

AN INVESTIGATION ON THE PROBLEM OF THINNING IN FINGERPRINT PROCESSING

ISAAC O. AVAZI OMEIZA

Department of Electrical and Electronic Engineering, University of Abuja, P.M.B. 117, Abuja,
Nigeria; isaacavazi@yahoo.com

Received: 04 October 2011

Accepted: 10 April 2012

Revised: 08 April 2012

Published: 19 December 2012

ABSTRACT

A high-integrity thinning procedure for binarised fingerprints is proposed in this paper. Several authors and software developers have approached the thinning problems in fingerprint-processing differently. Their approach produced in most cases, fingerprint skeletons with low reliability and thus require additional minutiae-pruning stage to discard the erroneous minutiae in the obtained skeletons. The work involves a careful blending of some already existing algorithms to achieve optimal performance in thinning binarised fingerprint images. The algorithms considered are as follows. The “Zhang and Suen” parallel algorithm for thinning digital patterns, the improved parallel thinning algorithm by Holt and company and template-based thinning algorithm by Stentiford and Mortimer. The idea of combining these stand-alone algorithms to improve the quality of obtained objects skeleton in general image processing was first suggested in a text by Parker in 1998. However, his work does not specifically address the fingerprint problem. This work has examined and proves the plausibility of this thinning approach in the particular case of fingerprint application domain. The thinning procedure obtained satisfactory skeletons for fingerprint applications.

Key words: Fingerprint, Thinning, Minutiae, Digital-pattern, Skeleton

1. INTRODUCTION

In the early twentieth century, fingerprint recognition became formally accepted as a valid identification method for humans and became a standard routine in forensics (Maltoni, Maio, Jain, Prabhakar, 2003). The fingerprints of individuals are said to be unique, even the fingerprints among twins are not the same (Bhanu and Tan, 2004). Individuality of fingerprints is captured by the local ridge structures (minute details referred to as minutiae) and their spatial distributions (Lee, and Gaensleen, 1991; Jain, Hong, Pankanti and Bolle, 1997a). Therefore, automatic fingerprint verification is usually achieved by minute detail matching instead of a pixel-wise matching or a ridge pattern matching of fingerprint images.

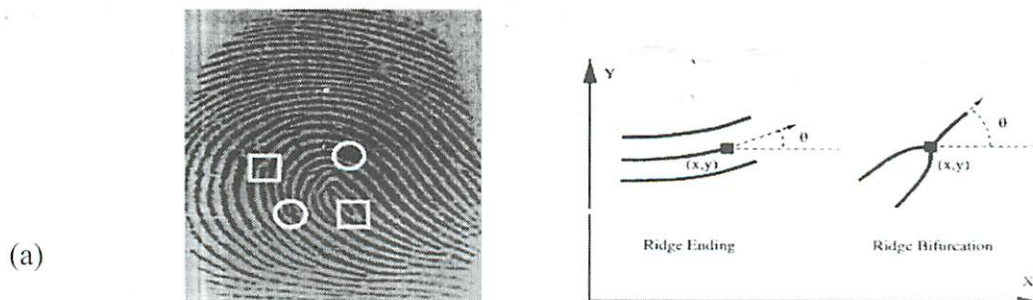


Figure 1: Examples of Minutiae (Hong, Wan, Jain,1998). (a) Minutiae Overlaid on a Fingerprint Image; Ridge Endings (□) And Bifurcation (○), (b) A Minutia Can Be Characterized By Its Position And Its Orientation.

Typically, automatic fingerprint identification and authentication systems rely on a comparison based on only two most prominent structures: ridge endings and ridge bifurcations (Jain, Hong, Pankanti and Bolle, 1997a). Fig. 1 shows examples of ridge endings and ridge bifurcations. Most of the existing automatic fingerprint verification systems are based on minutiae features (ridge bifurcation and ending ; see Fig. 1.). Such systems first detect the minutiae in a finger print image and then match the input minutiae set with the stored template (Halici, Jain, Erol, 1999).

Several approaches to automatic minutiae extraction have been proposed: although different from one another, most of these methods transform grey-scale finger-print images into binary images which clearly distinguishes the ridges from the valleys (ridge / valley segmentation). The binarised fingerprint images obtained are then subjected to a thinning process which allows for the ridge-line thickness to be reduced to one pixel.

Finally, a simple image scan allows the detection of pixels which corresponds to minutiae. In a single pixel thick skeleton, if a pixel in the thinned image has more than two neighbours, then the minutia is classified as a bifurcation , and if a pixel has only one neighbour, then the minutia is classified as an ending (Fig. 2). Hence the matching algorithm is designed to simply match minutiae endings only with minutiae endings and minutiae bifurcations only with minutiae bifurcations (Prabhakar, Jain, Pankanti, 2000). As such a potential minutia has the following three attributes: the $\{x, y\}$ position and the direction of the ridge on which it resides (θ) as shown in Fig. 1b. The minutiae points can be detected from the thinned image (Metre, 1993).

