Analysis and Performance Evaluation of Three-Phase Induction Motor Model with Zero Quadrature Axis Flux Component

G. A. Olarinoye[1*], M. F. Akorede[2], S. N. Okhuegbe[3]

[1]Department of Electrical Engineering Department, Ahmadu Bello University, Zaria, Nigeria.
[2]Department of Electrical & Electronics Engineering, University of Ilorin, Ilorin, Nigeria.
[3]Department of Energy Science and Engineering, University of Tennessee, Knoxville, USA.

ABSTRACT: Modeling of three phase induction motor has been extensively discussed in literature. Several models have been developed and applied by different authors to solve different problems. This paper presents a different model of the symmetrical three phase induction motor in which there is an alignment of the direct axis component of the rotor flux vector to direct axis of the reference frame. The alignment reduces the quadrature axis flux component to zero, removes the dependencies associated with the d-q components of the stator current and simplifies the rotor dynamics. The model is not commonly used in literature to study motor behavior when vector control is not the objective. The model is analyzed in the synchronous reference frame and simulated. Simulation results for torque, speed and flux in the motor are presented. Also presented are the waveforms of stator voltages and currents in both the d – q and a-b-c reference frames and the rotor d-q axis currents. These results provide a correct description of the three phase induction motor behavior under load and no-load conditions.

KEYWORDS: Three-phase induction motor, d-q components, reference frame, rotor flux vector, simulation

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I. INTRODUCTION

The three-phase induction motors continue to receive widespread attention because of advancements in semiconductor technology, development of cheap and fast digital signal processors and control strategies that enable the motor to perform tasks in new ways (Abad, 2016; El-Zohri and Mosbah, 2019; Richardson et al., 2021). They have low maintenance cost and are reliable (Aminu et al., 2021). They are now being increasingly used in industrial and transportation sectors of the economy and they are available in a wide variety of power range from fraction of a kW to thousands of kW (Ke et al., 2020; Li et al., 2021; Menghal and Laxmi, 2020; Kousalya and Singh, 2012). The modeling of the symmetrical three phase induction motor has been extensively discussed in literature.

Several models have been developed and applied by different authors. Some of these models include the equivalent circuit model, a-b-c reference frame models and the d-q reference frame model (Sharma et al., 2020; Saxena et al., 2017). The d-q model is the most popular because it not only allows for steady state, dynamic and transient studies but it also makes motor analysis easier and reduces solution computation time (Orille et al., 1999; Sanjay et al., 2015; Kuznyetsov, 2019). Some of the methods that have been applied to model the induction motor includes; numerical integration, finite element-based, complex vector representation etc. Kuznyetsov (2019) developed a numerical model of a three phase induction motor in its natural a-b-c reference frame. The method of average voltages at the integration step (AVIS) was applied. The behavior of the model was evaluated using AVIS and other numerical methods and the results were then compared. The focus of the work was on the calculation speed of the model. Menghal and Laxmi (2020) employed the d – q model of the three phase induction motor for transient and steady state studies under different load conditions. Different modes of operation were examined using artificial intelligence methods. A conventional three phase induction motor model derived in the arbitrary reference frame was presented, analysed in the rotor reference frame and simulated in Deb and Sarkar (2016).

This paper presents a different model of the symmetrical three phase induction motor in which the quadrature axis component of the rotor flux vector is reduced to zero because of an alignment of the flux vector to the direct axis of the reference frame. The aim of the work is to develop a solution for this model and use it to examine motor behaviour. The model is often applied in the vector control of three phase induction motor but this paper shows that the model can also be used to study the motor behavior when vector control is not the objective. The rest of the paper is divided into four sections. Section II presents the d-q reference frame model of the three phase induction motor with the d-axis aligned with the rotor flux vector while section III provides the solution strategy of the model. Section IV presents and discusses simulation results while the conclusion is presented in Section V.

II. MODEL OF THE THREE-PHASE INDUCTION MOTOR

The model with zero quadrature axis rotor flux component is presented in the following equations (Mohan, 2014);

$$V_{ds} = R_s i_{ds} + p\lambda_{ds} - \omega_e \lambda_{qs} \tag{1}$$

*Corresponding author: oyeguzeh@yahoo.com

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$$V_{qs} = R_s i_{qs} + p\lambda_{qs} + \omega_e \lambda_{ds} \quad (2)$$

