A Method for Solving the Voltage and Torque Equations of the Split-Phase Induction Machines

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ABSTRACT: Single phase induction machines have been the subject of many researches in recent times. The voltage and torque equations which describe the dynamic characteristics of these machines have been quoted in many papers, including the papers that present the simulation results of these model equations. The way and manner in which these equations are solved is not common in literature. This paper presents a detailed procedure of how these equations are to be solved with respect to the split-phase induction machine which is one of the different types of the single phase induction machines available in the market. In addition, these equations have been used to simulate the start-up response of the split phase induction motor on no-load. The free acceleration characteristics of the motor voltages, currents and electromagnetic torque have been plotted and discussed. The simulation results presented include the instantaneous torque-speed characteristics of the Split phase Induction machine. A block diagram of the method for the solution of the machine equations has also been presented.

KEYWORDS: Voltage equations, dynamic characteristics, split-phase, induction motor, torque

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NOMENCLATURE

V_s	Supply Voltage.
v_{qs}	Voltage applied across the running winding
v'_{ds}	Voltage applied across the starting winging referred to running winding
i_{qs}	Current in the running winding
i'_{ds}	Current in the starting winding referred to running winding
λ_{qs}	Flux linkage of the running winding
λ'_{ds}	Flux linkage of the starting winding referred to running winding
v'_{qr}	Voltage applied across the q – axis rotor winding referred to running winding
v'_{dr}	Voltage applied across the d – axis rotor winding referred to running winding
i'_{qr}	Current in the q – axis rotor winding referred to the running winding
i'_{dr}	Current in the d – axis rotor winding referred to running winding
λ'_{qr}	Flux linkage of the q – axis rotor winding referred to the running winding
λ'_{dr}	Flux linkage of the d – axis rotor winding referred to running winding
r_{qs}	Resistance of the running winding
r'_{ds}	Resistance of the starting winding referred to running winding
r'_r	Resistance of the rotor windings referred to running winding
w_r	Rotor speed
L_{mq}	Mutual inductance between the q – axis stator and rotor windings
L'_{md}	Mutual inductance between the d – axis stator and rotor windings referred to running winding
L_{lqs}	Leakage inductance of the running winding
L'_{lds}	Leakage inductance of the starting winding referred to running winding
L'_{lr}	Leakage inductance of the rotor winding referred to the running winding
T_e	Electromagnetic Torque
P	Number of poles on the machine
J	Total inertia of the machine
T_L	Load torque
p ‘	Derivative operator given as $\frac{d}{dt}$

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I. Introduction

Single phase induction machines (SPIMs) are the most popular machines the world over, (Fating et al, 2008), (Latt and Win, 2009). These motors are widely used in residential and commercial applications. These applications include pumps, compressors for refrigeration and air-conditioners, dish washers, vacuum cleaners, washing machines, clippers, hair dryers, fans, machine tools, and so on (Caruso et al, 2012). The split-phase induction machine is one of the several types of the single phase induction machines. Other types include the shaded pole, capacitor-start, permanent split capacitor machine and the capacitor-start, capacitor-run single phase induction machines.

The use of computer simulations to investigate the dynamic behavior of electrical machines has been demonstrated to be of practical importance to the engineer. The digital computer has been a powerful tool for the engineer especially because some of the simulation results obtainable are impossible to duplicate experimentally (Krause, 1965). Digital computer simulations of the dynamic behavior of the single phase induction machines are, of course, not uncommon in literature. However, an explicit algorithm for the solution of the voltage, flux linkage and torque equations of these machines leading up to the study of the transient performance of these machines has not been presented. This paper provides this information with special application to the split-phase induction motor. The embedded MATLAB function and other useful blocks from the SIMULINK library of the MATLAB software have been used for the program execution.

II. The Split-Phase Induction Machine

The split phase induction machine is a type of single phase Induction machine whose method of starting is defined by the difference in the ratio of winding resistance to inductance (*r/L*) between the starting and the running windings. It is required that the r/L ratio of the starting winding should be higher than that of the running winding for there to be a reasonable amount of starting torque, otherwise the machine will not start. This requirement produces the necessary phase shift between the currents in the two windings, thereby giving rise to a starting electromagnetic torque.

