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DEVELOPMENT OF A MATHEMATICAL MODEL FOR MANAGING MAGNITUDE AND RISK FACTORS OF INJURIES

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ABSTRACT

A mathematical model was developed for managing magnitude and risk factors of injuries in a manufacturing industry employing System Dynamics (SD) approach. Data were collected using an injury and illness investigation register. These were used to estimate and validate the parameters of the model. The principle of SD was employed to identify the relevant risk management safety-related components and their interrelationships. Results obtained from the experimental data depict a random periodic upwards and downwards trend in the magnitude of injuries; whereas the predicted injuries yielded an exponential decay in the number of injuries occurrence. The means and standard deviations of the observed and predicted injuries were 27 and 9.08; and 26 and 6.61 respectively. The corresponding values for observed and predicted preventions were 18 and 9.08; and 19 and 8.48 respectively. A comparison of predicted and observed injuries depict that the model is useful for providing a decision support and predicting the main variables required for managing magnitude and risk factors of injuries in a bottling plant.

Keywords: System dynamics, modeling, injury investigation, simulation, safety management

Nomenclature

F	= Factory hazardous conditions [Q]	γ_t	= Injury prevention rate [Q][T]$^{-1}$
f	= Injury proneness facto[Q](MHR)$^{-1}$	Y_t	= Prevented injury [Q]
α_t	= Injury rate [Q][T]$^{-1}$	q	= Workload [MHR]
X_t	= Magnitude of injury [Q]	X_p	= Potential hazards [Q]
E_p	= Expected prevention [Q]	h	= Risk factor of injury [T]$^{-1}$
λ	= Injury rate probability parameter [Q]	R	= Prevention goal [Q]
ρ_1	= Injury probability [Dimensionless]	X_n	= Pre-safety injury level [Q]
ρ_2	= Injury prevention probability [Dimensionless]	υ	= Safetyprogramme effectiveness factor [T]$^{-1}$

1.0 INTRODUCTION

Despite enormous advances in technology, human factor engineering (ergonomics), preventive medicine and other means to prevent accidents, the International Labour Organisation (ILO) and World Health Organisation (WHO) have observed that an estimated 120 million occupational injuries occur annually at workplaces worldwide (Adebiyi and Ajimotokan, 2010; Ajimotokan, 2009; Apurna et al., 2004; ILO, 2002; Saari, 1998).

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In the lower income countries such as those of the South Asia and Africa, injuries are one of the leading causes of adult mortality and a major contributor to disability (Brahmapurkar *et al.*, 2006; Murray and Lopez, 1996; Zwi, 1993; Feachem *et al.*, 1992; Smith and Barss, 1991).

Death, illness and injury on such scale impoverish individuals and their families, and challenge attempts to improve working conditions.

Nearly every type of work or industrial occupation has the potential of causing injuries. To prevent these, it is important to understand the factors that contribute to them. Research works have depicted that the major risk factors that contribute to the development of musculoskeletal disorders/injuries can be associated with a whole set of physical-ergonomic (force, frequency, awkward postures, extreme temperatures, vibrations etc), hazardous exposures, workplace and process design, psychosocial, individual and work-organizational factors (National Institute of Occupational Safety and Health, 1998).

For an effective preventive measure in reducing work-related injury rates, several studies have been conducted to examine specific risk factors for work-related injuries (Li *et al.* 1999; Zwerling *et al.*, 1996). More so, several quantitative and qualitative evaluation models have been developed over the years from classical statistics, These models include System Dynamics (SD) manufacturing safety model (Ajimotokan, 2009), vehicle safety management model (Kelvin, 2008), SD Safety programmed simulator (Charles-Owaba and Adebiyi, 2006), Efficiency index Charles-Owaba and Adebiyi, 2001), Accident rate model (Gerald, 2001), Safety sampling model (Aggrawal, 1990) among others. However, the accelerating economical, technological, social and environmental changes challenge the attempt to design and manage complex systems, which embody multiple feedback effects, long time delays and nonlinear responses to decisions (Kelvin, 2008; Morrison, 2007) that characterize real world problems (risk factors).

The aim of this study was to develop a SD model for risk management programme of manufacturing industry with emphasis on managing magnitude and risk factor of injuries.

2.0 MODEL DEVELOPMENT

To develop the model, data were collected from a bottling plant in Ilorin, Nigeria using an injury and illness investigation register and analyzed employing the accident investigation procedure. These were used to estimate the parameters of the model.