$$\tau_r \frac{d\lambda_r'}{dt} + \lambda_r' = L_m i_{ds} \quad (3)$$

$$\omega_s = \omega_r + \frac{R_r L_m i_{qs}}{L_r' \lambda_r'} \quad (4)$$

$$T_e = \frac{3P}{2}\frac{L_m}{L_r'}\left(\lambda_r' i_{qs}\right) \quad (5)$$

$$\lambda_{ds} = L_s i_{ds} + L_m i_{dr}' , \quad (6)$$

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr}' , \quad (7)$$

$$\lambda_r' = L_r' i_{dr}' + L_m i_{ds} , \quad (8)$$

$$0 = L_r' i_{qr}' + L_m i_{qs}, \quad (9)$$

$$T_e = J\left(\frac{2}{P}\right) p\omega_r + T_L \quad (10)$$

The derivation of model Eqs. (1) – (10) can be obtained in (Krause *et al*., 2013). V_{ds} and V_{qs} are the direct and quadrature axes voltages respectively. The direct and quadrature axes stator currents are represented by i_{ds} and i_{qs} respectively. i_{dr}' and i_{qr}' are respectively the direct and quadrature axis rotor currents. The flux linkages of the rotor and stator windings are λ_r', λ_{ds} and λ_{qs} respectively. R_s and R_r are the winding resistances of the stator and rotor respectively. T_e and T_L represent the electromagnetic torque and the load torque respectively. p is a differential operator and it is defined as;

$$p = \frac{d}{dt} \quad (11)$$

L_s and L_r' represent the self-inductances of the stator and rotor respectively. They are defined as follows;

$$L_s = L_{ls} + L_m \quad (12)$$

$$L_r' = L_{lr}' + L_m \quad (13)$$

where L_{lr}' and L_{ls} are the leakage inductances of the rotor and stator respectively. L_m is the mutual inductance between the rotor and stator circuits. ω_r is the rotor speed. P is defined as the number of stator poles. The dynamics of the rotor are controlled by the rotor time constant, τ_r defined as;

$$\tau_r = \frac{L_r'}{R_r} \quad (14)$$

If the rotor flux is maintained constant, then according to Eq. (5) the torque production will be dependent entirely on i_{qs}. The rotor flux magnitude λ_r', on the other hand, is proportional to i_{ds} as seen from Eq. (3). These relationships make the control of torque and, flux may be achieved independently as in the case of the direct current (DC) motor (Biswal and Satpathy, 2021).

The alignment of the direct axis with the rotor flux vector can be seen in Figure 1. It causes the rotor flux vector to rotate at the same speed as the *d-q* windings which is also the synchronous speed. Therefore, the rotor flux angle is calculated as;

$$\theta_e = \int_0^t \omega_e(\tau) d\tau \quad (15)$$

where ω_e is the supply frequency or the synchronous speed in rad/s. τ is the variable of integration. The flux angle has to be calculated at all times and used to transform the *a-b-c* variables to *d-q* quantities at every instant according to Eqs. (16) – (18) in order to maintain the alignment of the rotor flux vector with the *d*-axis (Richardson *et al*., 2021).

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} = \begin{bmatrix} \cos\theta_e & \sin\theta_e \\ -\sin\theta_e & \cos\theta_e \end{bmatrix}\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \quad (17)$$

$$\left.\begin{aligned} V_a &= V_m \cos\omega_e t \\ V_b &= V_m \cos(\omega_e t - 120^0) \\ V_c &= V_m \cos(\omega_e t - 240^0) \end{aligned}\right\} \quad (18)$$

The synchronous reference frame is chosen for the analysis of the induction motor because, in this reference frame, all voltages, currents and flux linkages are constants (or DC quantities) in balanced sinusoidal steady state.

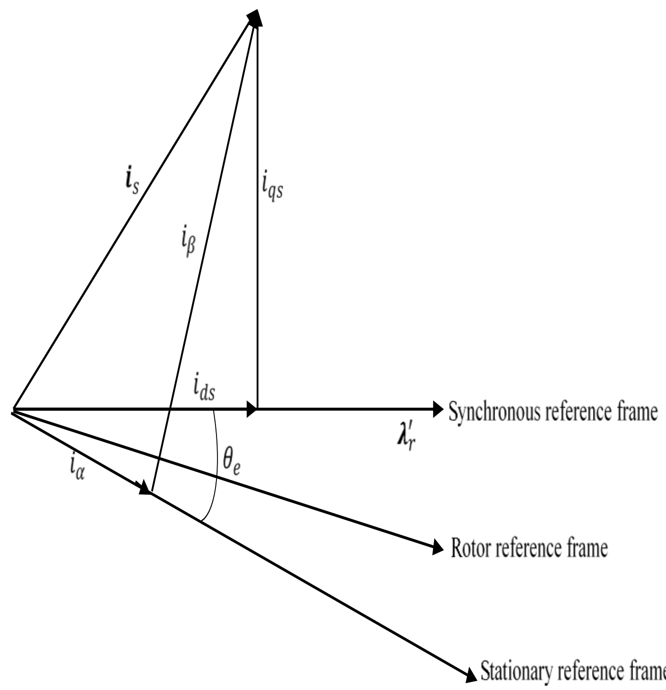

Figure 1: Rotor flux vector alignment with the d-axis.