Mathematically, starting torque

$$\mathrm{T_{st}} \propto \mathrm{R_F I_r I_s} \sin(\theta_r - \theta_s) \tag{1}$$

where I_r and θ_r are the running winding rms current and phase shift respectively. I_s and θ_s are the starting winding r.m.s current and phase shift respectively while R_F is the equivalent rotor resistance considering parallel magnetizing reactance (Lin et al, 2010). The difference in the *r/L* ratio is practically achieved by using a copper wire of thin cross-section for the starting winding while a copper wire of thick cross-section is usually used for the running winding. The rotor of this machine is assumed to be a squirrel cage rotor in which the rotor bars can be represented by two un-symmetrical windings whose axes are perpendicular to each other. For this type of SPIM, $V_{qs} = V_s$, $V_{ds} = V_s$. In addition $V'_{qr} = 0$, $V'_{dr} = 0$ since the rotor windings are short-circuited (Krause et al, 2002). V_s, V_{qs}, V_{ds}, V_{qr} and V_{dr} are defined in section 3. The running and the starting windings are spatially displaced 90^0 apart and are connected across the single phase supply as seen in Figure 1.

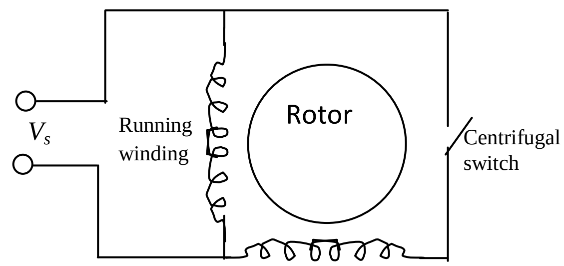

Figure 1: Schematic diagram of the Split-Phase Induction Motor

III. Equations of the Split Phase Induction Motor

The voltage, flux and electromagnetic torque equations are hereby written in equations (2)–(11).

Voltage Equations

$$v_{qs} = r_{qs} i_{qs} + p\lambda_{qs} \tag{2}$$

$$v'_{ds} = r'_{ds} i'_{ds} + p\lambda'_{ds} \tag{3}$$

$$v'_{qr} = r'_r i'_{qr} + p\lambda'_{qr} - w_r \lambda'_{dr} \tag{4}$$

$$v'_{dr} = r'_r i'_{dr} + p\lambda'_{dr} + w_r \lambda'_{qr} \tag{5}$$

Flux Linkage Equations

$$\lambda_{qs} = L_{lqs} i_{qs} + L_{mq}\left(i_{qs} + i'_{qr}\right) \tag{6}$$

$$\lambda'_{ds} = L'_{lds} i'_{ds} + L_{mq}\left(i'_{ds} + i'_{dr}\right) \tag{7}$$

$$\lambda'_{qr} = L'_{lr} i'_{qr} + L_{mq}\left(i_{qs} + i'_{qr}\right) \tag{8}$$

$$\lambda'_{dr} = L'_{lr} i'_{dr} + L_{mq}\left(i'_{ds} + i'_{dr}\right) \tag{9}$$

Electromagnetic Torque Equations

$$T_e = \left(\frac{P}{2}\right) L_{mq}\left(i_{qs} i'_{dr} - i'_{ds} i'_{qr}\right) \tag{10}$$

$$T_e = J\left(\frac{2}{P}\right) p\omega_r + T_L \tag{11}$$

The derivation of these equations is laborious and an insight to deriving them can be found in (Krause et al, 2002). It should be noted that the *q* –axis stator winding is the same as the running winding while the *d* – axis stator winding is the same as the starting winding. The *d-q* axis is a fictitious axis used to simplify the derivation of the machine equations. The *d-q* equivalent circuit diagrams of the machine are given in figures 2 (a) and (b) and they are easily obtained from equations (2) - (9).