As obvious, from the foregoing the success of the matching algorithm in the above described procedure depends heavily on the success of the thinning stage and the quality of the fingerprint skeletons obtained.

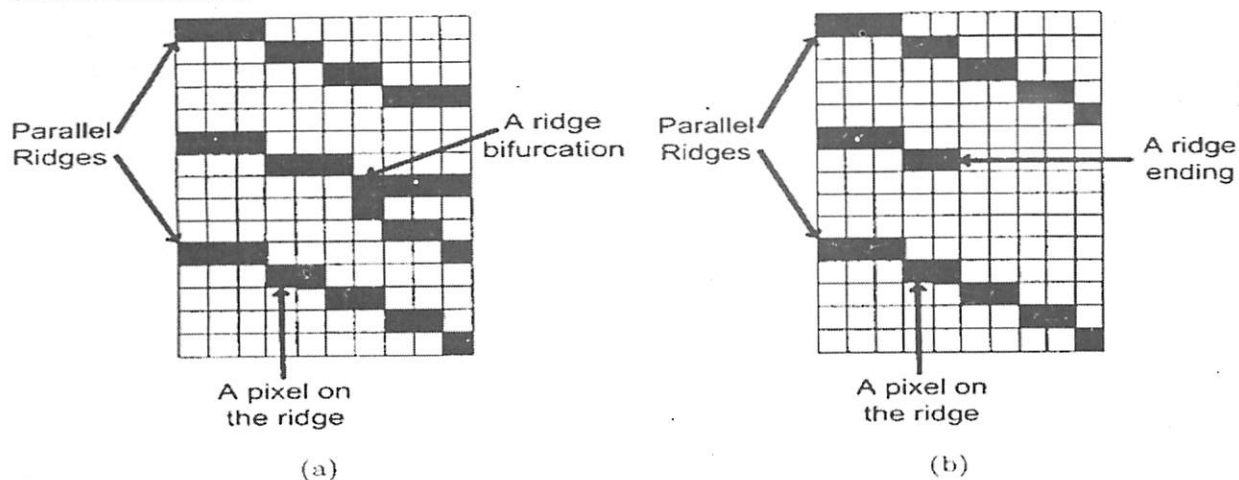


Figure 2: Examples of a Ridge Bifurcation and a Ridge Ending in a Thinned Fingerprint Image (Prabhakar, Jain, Pankanti, 2000)

2. THINNING NOTATIONS AND DEFINITIONS

According to Parker (1997), thinning can be defined as the act of identifying those pixels belonging to an object that are essential for communicating the objects shape or simply the act of identifying the skeleton. A large number of thinning approaches are available in the literature (Lam, Lee and Suen, 1992), due to the central role of these processing step in many applications: character recognition, document analysis, map and draws vectorisation etc. Of the several papers on the subject of thinning in print, the vast majority are concerned with the implementation of a variation on an existing method, where the innovation is usually related to the performance of the algorithm. Many of the

more recent thinning algorithms were designed to achieve an improvement on an existing method while often leaving the basic principles alone (Parker. (1997).

For fingerprint processing, Hung (1993) used the algorithm by Arcelli and Baja (1984); Ratha et al (1995) adapted a technique available in Sakai, Nagao and Matsushima (1972); Metre (1993) employed the parallel algorithm described in Tamura (1978). Finally, Coetzee and Botha (1993) used the Baruch's method in Baruch (1988). Unfortunately, the ridge boundary aberrations in the skeletons obtained by the aforementioned authors have an adverse impact on the skeleton, resulting in "hairy" growth (spikes) which lead to spurious ridge bifurcations and endings (false minutiae), hence the skeleton needed to be smoothed before minutiae points can be extracted (Ratha, Chen and Jain, 1995). Simple structural rules may be used to detect many of the false minutiae that usually affect thinned binary fingerprint images. Xiao and Raafat (1991), identified the most common false minutiae and introduced an ad hoc approach to remove them (Maltoni, Maio, Jain, 2003 and Prabhakar, 2003). This minutiae pruning stage can be avoided or the use minimized if the thinning method had produced high quality skeletons, hence the need for this work.

2.1 Concepts of Connectivity and Adjacency in Digital Pictures

A pixel in a digital picture is spatially close to several other pixels. If we associate each pixel with a lattice point (i.e. a point with integer coordinates) in the plane as in Figs.3a and 3b ; two lattice points in the plane are said to be 8-adjacent if they are distinct and each coordinate of one differs from the corresponding coordinate of the other by at most 1, two lattice points are 4-adjacent if they are 8-adjacent and differ in at most one of their coordinates (Kong and Rosenfield, 1989). In the context of bi-level or binary pictures a lattice point associated with a pixel that has value 1 is called a black point; a lattice point associated with value 0 is called a white point. In digital topology paradoxes are avoided by using different adjacency relations for black and white points (Kong and Rosenfield, 1989). So in practice, 8-adjacency is used for black points and 4-adjacency for the white, or vice versa.