The SD methodology by Burn (1975) and Forrester (1973) as adopted by Charles-Owaba and Adebiyi (2006), and Adebiyi (2006) comprising the following five (5) general steps were

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determination of model equations from the diagram and Implementation of simulation from model equation

The model was delineated based on following assumptions: consistency of operational conditions guaranteed, safety culture is recognized integral system of the manufacturing system and stability of establishment and government policies.

The behaviour of complex, real-world problem is a continuous, dynamic flow, which can only be explained in causal terms after its decomposition into discrete events. More so, the central concept to SD theory is the representation of system structure in terms of flows. A flow diagram depicts the dynamic relationships of the components of a multiple feedback system. Applying this approach and relying on subject matter experts' opinion, the causal loop and flow diagrams were delineated employing a software application called "Vensim" developed by Vantana Systems for SD synthesis. The resulting causal loop diagram and flow diagram are depicted in Figures 1 and 2.

All the various types of quantities or couplings identified and applied in the flow diagram are summarized in Table 1.

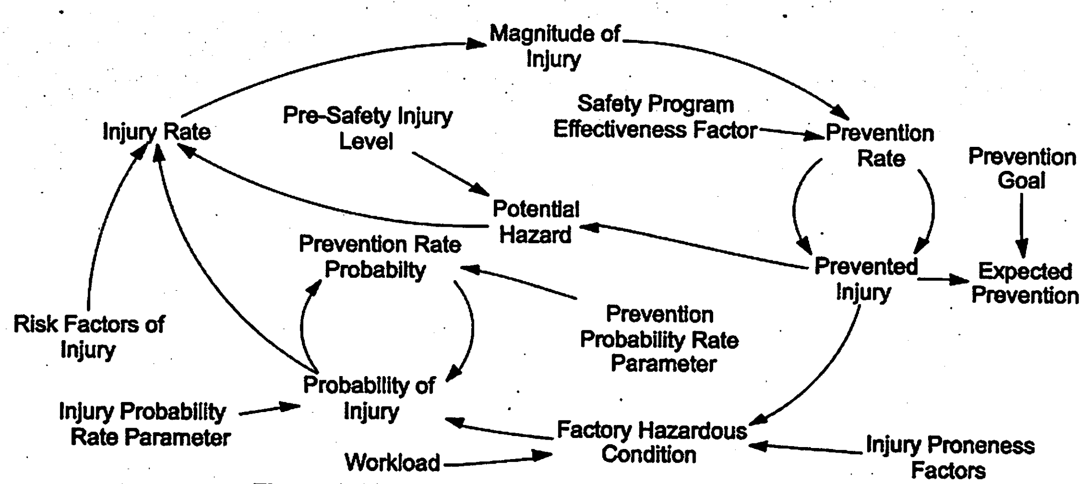

Figure 1: Manufacturing Safety Causal Loop Diagram

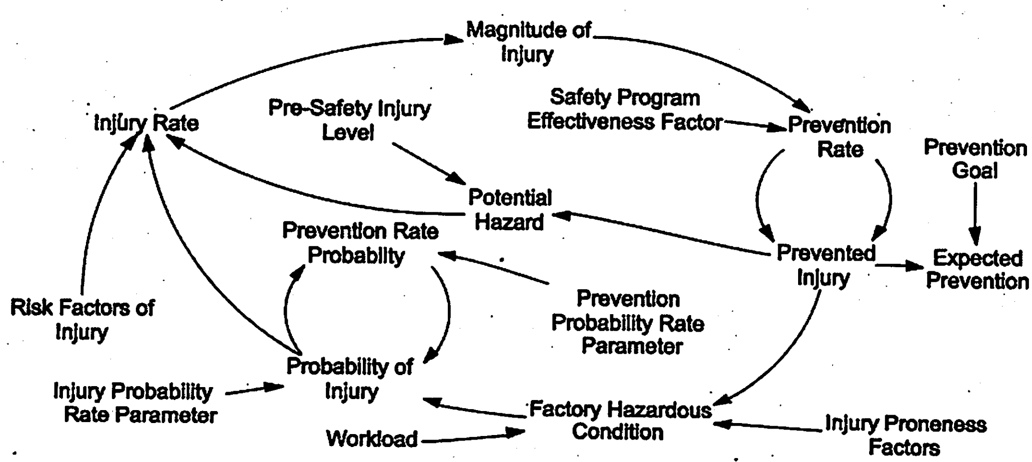

Figure 2: Manufacturing Safety Model Flow Diagram

Note. The rectangles represent a stock of magnitude of injury and prevented injury;the thick arrows with valves and cloud symbols represent flows of injury and prevention rates and the thinner arrows indicate causal influence.