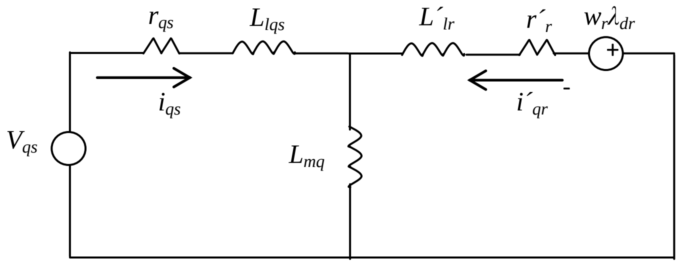

Figure 2(a): q-axis equivalent circuit of the machine

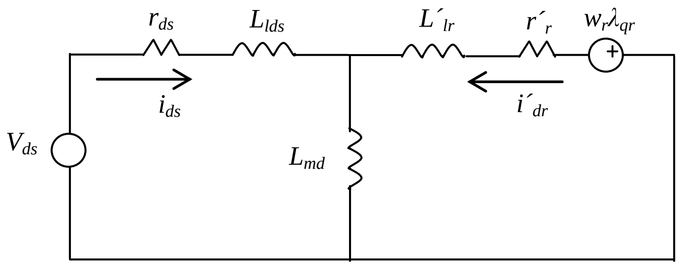

Figure 2(b): d-axis equivalent circuit of the machine

IV. SOLUTION OF MODEL EQUATIONS

A detailed procedure of the way to solve the split-phase induction machine equations is hereby set forth.
Equations (2) – (11) may be re-written as follows;

$$\lambda_{qs} = \int_{t_0}^{t_f} \left(v_{qs} - r_{qs} i_{qs}\right) dt + \lambda_{qs}(t_0) \tag{12}$$

$$\lambda'_{ds} = \int_{t_0}^{t_f} \left(v'_{ds} - r'_{ds} i'_{ds}\right) dt + \lambda'_{ds}(t_0) \tag{13}$$

$$\lambda'_{qr} = \int_{t_0}^{t_f} \left(v'_{qr} - r'_r i'_{qr} + w_r \lambda'_{dr}\right) dt + \lambda'_{qr}(t_0) \tag{14}$$

$$\lambda'_{dr} = \int_{t_0}^{t_f} \left(v'_{dr} - r'_r i'_{dr} - w_r \lambda'_{qr}\right) dt + \lambda'_{dr}(t_0) \tag{15}$$

where the reciprocal of operator p has been replaced by the integrator.

$$\lambda_{qs} = L_{lqs} i_{qs} + \lambda_{mq} \tag{16}$$

$$\lambda'_{ds} = L'_{lds} i'_{ds} + \lambda'_{md} \tag{17}$$

$$\lambda'_{qr} = L'_{lqr} i'_{qr} + \lambda_{mq} \tag{18}$$

$$\lambda'_{dr} = L'_{ldr} i'_{dr} + \lambda'_{md} \tag{19}$$

where

$$\lambda_{mq} = L_{mq}\left(i_{qs} + i'_{qr}\right) \tag{20}$$

$$\lambda'_{md} = L'_{md}\left(i'_{ds} + i'_{dr}\right) \tag{21}$$

From equations (16) – (19), stator and rotor currents may be expressed in terms of flux linkages as follows:

$$i_{qs} = \frac{1}{L_{lqs}}\left(\lambda_{qs} - \lambda_{mq}\right) \tag{22}$$

$$i'_{ds} = \frac{1}{L'_{lds}}\left(\lambda'_{ds} - \lambda'_{md}\right) \tag{23}$$

$$i'_{qr} = \frac{1}{L'_{lr}}\left(\lambda'_{qr} - \lambda_{mq}\right) \tag{24}$$

$$i'_{dr} = \frac{1}{L'_{lr}}\left(\lambda'_{dr} - \lambda'_{md}\right) \tag{25}$$

When equations (22) – (25) are substituted for currents in equations (20) and (21), the following expression can be obtained for λ_{mq} in terms of the q-axis stator and rotor flux linkages, q-axis mutual inductance and the leakage inductances of the q-axis stator and rotor windings.

$$\lambda_{mq} = \left[\frac{1}{L_{mq}} + \frac{1}{L_{lqs}} + \frac{1}{L'_{lr}}\right]^{-1} \left(\frac{\lambda_{qs}}{L_{lqs}} + \frac{\lambda'_{qr}}{L'_{lr}}\right) \tag{26}$$