III. SOLUTION TO THE MODEL

In solving the model of Eqns. (1) - (10), the stator and rotor flux linkages λ_{ds}, λ_{qs} and λ_r' are often considered as state variables. Voltages serve as inputs and the currents i_{ds}, i_{qs}, i_{dr}' and i_{qr}' are then calculated from the flux linkages as follows from Eqs. (6) - (9).

$$\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix}^{-1}\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ 0 \\ \lambda_r' \end{bmatrix} \quad (19)$$

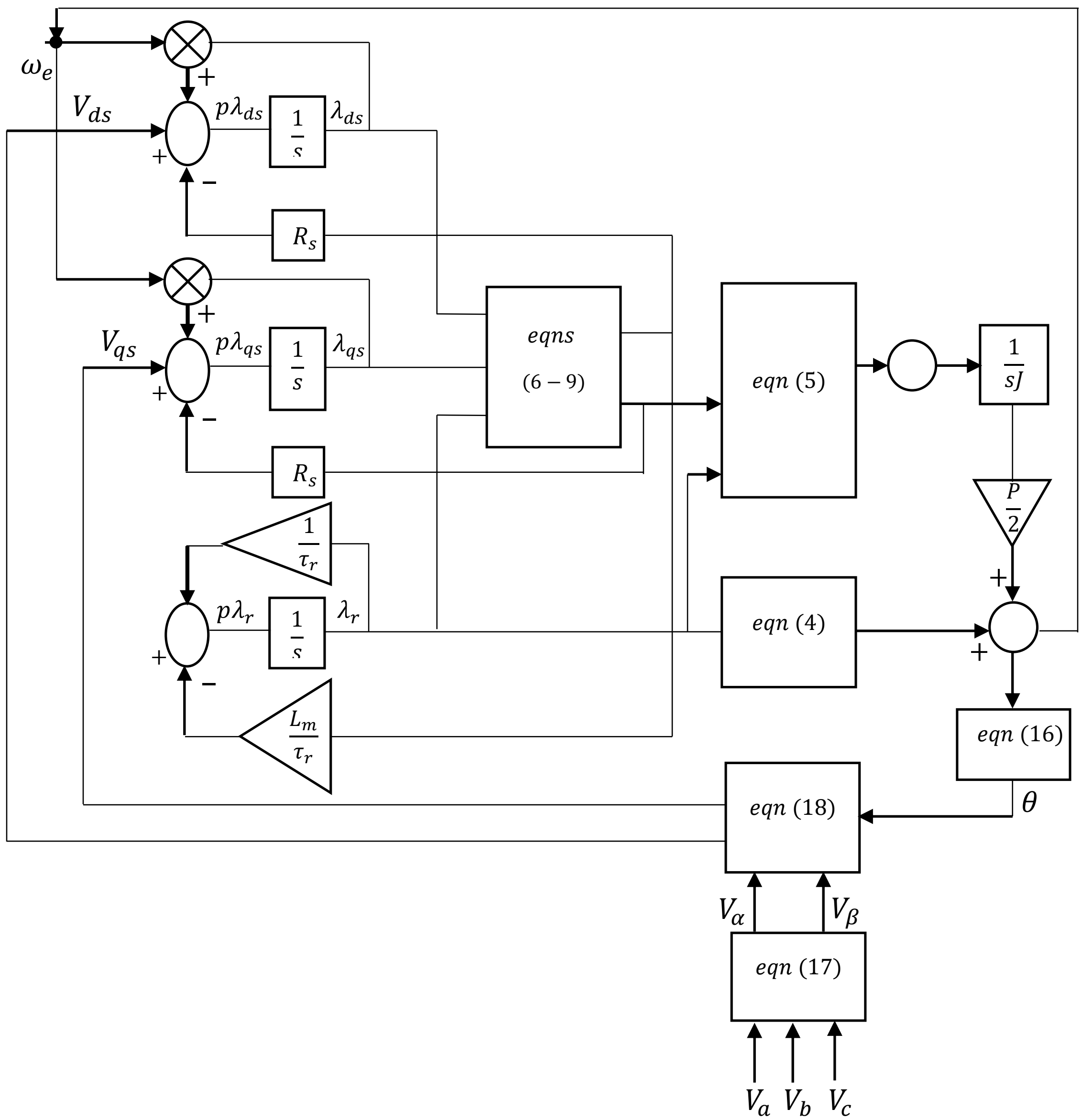

Figure 2: Solution to the model in block diagram form.

