RC beam with respect to time. The results show that the probabilistic analysis considering fuzzy and non-fuzzy gives equivalent results. The flexural resistance decreases sharply in the period from the 5th to the 25th year. This degradation is shown in Table 3 as follows.

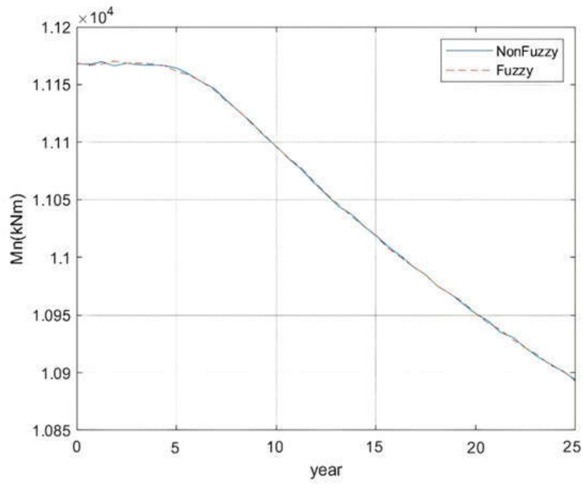


Figure 7: Degradation of flexural resistance of the beam

Table 3. Degradation of the moment resistance

M_n	Year					
	0	5	10	15	20	25
Fuzzy	11168.48	11135.73	11095.78	11017.49	10951.81	10892.30
Non-fuzzy	11168.67	11137.57	11095.63	11017.16	10952.39	10892.22

Figure 8 shows the probability of failure of the structure with respect to time. It can be seen that the probabilistic analysis considering fuzzy and non-fuzzy also gives equivalent results. The probability of failure after the onset of corrosion increases significantly. In the period from the 5th to the 25th year, the probability of failure increases by 2 times. This degradation is shown in Table 4 as follows.

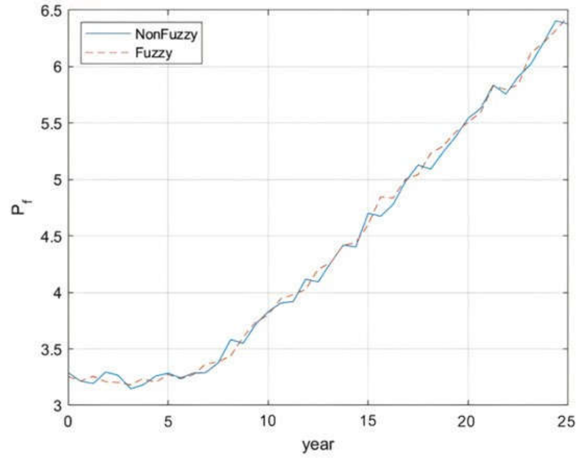


Figure 8. Probability of failure of the structure

Table 4. Probability of failure of the structure

P_f	Year					
	0	5	10	15	20	25
Fuzzy	3.262	3.416	3.797	4.686	5.513	6.44
Non-fuzzy	3.281	3.385	3.809	4.739	5.446	6.493

Figure 9 shows the probability of the reliability index to time. The data also showed that the probabilistic analysis considering fuzzy and non-fuzzy gives equivalent results. The probability of the reliability index decreases gradually in the period from the 5th to the 25th year. This degradation is shown in Table 5 as follows.

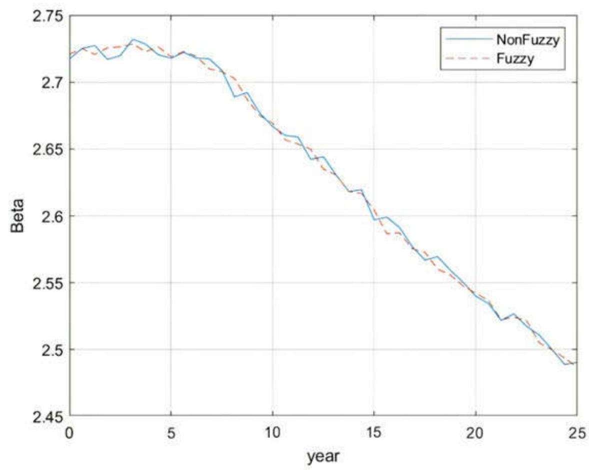


Figure 9. Reliability Index

Table 5. Probability of the reliability index

Beta	Year					
	0	5	10	15	20	25
Fuzzy	2.720	2.705	2.670	2.598	2.542	2.487
Non-fuzzy	2.718	2.708	2.669	2.594	2.546	2.484

VII. CONCLUSIONS

This study established the theoretical basis and applied fuzzy random and probabilistic theory in evaluating the moment capacity, the failure probability and predicting the service life of RC bridge structures. The prediction process incorporated the uncertainty of input parameters and employed Monte Carlo simulation. The methodology was applied to predict the service life of an old RC bridge structure, considering reinforcement corrosion due to chloride intrusion. The results show that the distribution law of the initiation time of steel corrosion is close to the normal distribution law - logarithm. The prediction results using fuzzy and non-fuzzy probabilistic analyses for the initial corrosion time, moment capacity degradation, failure probability, and reliability index of RC bridge structures are comparable.

i. The predicted initial corrosion times of RC bridge structures using fuzzy and non-fuzzy probabilistic analysis are 29.96 years and 29.94 years, respectively.

ii. The flexural strength of RC bridge structures using fuzzy and non-fuzzy probabilistic analysis gives similar results. The flexural strength of the RC structure declines sharply from the 5th year.

iii. The analysis results show that the failure probability of RC bridge structures increases twofold after 25 years.

The proposed model's reliability can be enhanced by incorporating more comprehensive information on the input parameters used to calculate durability and resistance, particularly empirical data obtained from field experiments. Consequently, the computational predictions of the lifespan and resistance of RC structures will be more accurate in real-world applications.

AUTHOR CONTRIBUTIONS

T. T. Tran: Conceptualization, Methodology, Validation, Writing – original draft. **T. M. Tran:** Computational analysis, Writing – review & editing. **X. T. Nguyen:** Supervision, writing – original draft, Writing – review & editing. **V. T. Nguyen:** Writing – original draft. **B. T. Vu:** Writing – review & editing.

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