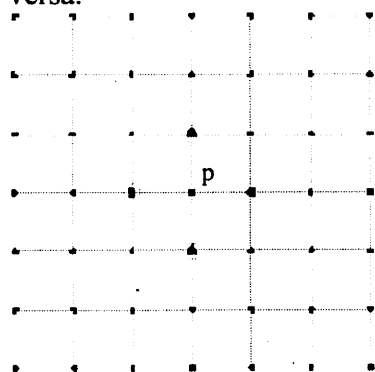


Fig. 3a: The 4-neighbours of a point p

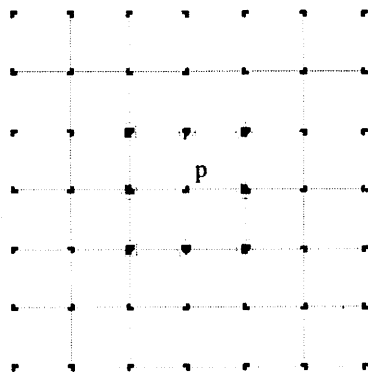


Fig. 3b: The 8-neighbours of a point p

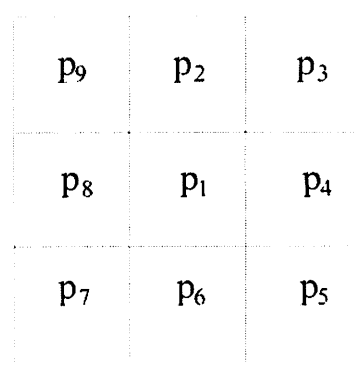


Fig.3c Pixel neighbourhood definitions

Definition 2.1 A (conventional) digital picture is quadruple (V, m, n, B) , where $V = \mathbb{Z}^2$ or $V = \mathbb{Z}^3$, $B \subseteq V$, and where $(m, n) = (4,8)$ or $(8,4)$ if $V = \mathbb{Z}^2$

The digital picture $P = (V, m, n, B)$ is called two-dimensional if $V = \mathbb{Z}^2$. The elements of V are called the points of the digital picture. The points in B are called the black points of the picture; the points in $V - B$ are called the white points of the picture. Usually B is a finite set, if so P is said to be finite.

Two black points in a digital picture (V, m, n, B) are said to be adjacent if they are m-adjacent and two white points or a black point and a white point is said to be adjacent if they are n-adjacent. A

digital picture (I, m, n, B) is also called an (m, n) digital picture. A black point is said to be isolated if it is not adjacent to any other black point. A black point is called a border point if it is adjacent to one or more white points; otherwise it is called an interior point.

2.2 Two Dimensional Thinning

Image thinning is a common pre-processing operation in pattern recognition. Its goal is to reduce the set of black points to a “skeleton” in a “topology-preserving” way.

Criterion 2.1

Let $P = (Z^2, m, n, B)$ be a two dimensional digital picture, then deletion of the points in a subset D of B preserves topology if and only if (Kong and Rosenfield, 1989)

1. each black component of P contains exactly one black component of P' , and
2. each white component of P' contains exactly one component of P , where P' is the digital picture $(Z^2, m, n, B-D)$

It is usual to thin a binary image as an $(8, 4)$ digital picture rather than as a $(4, 8)$ digital picture, because thinner skeletons are then obtained. For $(8, 4)$ digital pictures criterion 2.1 is equivalent to Hilditch's concept of connectivity preservation (Hilditch, 1969). A black point in a two-dimensional digital picture is called a simple point if its deletion preserves topology in the sense of criterion 2.1, thus p is a simple point of an (m, n) digital picture if and only if the number of black components and the number of white components stay the same when p is deleted. In particular, only isolated border points can be simple. From the works of Rosenfield (1970), Mylopoulos and Pavlidis (1971), Kong and Rosenfield deduced the following theorem.

Theorem 2.1 (Theorem 4.1 in Kong and Rosenfield, 1989)

Let P be a non-isolated border point in an $(8, 4)$ or $(4, 8)$ digital picture, Let B be the black point set of the digital picture and $B' = B - \{P\}$, while $N(P)$ represents the 3×3 neighbourhood of P . Then the following are equivalent

1. P is a simple point.
2. P is adjacent to just one component of $N(P) \cap B'$.
3. P is adjacent to just one component of $N(P) - B$.

This theorem shows that only an examination of the 3×3 neighbourhood $N(P)$ of P is sufficient to determine whether or not a point P is simple and as such can be deleted.