The electromagnetic torque is calculated from Eqns. (5) and (10). The solution to the model is illustrated in Figure 2. An observation of the figure shows that the synchronous speed has to be calculated using Eq. (4) and the rotor flux angle is then calculated from Eq. (16). This model is easier and quicker to solve than the d – q reference frame model in which there is no alignment. The Simulink block created in MATLAB to implement the solution diagram is provided in Figure 3.

Table 1 contains the parameters of the symmetrical three phase induction motor applied in the analysis.

Table 1: Three-phase induction motor parameters (Mohan, 2014).

Parameters	Values
Rated power	2.4 kW
Line Voltage	460 V
Rated speed	1750 rpm
Frequency	60 Hz
Number of poles	4
Stator resistance	1.77 Ω
Rotor resistance	1.34 Ω
Stator leakage reactance	5.25 Ω
Rotor leakage reactance	4.57 Ω
Mutual reactance	1.39 Ω

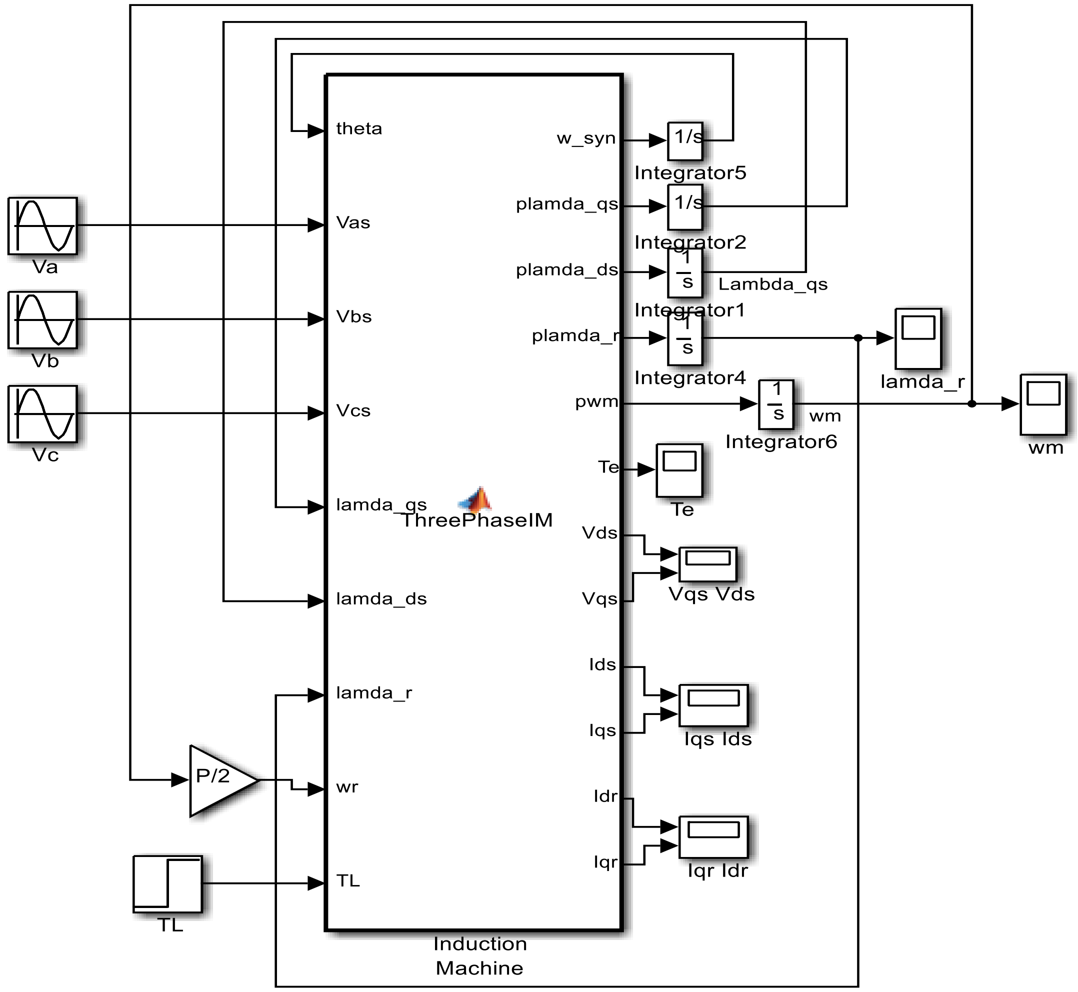

Figure 3: MATLAB/Simulink block to implement the solution diagram.