Similarly, for λ_{md}, an expression can be obtained as follows:

$$\lambda'_{md} = \left[\frac{1}{L'_{md}} + \frac{1}{L'_{lds}} + \frac{1}{L'_{lr}}\right]^{-1} \left(\frac{\lambda'_{ds}}{L_{lds}} + \frac{\lambda'_{dr}}{L'_{lr}}\right) \tag{27}$$

The electromagnetic torque equation (10) is quite straight-forward and needs no modification. However, the rotor speed may be obtained from equation (11) as follows;

$$\omega_r = \int_{t_0}^{t_f} \left(\left(\frac{1}{J}\right)\left(\frac{P}{2}\right)\left(T_e - T_L\right)\right) dt + \omega_r(t_0) \tag{28}$$

Equations (12) – (28) along with equation (10) will now form the model equations for use in MATLAB/SIMULINK in order to simulate the machine model. Time t_0, the lower limit of the integral equations, is the initial simulation time while time t_f is the final simulation time. Both these parameters can be assigned values in the simulation parameters dialogue box in the Simulink model. The parameters $\lambda_{qs}(t_0)$, $\lambda'_{ds}(t_0)$, $\lambda'_{qr}(t_0)$, $\lambda'_{dr}(t_0)$, $\omega_r(t_0)$ are the initial values for the running winding flux linkage, referred starting winding flux linkage, q-axis rotor flux linkage, d-axis rotor flux linkage and rotor speed respectively. These values can also be assigned in the integrator block of the Simulink model. They have been set to zero in this paper. Figure A1 in Appendix gives the block diagram of the flow of the solution of the algorithm that has been set forth. Figure A2 gives the Simulink model constructed for the machine.

V. SIMULATION RESULTS

The simulation results are set forth here to illustrate the validity of the algorithm of the solution of the machine equations and the SIMULINK model of the split phase induction machine. The single phase induction machine that was used for the simulations is a ¼ hp *110* volts, *60*Hz, *4*–pole induction machine with the following parameters (Krause et al, 2002), (Ong, 1998).

r_{qs} = 2.02Ω, r_{ds} = 7.14Ω, L_{lqs} = 0.0074H, L_{lds} = 0.0085H, L_{mq} = 0.1772H, L_{md} = 0.2467H, N_{dq} = 1.18, r'_{qr} = 4.12Ω, r'_{dr} = 5.74Ω, L'_{lr} = 0.056, J = 0.0146 kgm^2

VI. Free Acceleration Characteristics

The voltage applied to the machine was assumed to be equal to $V_m \cos \omega t$ where $V_m = \sqrt{2}V_S$. The effect of opening and re-closing of the centrifugal switch has been omitted in the simulations in order to simplify the solution. Initial and final simulation times have been chosen as 0 and 2.5 seconds respectively.