An alternative approach to the notion of a simple point was provided by Hilditch (1969); where a crossing number $\chi(p)$ was defined to be the number of times one crosses over from a white point to a black point when all the 8-neighbours of p are visited in cyclic order, starting at a 4-neighbour of p and returning to the starting point. In a digital picture $(Z^2, 8, 4, B)$ the Hilditch crossing number $\chi(p)$ is equal to the number of components of $B \cap N(P) - \{P\}$, except that if P is an interior point then $\chi(p) = 0$. It also follows from theorem 2 that in an $(8, 4)$ digital picture a black point P is simple if and only if $\chi(p) = 1$. Hilditch's crossing number $\chi(p)$ is equal to the connectivity number $N(P)$ proposed by Yokoi et al. (1973), defined as follows.

$$N(P) = \sum_{ks} N_k - (N_k \cdot N_{k+1} \cdot N_{k+2}) \tag{1}$$

Where N_k is the colour value of one of the eight neighbours of the pixel P and $S = \{1, 3, 5, 7\}$. N_1 is the color value of the pixel to the right of the central pixel P , with the neighbouring pixels numbered in counter-clockwise order around the center. The value of N_k is one (1) if the pixel is white (background) and zero if black (object). The center pixel is N_0 , and $N_k = N_{k-8}$ if $k > 8$.

3. THE INVESTIGATED THINNING ALGORITHMS

The major part of the proposed composite thinning algorithm is the Zhang-Suen parallel algorithm for thinning binary pictures (Zhang and Suen, 1984). It was selected for the purpose of good speed. In parallel picture processing, the new value given to a point at the n th iteration depends on its own value as well as those of its eight neighbours at the $(n-1)$ th iteration, so that all picture points can be processed simultaneously.

3.1 The Zhang and Suen Algorithm

In this section the algorithm is presented. In the following discussion, it is assumed that objects points have value 1 and background points have value 0. The method consists of two basic steps applied to border pixels of the given objects in successive passes. A border pixel is any pixel with value 1 and having at least one 8-neighbour valued 0.

With reference to the 8-neighbourhood definition showed in Fig.3c,

The first step flags a border pixel p for deletion if the following conditions are satisfied (Gonzalez and Woods, 2002).

- (a) $2 \leq N(P_i) \leq 6$
- (b) $C(P_i) \leq 1$
- (c) $P_2 * P_4 * P_6 = 0$
- (d) $P_4 * P_6 * P_8 = 0$

where $N(P_i)$ is the number of non zero neighbours of P_i , and $C(P_i)$ is the number of 0 to 1 transitions in the ordered sequence $P_2, P_4, \dots, P_8, P_6$.

In the second step, conditions (a) and (b) remain the same, but conditions (c) and (d) are replaced with

- (c') $P_2 * P_4 * P_8 = 0$
- (d') $P_2 * P_6 * P_8 = 0$

Step 1 is applied to every border pixel in a binary region under consideration. If one or more of the conditions (a) through (d) is not met, the pixel in question is left as an object pixel, if all the conditions are satisfied; the pixel is marked for deletion. However, that point is not deleted from the foreground until all border points in the image have been similarly examined. This prevents altering the structure of the global data during execution of the algorithm and ensures that a new value given to a point at the n th iteration depends on its own value as well as those of its 8-neighbours at the $(n-1)$ th iteration, only.

In effect, one iteration of the thinning algorithm consists of: (i) applying step 1 to mark border points for deletion; (ii) deleting the flagged points; (iii) applying step 2 to flag the remaining border points for deletion; and (iv) deleting the flagged points. This basic procedure is applied iteratively until a pass exists in which no pixel is marked for deletion, then the algorithm terminates, yielding the skeleton of the region.

3.2 Stair-Case Elimination

An improvement of the above algorithm was suggested by Holt et al (1987), that is faster and does not involve sub-iterations. First, the two sub-iterations are written as logical expressions which use 3x3 neighbourhood about the pixel concerned. The first sub-iteration can be written as:

$$vC \text{ And } (\sim\text{edge } C \text{ Or } (vE \text{ And } vS \text{ And } (vN \text{ Or } W))) \quad (2)$$

which is the condition under which the center pixel C survives the first sub-iterations. The v function gives the value of the pixel (1 = true for an object pixel, 0 = false for background), and the edge

function is true if C is on the edge of the object—this corresponds to having between two and six neighbours and connectivity number = 1. The letters E, S, N and W correspond to pixels in a particular direction from the center pixel C; E means east, S means south etc.