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Table 1: Types of Quantities or Couplings

Quantities	SD Simulation quantities
Rates	*Injuryoccurrence *Preventions
Auxiliary	·Factory hazardous condition ·Injury prevention goal ·Expected prevention ·Prevention probability ·Potential hazard ·Probability of injury
Input parameter	*Injury causing probability *Injury proneness factor *Workload *Prevention probability parameter *Pre-safety injury level
State	*Magnitude of injury ·*Number of prevented injury

2.1 Development of Model Equations

The procedural step for developing model equation as presented by Burns (1975), Charles-Owaba and Adebiyi (2006) and Adebiyi (2006) were employed. These are: identify the polarity of each feedback loop therefore possible behaviour of each loop; write the state equation of each loop with respect to connected rate of flow; write the rate equations in terms of exogenous variables, state variables parameters and auxiliaries, which relate to the particular rate of interest using dimensional consistency; write all the auxiliary equations also using the dimensional consistency principle; substitute for the auxiliary equations of step four in the rate equations of step three; and substitute for rate equations of step three in the state equation of step three.

Applying above steps, using the safety programme flow diagram (Figure 2): the injury rate is the rate of change of magnitude of injury; that is,

$$\alpha_t = \frac{dX_t}{dt} = \phi(X_t, X_p, \rho_1, h) \quad (1)$$

where X_t is the periodic magnitude of injury, X_p is potential hazard, is probability of injury and h is risk factor of injury. And

Injury prevention rate $\gamma_t = \frac{dY_t}{dt} = \phi(E_p, Y_t, \upsilon, \rho_2)$ (2)

Where Y_t is prevented injury, E_p expected prevention, = safety program effectiveness factor and is injury prevention probability.

Considering the injury occurrence rate, the potential hazard, Xp define the difference between the pre-safety injury level Xn and the prevented injury Yt at any instant t of the safety program; that is,

$$X_p = \phi(X_n, Y_t) \quad (3)$$

Thus,

$$X_p = X_n - Y_t \quad (4)$$

The probability of injury depends on the hazardous condition F (variable) and the injury rate occurrence parameter. Duzgun and Einsten (2004) as adopted by Charles-Owaba and Adebiyi (2006) reported that the events occurring to a Poisson process at a time function or variable between the events has exponential distribution. Thus, the exponential distribution by Duzgun and Einsten (2004) and Clemens (2003) was adopted. The probability of accidents is thus given as

$$\rho_1 = 1 - e^{-\lambda t} \quad (5)$$

where P_1 is the probability of experiencing one or more random occurrence of injury at any period of time t, is the reciprocal of potential hazard before safety programme:

$$\lambda = 1/X_p \quad (6)$$

For this work, the variable is the hazardous condition F which is a function of man-hours of worker q, the injury proneness factor f, and the prevented injury Y_t. Therefore, mathematically express as;

$$F = qf - Y_t \quad (7)$$

This implies that, p_1

$$F = f(q, f, Y_t) \quad (8)$$

Substituting F for t in equation (5):

$$\rho_1 = 1 - e^{-\lambda(qf - Y_t)} \quad (9)$$

To measure and transmit information at any rate, some time interval is necessary. Therefore, the risk factor of injury, h is the proportion of injury occurrence per unit time and is the inverse of the time interval of observed injuries.