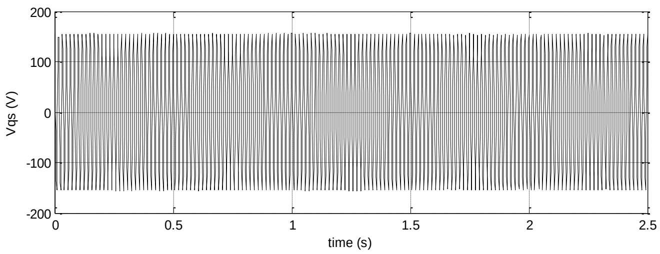

Figure 3: Voltage applied to running winding

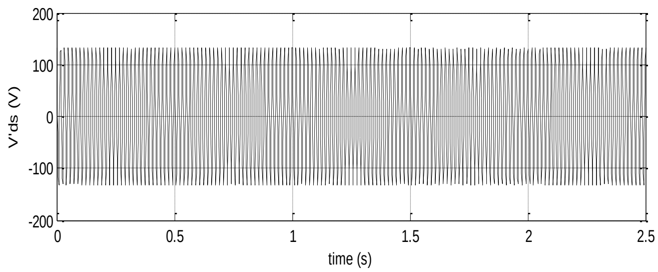

Figure 4:Voltage applied to auxiliary winding

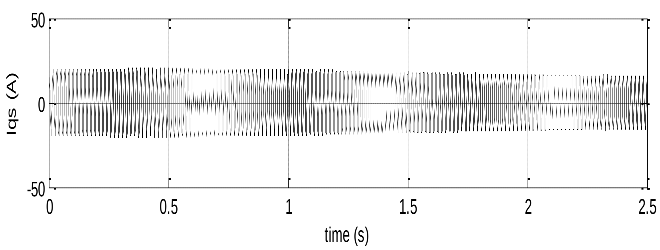

Figure 5: Current in the q-axis stator winding

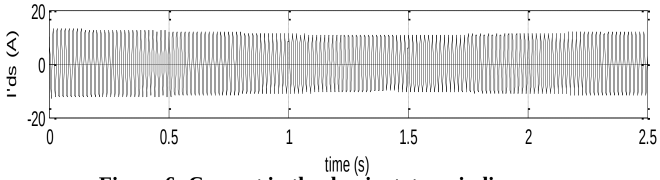

Figure 6: Current in the d-axis stator winding

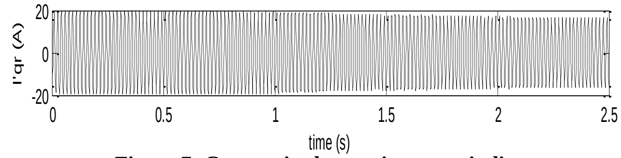

Figure 7: Current in the q-axis rotor winding

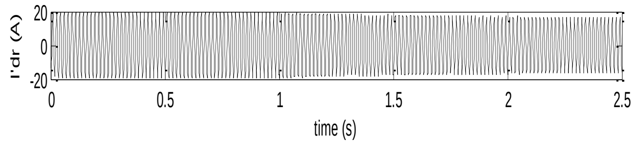

Figure 8: Current in the d-axis rotor winding

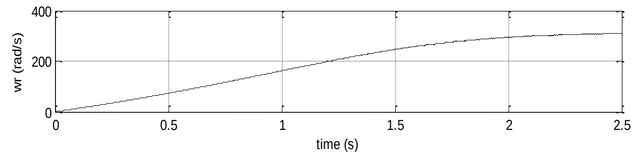

Figure 9:Rotor speed

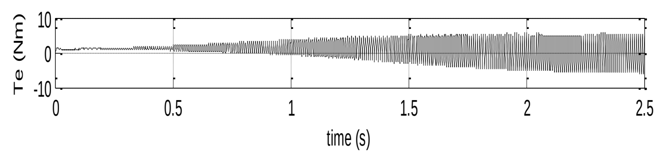

Figure 10:Electromagnetic torque in the machine

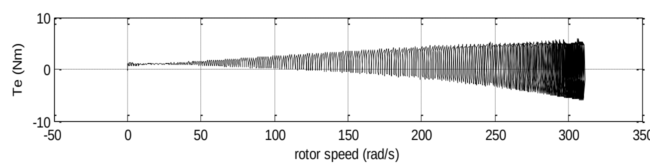

Figure 11: Dynamic torque-speed characteristics of the machine

VII. Discussion of Results

The free acceleration characteristics of the split phase Induction machine are given in Figures 3 – 11 and they have been obtained at no-load. The results compare favourably well with the results obtained in (Ong, 1998). The voltages V_{qs} and V_{ds} across the main and auxiliary windings respectively are shown in figures 3 and 4.They are sinusoidal functions of time as expected. It can be observed that the voltage waveforms are unequal in magnitude. The reason for this is the turns ratio between the two windings. In figures 5 - 8, the stator and rotor currents I_{qs}, I_{ds}, I'_{qr} and I'_{dr} are seen to take the form of the supply voltage which is sinusoidal in nature. The running or main winding current is observed in Figure 5 to be higher in magnitude than that of the starting winding shown in figure 6 which is expected. The reason is because the running winding resistance is lower than that of the starting winding.