The second sub-iteration is written as :

$$vC \text{ And } (\sim\text{edge C Or } (vW \text{ And } vN \text{ And } (vS \text{ Or } vE))) \quad (3)$$

Holt et al further combined the two expressions for survival (eqns. 1 and 2) with a connectedness-preserving condition (needed for parallel execution) and came up with the following single expression for pixel survival.

$$vC \text{ And } (\sim\text{edge C Or } (\text{edge E And } vN \text{ And } vS) \text{ Or } (\text{edge S And } vW \text{ And } vE) \text{ Or } (\text{edge E And } \text{edge SE And } \text{edge S})) \quad (4)$$

Unfortunately, when thinning has been completed with the Zhang-Suen procedure, unnecessary skeletal elements still remains which cannot be removed using the algorithm (Holt et al,1987 and Parker,1997). For example, with the pattern

```

0 1 0 0
0 1 1 0
0 0 1 0
    
```

either element of the middle line may be removed without changing the connectedness of the overall picture. Basically, the central pixel in one of the following windows can be deleted:

```

0 1 x      x 1 0      0 x x      x x 0
1 1 x      x 1 1      x 1 1      1 1 x
x x 0      0 x x      x 1 0      0 1 x
    
```

This means that a foreground pixel can be removed or deleted if it has a window of the forms enumerated above where the value of the unspecified neighbours may be either 0 or 1. To ensure that a hole is not created, the condition is added that one of the unknown side neighbours be 0.

The condition for a foreground pixel survival in the stair case elimination procedure is thus given by

$$vC \text{ And } \sim(vN \text{ And } ((vE \text{ And } \sim vNE \text{ And } \sim vSW \text{ And } (\sim vW \text{ Or } \sim vS)) \text{ Or } (vW \text{ And } \sim vNW \text{ And } \sim vSE \text{ And } (\sim vE \text{ Or } \sim vS)))) \quad (5)$$

After elements have been removed in a pass using the northward bias scheme above, it is necessary to repeat the operation using a southward bias. This is done by using the same expression but with north and south interchanged.

3.3 Prevention of Line Fuzz or Spurious Projections

While the above algorithms are robust enough to handle the issues of connectivity and speed of computation, they give little attention to the problems of line fuzz or hairs (spurious projections) in the thinned image objects. The latter is a very important issue in fingerprints skeletons as spurious projections will lead to the creation of false minutiae (ridge ends and bifurcations) where they do not actually exist in the original image. A thinning algorithm that provides remedy against these artifacts

is the algorithm by Stentiford and Mortimer (1983), though its primary focus was the thinning of hand printed characters for optical character recognition. The basic algorithm can be summarized as follows (Stentiford and Mortimer(1983).

- Step 1: Scan matrix M across character according to Table 1 and identify next fit position.
- Step 2: If the central element at a fit is not an end point and has connectivity value one, then mark it for deletion.
- Step 3: Repeat steps 1 and 2 for all fit positions
- Step 4: Repeat steps 1 to 3 for each of M_2, M_3, M_4
- Step 5: Delete all marked elements
- Step 6: If one or more elements are deleted in step 4 then return to step 1.
- Step 7: Exit.

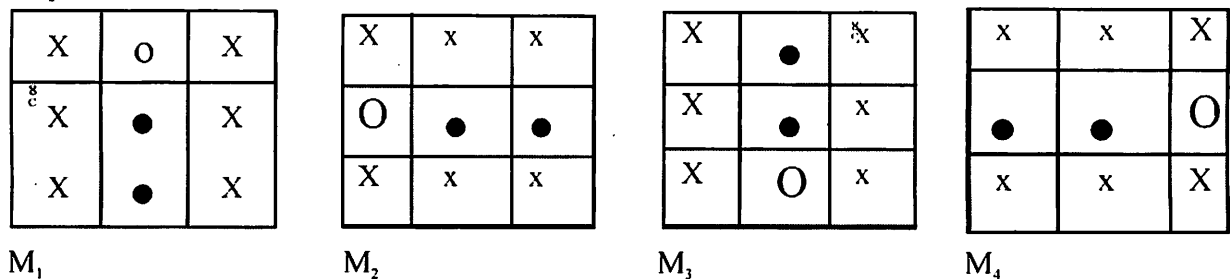


Figure 4: Thinning Matrices (Stentiford and Mortimer, 1983).

The results of such a thinning process are sensitive both to the order of application of the M_i s and also the direction of scan. An ordering which minimizes spurious tail production is given in table 1(Stentiford and Mortimer, 1983). Stentiford and Mortimer suggested a preprocessing stage to minimize thinning artefacts. A smoothing step before thinning is suggested to remove small irregularities in the binary image which may cause line fuzz or hairs. Basically, a pass is made over all pixels, deleting those having two or fewer black neighbours and having a connectivity number of 1.