From the flow diagram, injury rate is

$$\alpha_t = (X_p - X_t)\rho_1 h \quad (10)$$

From equation (4) and (9) $X_p = X_n - Y_t$ and $\rho_1 = 1 - e^{-\lambda(qf-Y_t)}$

Putting equation (4) and (9) into equation (10):

$$\alpha_t = \left[(X_n - Y_t - X_t)(1 - e^{-\lambda(qf-Y_t)})h\right] \quad (11)$$

Substituting equation (11) into equation (1):

$$\alpha_t = \frac{dX_t}{dt} = \left[(X_n - Y_t - X_t)(1 - e^{-\lambda(qf-Y_t)})h\right] \quad (12)$$

Using the principle of integrating factor to solve equation (12), the number of injury is obtained as:

$$X_t = X_n - Y_t\left(1 - e^{-ht(1-e^{-\lambda(qf-Y_t)})}\right) + X_0 e^{-ht(1-e^{-\lambda(qf-Y_t)})} \quad (13)$$

Thus, the periodic magnitude of injury X_t observed was derived in term of plant workload, injury proneness factor, pre-safety injury level, risk factor, injury rate probability parameter and number of prevented injury.

Considering the prevention loop of the model flow diagram, the prevention probability is given as:

$$\rho_2 = 1 - \rho_1 \quad (14)$$

Putting equation (9) into equation

$$\rho_2 = e^{-\lambda(qf-Y_t)} \quad (15)$$

From equation (2), prevention rate can be expressed as:

$$\gamma_t = \left(\frac{E_p - Y_t}{\upsilon}\right)\rho_2 \quad (16)$$

Therefore, substituting equation (15) into equation (16):

$$\frac{dY_t}{dt} = \left[\frac{E_p - Y_t}{\upsilon}\right] e^{-\lambda(qf-Y_t)} \quad (17)$$

Integrating equation (17), the equation for the number of prevented injury is obtained as

$$Y_t = E_p\left[1 - e^{-e^{-\left[\lambda(qf-Y_t)\right]_0^t}}\right] + Y_0 e^{-e^{-\left[\lambda(qf-Y_t)\right]_0^t}} \quad (18)$$

3.0 RESULTS AND DISCUSSION

A mathematical model was formulated from manufacturing safety progamme flow diagram. The model comprises two models dynamically linked systems: the magnitude of injury (X_t) and prevented injury (Y_t), such that the magnitude of injury depends on the factory hazardous conditions, risk factor of injury, potential hazard and probability of injury. Also, the number of prevented injuries depends on the expected prevention, effectiveness of safety programme activities and injury prevention probability. The model has three (3) driving variables; namely: safety programme effectiveness factor, pre-safety injury level Xn and risk factor of injury (h). The observed data and the standard run of the model with input parameter estimates for 12 months period are depicted by Figures 3 and 4, which show that there is good agreement between predicted and observed injuries. At the beginning of the programme, these parameters encounter a significant increase due to high pre-safety injury level.

The exponential decay in magnitude of injuries and corresponding growth in number of predicted preventions as shown in Figures 3 and 4 respectively, are expected. These are expected because an effective safety program reduces hazardous conditions, improves safety and performance.

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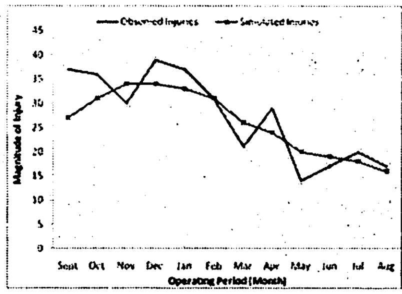

Figure 3: Comparison of Variations of Observed and Simulated Injuries Vs Operating Period

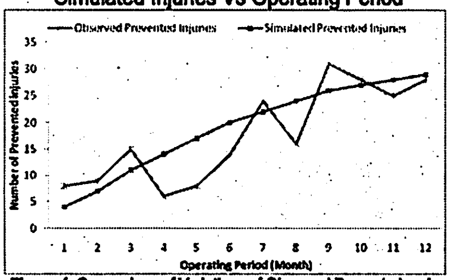

Figure 4: Comparison of Variations of Observed Prevented and Simulated Prevented Injuries Vs Operating Period

4.0 CONCLUSIONS

A system dynamic approach that captures a transition to world-class safety performance was conceptualized and used to develop a model for managing magnitude and risk factors of injuries in a manufacturing industry. An accident investigation register was administered to capture needed data to estimate the model parameters and identify risk factors of injuries. The principles of system dynamics were used to identify the relevant risk management safety-related components of a manufacturing industry. Applying the concept of causality analysis, a causal loop diagram indicating how injury prevention activities may eliminate hazardous condition was delineated and conceptualized into a flow diagram from which a set of differential equations were formulated. A comparison of predicted and observed injuries depict that a satisfactory agreement was found between the observed data and the model predictions.

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