A critical look at the current waveforms will reveal the existence of a phase shift between the currents of the running and the starting winding. It has a small value but it is sufficient to produce a starting torque in the machine. In practical terms, the higher the *r/L* ratio of the starting winding relative to the running winding, the higher will be the phase shift and hence the starting torque. The rotor currents I'_{qr} and I'_{dr} as seen in figures 7 and 8 are higher at low rotor speeds and reduce to a lower value at around 2s when the rotor speed depicted in figure 9 approaches synchronous speed. At approximately synchronous speed, the rotor currents and hence motor torque is seen to have an average value that is approximately equal to zero. It is seen from figures 10 and 11 that the electromagnetic starting torque of this machine is not zero at the point of starting.

The deliberate asymmetry between the starting and running windings is the reason for this. The speed of the machine is therefore seen to rise from zero to a value very close to the synchronous speed given as $\omega = 2\pi f = 377.04$ rad/s in about 2.5s. See figure 9. The electromagnetic torque can be seen to have both an average value component and a pulsating component. The average component has a steady value. The pulsating component, on the other hand, varies sinusoidally with time at a frequency double that of the supply voltage. A steady state analysis of the machine will make a better distinction between the average and pulsating torque components of the instantaneous electromagnetic torque produced in the machine.

VIII. Conclusion

A method for solving the equation which describes the dynamic behavior of the split phase induction motor has been explicitly presented. The solutions to these equations have been implemented using a Simulink Model of the machine, created in the MATLAB Simulink editor. The results have been presented in the form of graphs of stator and rotor currents and motor torque versus time. The instantaneous torque–speed characteristics of the machine have also been plotted. The results are consistent with the theoretical description of the split phase induction machine as provided in existing literatures. It has been observed that most papers go on to give simulation results arising from the analysis of dynamic performance of the single phase induction machines while assuming that readers know how the model equations are solved. The knowledge of the way these equations are solved is extremely important especially for beginners who are interested in doing research in the area of rotating machine analysis. This paper therefore seeks to make known, one of the ways of solving the voltage and torque equations of the split phase induction machine, to beginners interested in the analysis of the dynamic performance of single phase induction machines.