Table 1: Matrix Ordering and Scan Directions

Order of Application	Direction of Single Scan Line	Direction of Successive Scan Lines
M_1	Left to Right	Downwards
M_2	Upwards	Left to Right
M_3	Right to Left	Upwards
M_4	Downwards	Right to Left

Another preprocessing step is suggested for hole removal. Hole removal is done by raster scanning the optical character to be thinned with six patterns of bits. Three of these (H_i) will fit all holes containing one or two elements and the remaining three (I_i) will fit the same size of hole but only if the hole is embedded at least two elements deep in a limb. The detected holes are removed by either merging the hole with the adjacent white areas by the removal of black elements or filling it with additional black elements. A final preprocessing stage (known as acute angle emphasis) involves the detection of upward and downward acute angles between limbs by again scanning the character to be

thinned with patterns of bits. Five of these masks denoted D_i s are designed to fit the sharpest forms of downward pointing acute angle and a reflected set (U_i s) is designed to fit upward pointing acute angles. After a fit is found the central black element is deleted and the process is repeated two times more. The second and third iterations are only carried out if the preceding iteration effected a deletion (Stentiford and Mortimer, 1983).

4. EXPERIMENTAL RESULTS

For minutiae-based fingerprint matching a thinning step is required before minutiae extraction in order to simplify or reduce the complexity of the latter task. However, the quality of the obtained skeleton will grossly affect the success of the matching stage. Several authors have had to utilize some intuitive heuristics to eliminate false minutiae (Jain et al, 1997a; Jain et al, 1997b; Maio and Maltoni, 1997; Ratha et al, 1996). In simple approaches they are eliminated using some distance criteria, e.g. minutiae which are too close to each other are discarded (Hung, 1993; Xiao and Raafat, 1991). The method for minutiae pruning is usually ad-hoc; as such, the overall reliability of the minutiae extraction stage would be greatly increased, if by careful selection of a suitable thinning or skeletonizing procedure, the production of false minutiae (ridge ends and bifurcations) is minimized.

The experiment involves the digitization of ink-dabbed fingerprints with a flat-bed document scanner. Binarising the grey scale fingerprints, Coding in C programming language the Stentiford and Mortimers' (1983) thinning algorithm, the Zhang and Suen (1984) thinning algorithm and , and the Holt's et al (1987) improved parallel thinning algorithm. The codes were tested with binarised fingerprints as input data and evaluating the output thinned images in the light of the existence or generation of false minutiae points and the effective localization of true minutiae positions in the thinned fingerprints. Each of the three algorithms were tested in-turn as stand-alone algorithms and a combination of the three in different orders. The need for the inclusion of a topology-preserving connectivity measure in thinning of binary pictures as it affects fingerprints was also examined.

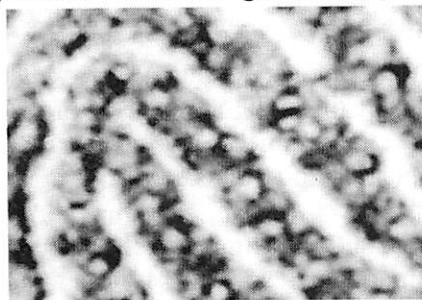


Fig. 5 Original Fingerprint
 Fingerprint of Fig. 7.

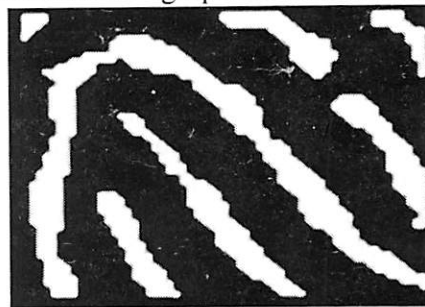


Fig.6 Binarised Version of the Original

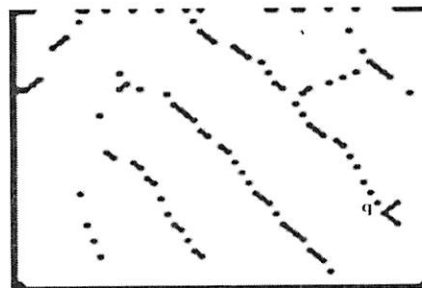


Fig.7a. Thinning without topology preservation

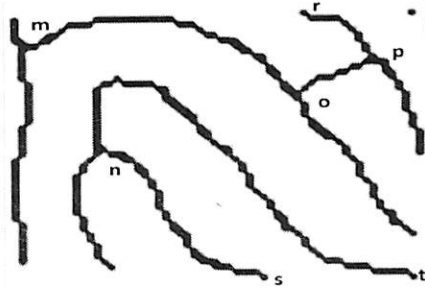


Fig. 7b. Thinning using only the Zhang-Suen's algorithm

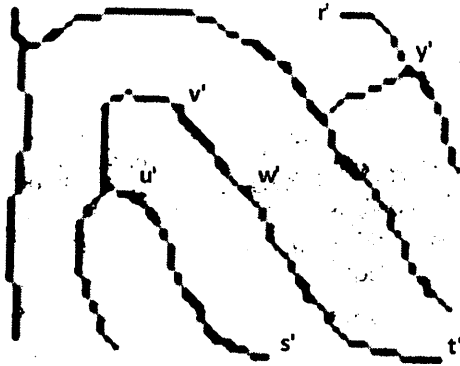


Fig. 7c. Thinning, using both Stentiford's smoothing procedure and Zhang-Suen's algorithm,