IV. RESULTS AND DISCUSSION

The solution diagram in Section III was implemented in the Simulink environment of the MATLAB software. Results are presented and discussed in this section. Figure 4 shows the results of motor behavior when load torque was applied to the model.

It can be observed that with the motor initially unloaded, a rated load of 12.644Nm was applied at steady state at a time of 1s and this load was stepped down to half its value at 1.5s. The load torque was further stepped down to 0Nm at 2s till the end of simulation time of 2.5s. The speed of the motor is seen in Figure 4 (b) to rise from 0 rad/s to a steady state value of 185.5 rad/s in 0.33 secs. Oscillations in response to the sudden application and reductions in load torque can be observed at simulation times of 1s, 1.5s and 2s. The steady state speed rises slightly with each reduction in load. The induced torque is provided in Figure 4 (c).

It can be seen to oscillate during the acceleration time. It can also be observed that the induced torque reduces to zero at steady state under no-load condition. It equals the load torque in steady state after experiencing transients during changes in load torque in accordance with the motor dynamics given by Eq. (10). The rotor flux profile is shown in Figure 4 (d). It is constant at a value of 0.96 Wb in steady state at no load. This is so because the three phase induction motor is designed to operate at the rated flux (i.e. 0.96 Wb in this case) regardless of changes in load torque. The rotor flux can be observed to be fairly constant during changes in load. The three phase voltages and currents are presented for 3 cycles between 0.99s and 1.04s of simulation time in Figure 5 (a) and (b) respectively. The voltages have a peak value of 375.6 V and frequency of 377 rad/s. The currents have the sinusoidal form of the voltages. The three phase currents can be observed to rise in response to the increase in load torque. They are identical in magnitude and frequency but differ in phase by 120°.

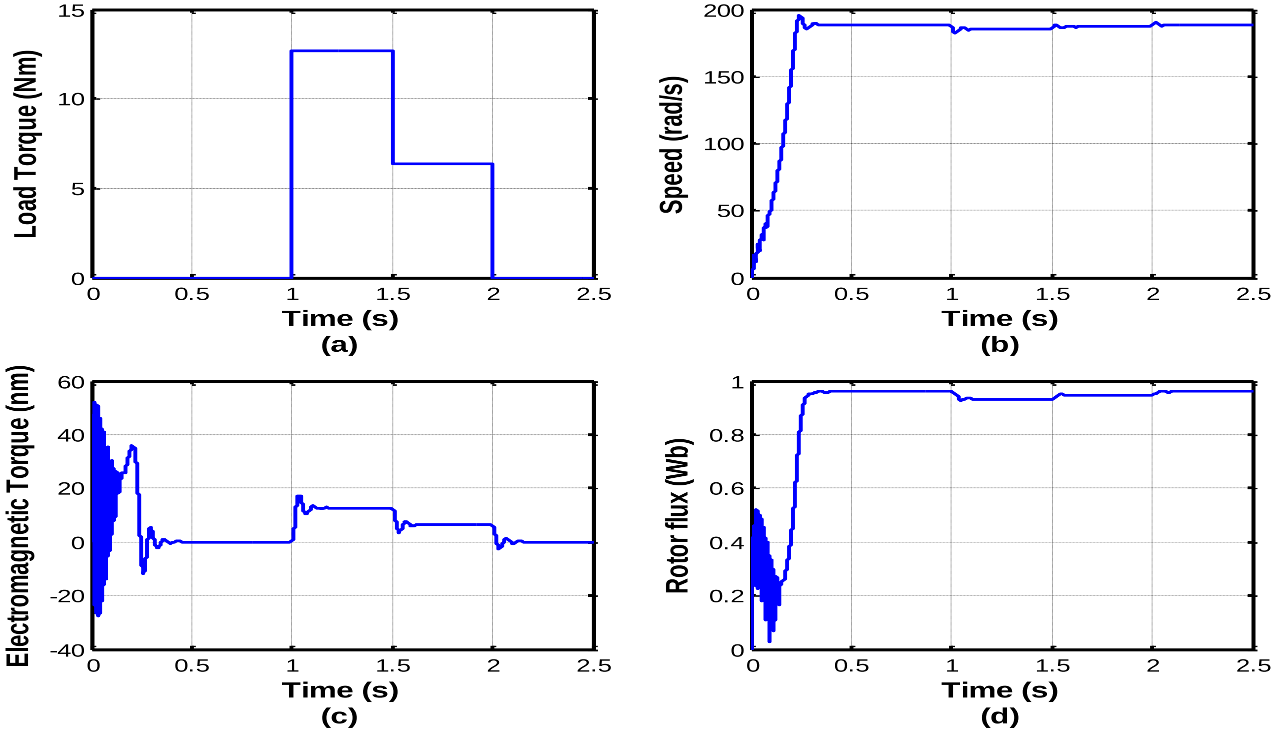

Figure 4: Simulation results of the motor behavior (a) Load torque with Time (b) Speed with Time (c) Electromagnetic torque with Time (d) Rotor flux with Time.