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APPENDIX

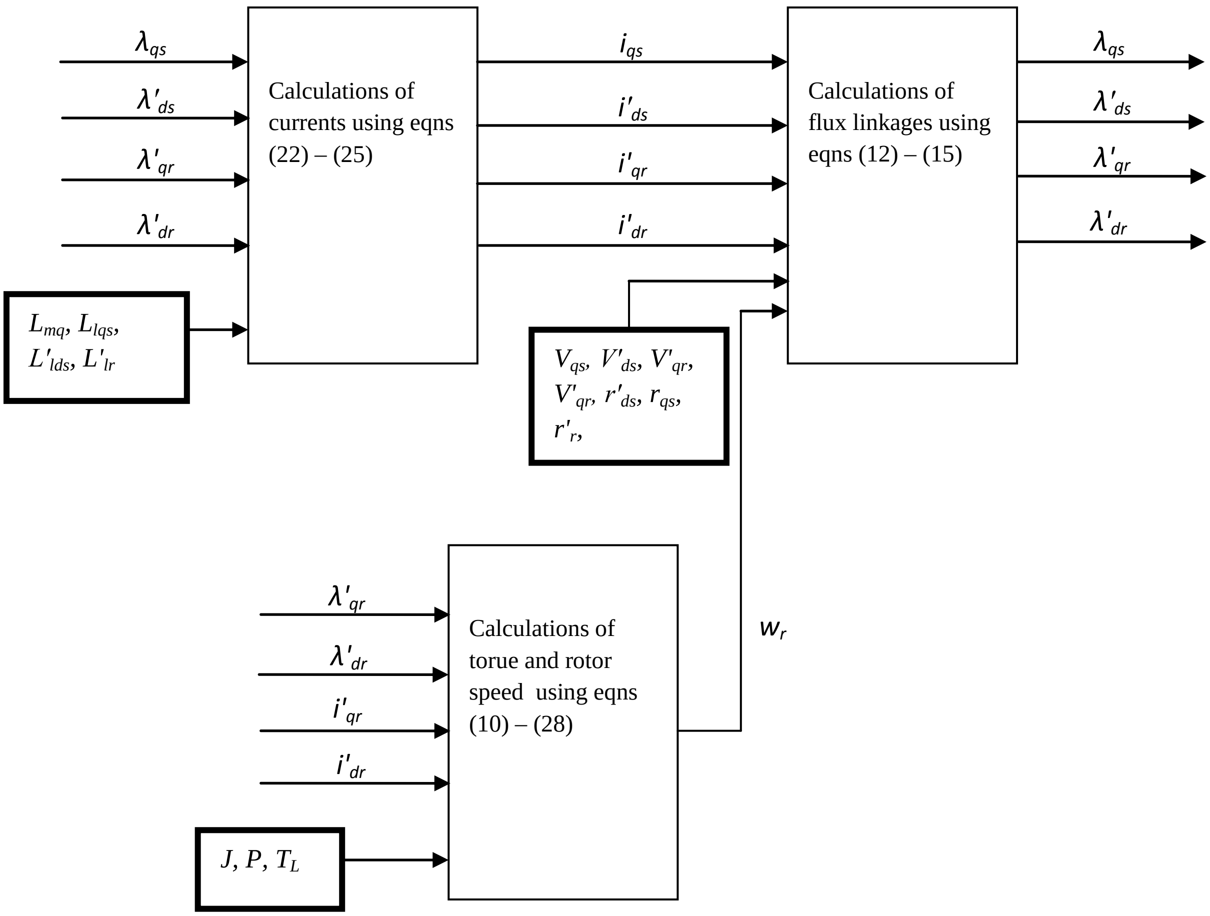

Figure A1: Block Diagram of the solution of model equations

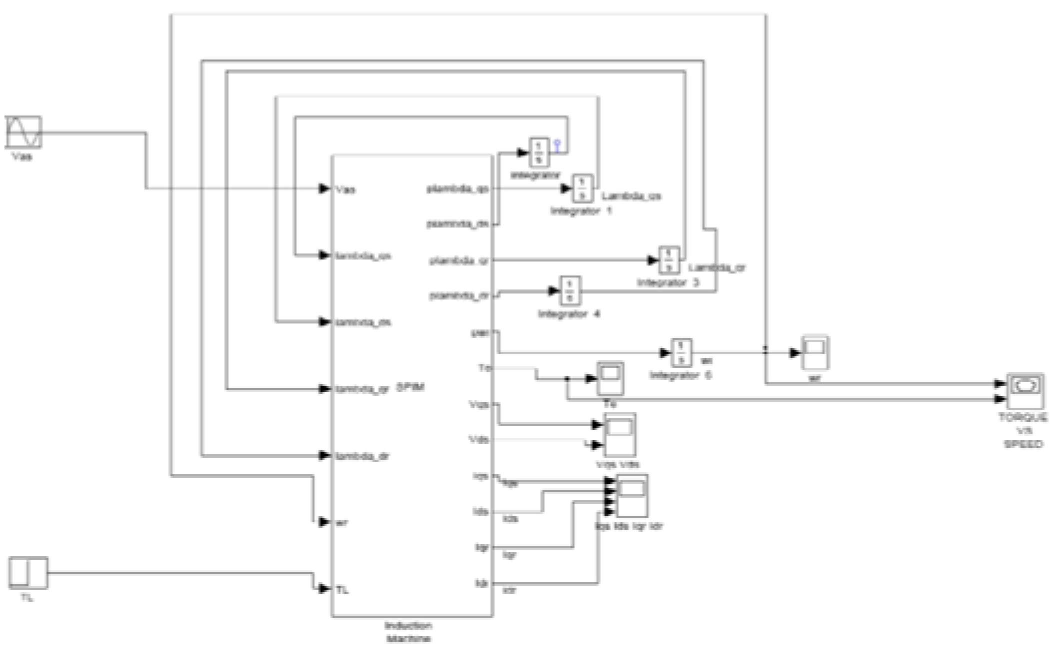

Figure A2: Simulink model of the split phase induction machine