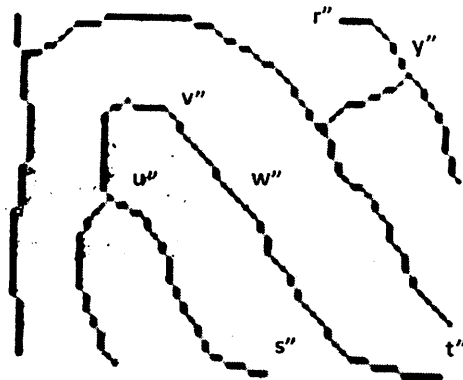


Fig. 7d. Thinning with a combination of Stentiford's smoothing procedure, Zhang-Suen's algorithm and Holt et al's staircase removal scheme.

Figure 5 shows a fingerprint image binarised as in Figure 6. Figure 7a shows an attempt to thin the binary image of Figure 6 without paying attention to the concept of topology preservation. From Figure 7a, it is observed that the skeleton is disjointed and possesses several false ridge ends while a false bifurcation point is created at point q. Figure 7b is a thinning attempt with Zhang and Suen thinning algorithm which incorporates Yokoi's connectivity measure. This result shows that the incorporation of a topology-preserving connectivity measure results in a good thinned image with bifurcation points m, n, o, p, duly represented, though not well localized. Figure 7c uses the Stentiford's smoothing procedure for preprocessing before using the previous Zhang & Suen method. Comparing Figure 7b with Figure 7c, the inclusion of a Stentiford pre-processing step reduced the "limbs" at points r, s and t to their new positions r', s', t' and eliminated the isolated minutia at point L.

The result of Figure 7c shows substantial amount of stair casing at point u', v', w', y'. This makes the exact bifurcation points around points u' and y' for instance difficult to resolve. Moreover, one of the ridge pixels around the point w' for example, has three eight-connected neighbours when it is not a bifurcation point. This is contrary to the requirement that ridge ends, ordinary ridge pixels and bifurcation-point pixels should have only one, only two and at least three eight-connected neighbours respectively (Prabhakar, Jain and Pankanti, 2000). In Figure 7d these ambiguities were resolved through the inclusion of the stair case elimination procedure of Holt and Company as a post-processing step after implementing the Zhang-Suen thinning procedure. This is evident when points u', w' and y' in Fig. 7c are compared with their equivalent points u'', w'' and y'' in Figure 7d.

5. CONCLUSION

In this paper, an investigation has been carried out on the 'thinning problem' of fingerprint processing. Experiment shows that a principal requirement in the successful thinning of binarised fingerprints is that the thinning procedure should include a topology-preserving connectivity measure like the Hilditch's crossing number or the Yokoi's connectivity measure. In particular, the well-known Zhang-Suen thinning scheme needs some refinement if it is to produce suitable skeletons for fingerprint images that will not possess several false minutiae and as such call for an additional minutiae pruning steps which at present are based on ad hoc methods. From the experiments, it can be concluded that the Zhang-Suen method can be used along with the pre-smoothing methods of the Stentiford's algorithm for better localisation of the ridge ends. Also, the inclusion of the stair case elimination procedure of Holt & company will effectively resolve

ambiguities in the true positions of the bifurcations. On the whole a careful blending of suitable aspects of the Stentiford's, Zhang-Suen and Holts thinning procedures will result in an efficient thinning scheme for fingerprint processing. Judging from our experiments the need for the stair case elimination procedure seems to be more paramount when compared with the requirement for stentiford's pre-smoothing procedure; in the specific case of thinning or generating high-integrity skeletons for binarised-fingerprints.