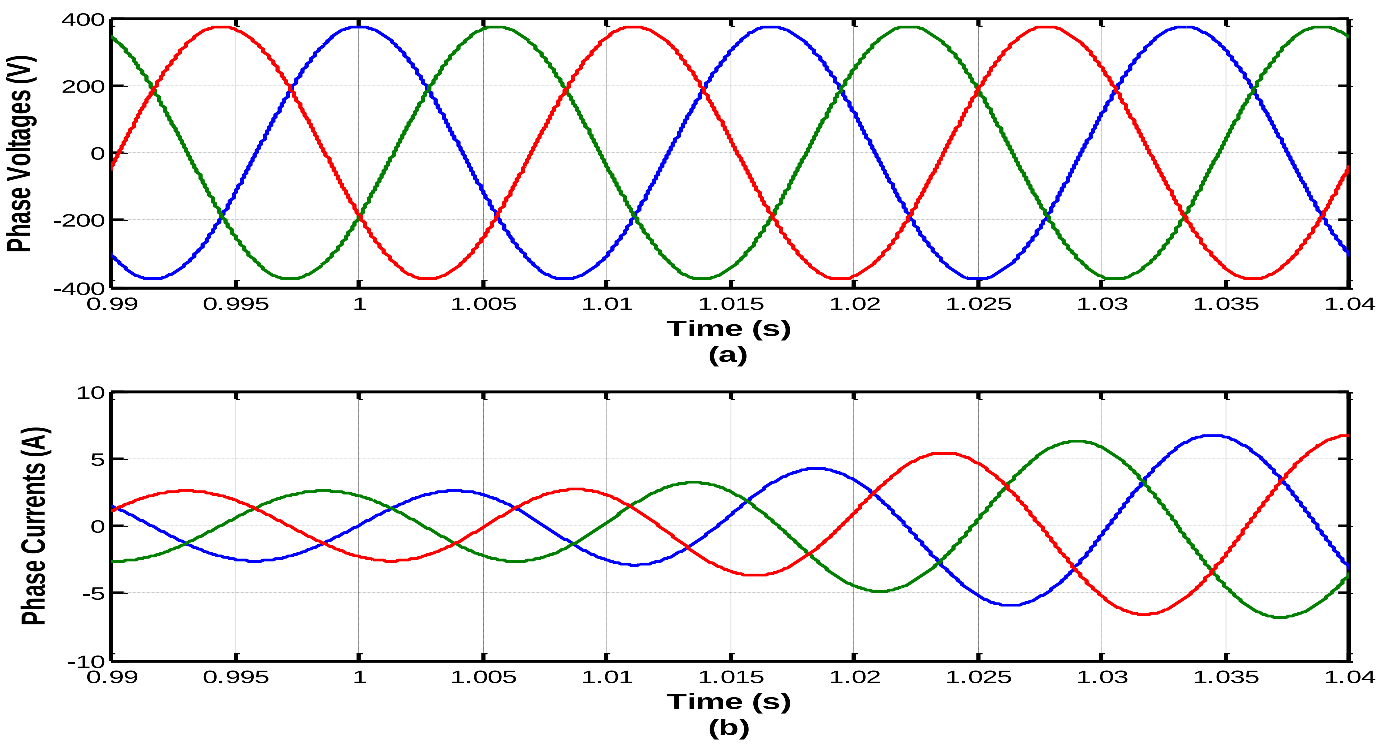

Figure 5: Simulation results of stator phase voltages and currents.

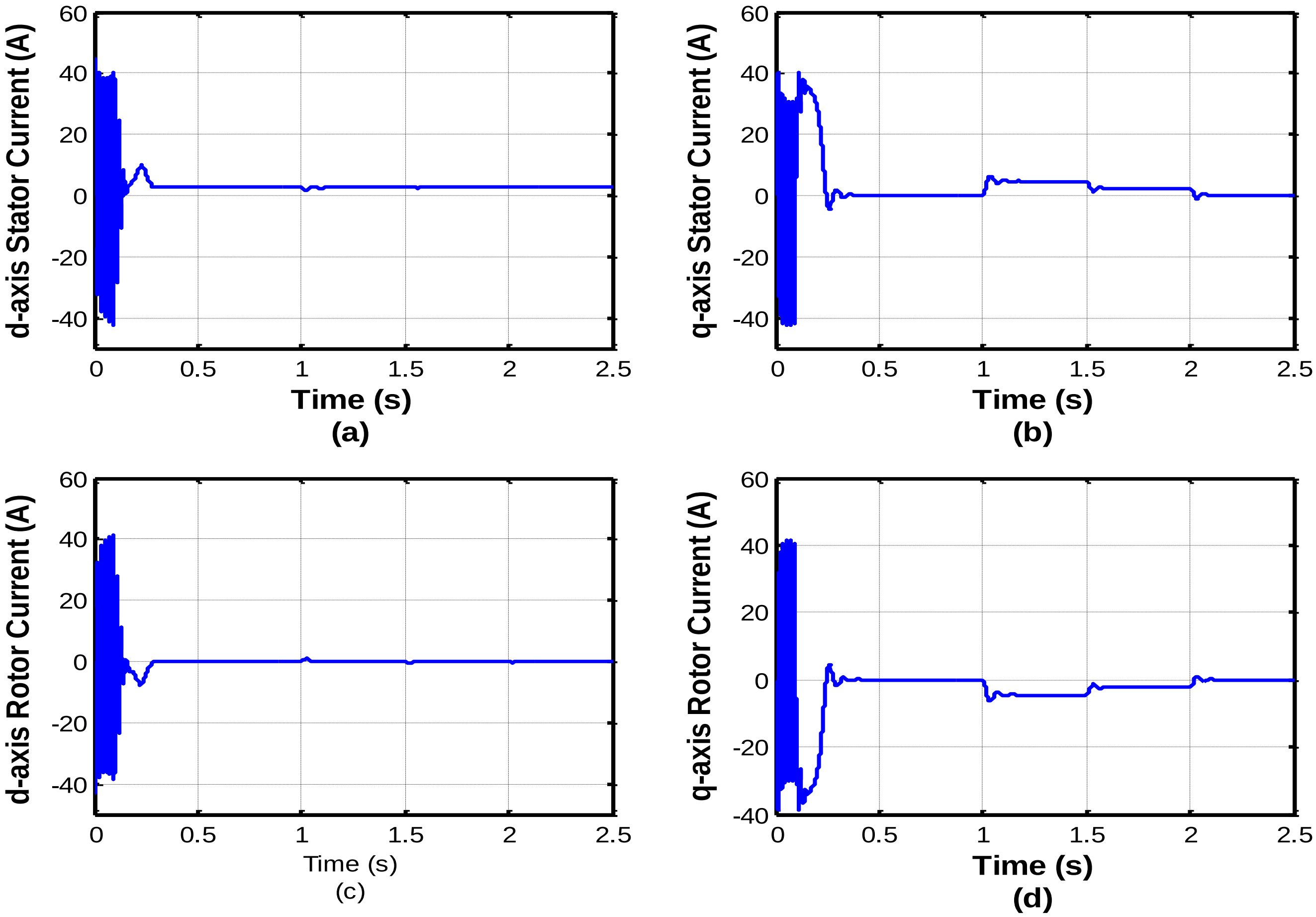

Figure 6: Simulation results of *d* and *q*-axis stator and rotor currents.