REFERENCES

- Arcelli, C. and Baja, G. S. D. (1984). A Width Independent Fast Thinning Algorithm. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 4 (7): 463-474.
- Baruch, O. (1988). Line Thinning by Line Following. *Pattern Recognition Letters*, 8 (4): 271-276.
- Bhanu, B. and Tan, X. (2004). *Computational Algorithms for Fingerprint Recognition*, Kluwer Academic Publishers, U.S.A.
- Coetzee, L. and Botha, E. C. (1993). Fingerprint Recognition in Low Quality Images. *Pattern Recognition*, 26 (10): 1441-1460.
- Gonzalez, R.C and Woods E.R. (2000). *Digital Image Processing*, Pearson Education Inc, India.
- Halici, U., Jain, L. C., Erol, A. (1999). Introduction to Fingerprint Recognition. In *Intelligent Biometric Techniques in Fingerprint and Face Recognition*. Jain, L. C., Halici, U., Hayashi, I. and Lee, S. B. (Eds), CRC Press, Boca Raton, FL, pp. 3-34.
- Holt, C. M.; Stewart, A.; Clint, M. and Perrott, R. H. (1987). An Improved Parallel Thinning Algorithm. *Communications of the ACM*, 30 (2): 156-160.
- Hong, L.; Wan, Y. and Jain, A. K. (1998) Fingerprint Enhancement Algorithms and Performance Evaluation. *IEEE Trans on Pattern, Analysis and Machine Intelligence*, 20(8):777-789.
- Hung, D. C. D. (1993). Enhancement and Feature Purification of Fingerprint Images. *Pattern Recognition*, 26 (11): 1661-1671.
- Jain, A. K.; Hong L.; Pankanti S. and Bolle, R. (1997a), An Identity Authentication System Using Fingerprints. *Proceedings of the IEEE*, 85 (9).
- Jain, A. K.; Hong, L. and Bolle, R. (1997b). On-Line Fingerprint Verification. *Trans on Pattern Analysis and Machine Intelligence*, 19 (4): 302-314.
- Kong, T. Y. and Rosenfield, A. (1989). Digital Topology: Introduction and Survey, *Computer Vision, Graphics. and Image Processing*, 48: 357-393.
- Lam, L.; Lee, S. W. and Suen, C. Y. (1992). Thinning Methodologies: A comprehensive Survey. *IEEE*, 14 (9): 869-885.
- Lee, H. C. and Gaensleen, R. E. (1991). *Advances in Fingerprint Technology*. Elsevier, New York.

- Maltoni D., Maio D., Jain A.K., Prabhakar S. (2003), Handbook of Fingerprint Recognition. Springer Science + Business Media, Inc, U.S.A.
- Maio, D. and Maltoni, D. (1997). Direct Grey-Scale Minutiae Detection in Fingerprints. IEEE Transactions on Pattern Analysis and Machine Intelligence, 19 (1).
- Mehetre, B. M. (1993), Fingerprint Image Analysis for Automatic Identification, Machine Vision and Applications.
- Mylopoulos, J. and Pavlidis, T. (1971). On the Topological Properties of Quantised Spaces I: The Notion of Dimension, J. Assoc. Comput. Mach., 18: 239-246.
- Parker, J. R. (1997). Algorithms for Image Processing and Computer Vision. John Wiley & Sons, USA.
- Prabhakar, S.; Jain, A. K. and Pankanti, S. (2000). Learning Fingerprint Minutiae Location and Type. Proceedings of the 15th International Conference on Pattern Recognition, Barcelona.
- Ratha, N. K.; Chen, S. Y. and Jain, A. K. (1995). Adaptive Flow Orientation-Based Feature Extraction in Fingerprint Images. Pattern Recognition, 28 (11): 1657-1672.
- Ratha, N. K.; Karu, K.; Shaouyun, C. and Jain, A. K. (1996). A Real-Time Matching System for Large Fingerprint Databases. IEEE Transaction on Pattern Analysis and Machine Intelligence, 18 (8): 799-813.
- Rosenfield, A. (1970). Connectivity in Digital Pictures, ACM, 17 (1).
- Ronse, C. (1986). A Topological Characterization of Thinning. Theoret. Comput. Sci. 43: 31-41.
- Sakai, T., Nagao, M., Matsushima, H. (1972). Extraction of Invariant Picture Sub-structures by Computer. Computer Graphics and Image Processing, 1 (1): 81-96.
- Stefanelli, R. (1986). A Comment on an Investigation into the Skeletonization Approach of Hilditch. Pattern Recognition, 19 (1): 13-14.
- Stentiford, F. W. M. and Mortiner, G. R. (1983). Some New Heuristics for Thinning Binary Handprinted Characters for OCR. IEEE Transactions on Systems, Man and Cybernetics, 13 (1): 81-84.
- Tamura, H. (1978). A Comparison of Line Thinning Algorithms from Digital Topology Viewpoints. In: Proceedings of the 4th International Conference on Pattern Recognition: 715-719.
- Xiao, Q. and Raafat, H. (1991). Fingerprint Image Post-Processing: A Combined Statistical and Structural Approach. Pattern Recognition, 24 (10): 985-992.
- Yokoi, S.; Toriwaki, J. and Fukumura, T. (1973). Topological Properties in Digitized Binary Pictures. Systems Computer Controls, 4: 32-39.
- Zhang, T. Y. and Suen, C. Y. (1984). A Fast Parallel Algorithm for Thinning Digital Patterns. Communications of the ACM, 27 (3): 236-239.