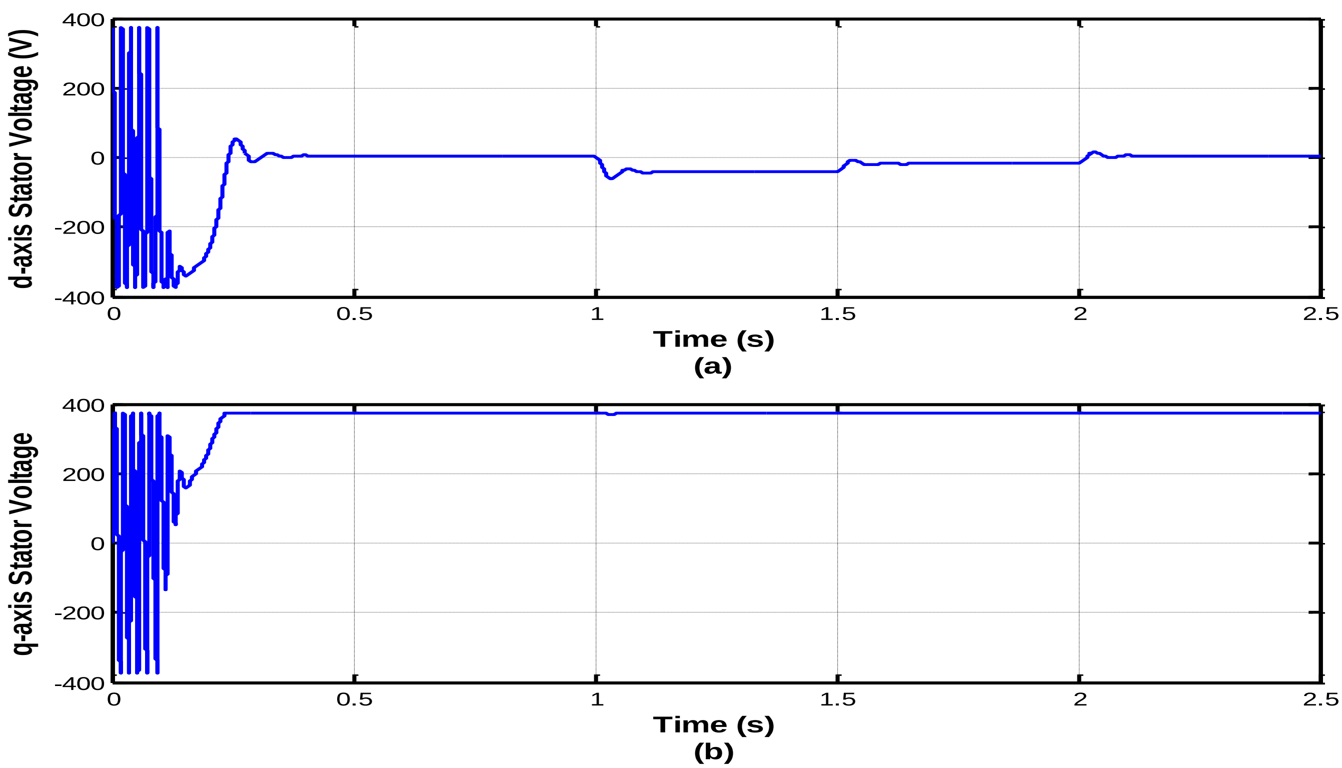

Figure 7: Direct and quadrature axis voltages in the stator.

The direct and quadrature axis currents of the stator and rotor are presented in Figure 6 (a) – (d). They are DC currents during the steady state period. The direct axis current in the stator is constant and positive regardless of load changes which is expected because it is in phase with the rotor flux according to Eq. (3). The quadrature axis stator current is positive and its value changes with load applied on the motor which is expected as well because it is proportional to the electromagnetic torque produced according to Eq. (5). This result demonstrates the decoupling of the direct and quadrature axis stator currents from each other. The rotor currents in the *d-q* reference frame are similar to their stator counterparts except that they flow in opposite directions.

The direct axis voltage in the stator is shown in Figure 7 (a). Its value is 0 V when no load was applied on the motor. On load, its magnitude changes in direct proportion to the load applied. The quadrature axis stator voltage is shown in Figure 7 (b). Its value is the same as the peak value of the phase voltages applied to the model. It can be seen that oscillations occur in both the direct and quadrature axis voltages because of the dynamics of alignment in accordance to Eqs. (4) and (16). The oscillations appear during the acceleration time between 0 and 0.33 s.

IV. CONCLUSION

A *d-q* reference frame model of the symmetrical three phase induction motor with zero quadrature axis rotor flux component was presented and simulated in this paper. The analysis of the *d-q* reference frame model was performed in the synchronous reference frame. Simulation results for speed, torque and flux have been presented. Those of stator phase voltages and currents, direct and quadrature axis currents and voltages have been presented as well. A rated load of 12.644Nm was applied at steady state and was stepped down first to half its value for 0.5s and second, to 0 Nm for 0.5s as well. The steady state motor speed was observed to be 185.5 rad/s (mechanical). Oscillations occurred during changes in load torque.

Rotor flux magnitude was fairly constant at a value of 0.96 Wb at steady state. The *d*- axis current in the stator maintained a constant value of 2.6A in steady state while the *q*-axis current changed in direct proportion to the load applied. The *d*- axis stator voltage, on the other, has a magnitude whose value changed in direct proportion, as well, to the load while the *q*-axis stator voltage magnitude was observed to be equal to 375.6 V which is the peak value of the phase voltages applied to the model. The model effectively decouples the direct axis current from the quadrature axis current in the stator and by extension enables the independent control of torque and flux thereby providing opportunities for easier speed control of three phase ac induction motors.

This work has applied the model for the purpose of examining motor behaviour when it is standing alone. This is a remarkable shift from what obtains in literature where such model is typically applied to investigate the behavior of a vector control system that includes the three phase induction. The results obtained show that the model is a viable alternative to the commonly used *d-q* reference frame model with no alignment, in the synchronous reference frame. The results also show that the *d-q* axis voltages and currents exhibit oscillations during motor acceleration time because of the dynamics of alignment but attain constant values during the time that motor speed is constant. The model is especially recommended for use in studying the behaviour of the motor where vector control is not the objective.

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