"SEQUENCING FOR BATCH PRODUCTION IN A GROUP FLOWLINE MACHINE SHOP"

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ABSTRACT
The purpose of the paper is to develop a useful technique for sequencing batches of components through machine shops arranged under the group flowline production system. The approach is to apply a modified version of Petrov's group flowline technique for machining components which follow a unidirectional route. An outline of the use of the technique is given and a numerical example is included at the end of the paper to demonstrate the application of the new method

1. INTRODUCTION

A number of techniques for sequencing different "batches of components through machine shops have been developed. Some of these techniques do not appear to be as successful as others, although nearly all of them present interesting concepts from the points of view of both practice and research. However, in the majority of cases, these techniques are either restricted to a certain number of machines or jobs that can be made available at a time; or are suited to situations where the machines are either identical or are arranged in a functional manner, e.g. Baker [1], Eilon and Pace [2].

One attractive and simple technique for solving the batch sequence problem, in a group flow line situation, that is known to the author is that developed by Petrov [3]. A modified version of this technique and the rules that govern its application 1S been discussed in this paper.

However, before discussing the technique and the rules governing its application, it is necessary firstly, to understand the nature of the problem in which it will be applied and secondly to be aware of the objective to be achieved.

2. THE PROBLEM AND OBJECTIVE

Assume that there exists a flow line system through which all the components machined follow a unidirectional (one directional) route. The components, p different types, are issued one after another for machine operations on m machines. Let T_{ij} be the processing time for a batch of the ith component (i=1,2,....,p) in the j^{th} operation (j=1,2,..,m). This processing time per batch is determined from the simple relationship:

Processing time per batch = operation time per batch + set-up time per batch.

i.e. $T_{ij} = Q_i \, t_{pij} + t_{q.uij}$ (1)

where Q_i is the batch size for the ith component. t_{pij} is the piece time for the j^{th} operation on the ith component in hours. $t_{q.uij}$ is the set-up time for j^{th} operation on a batch of the ith components in hours.

Since in a process of this nature, the piece (ie. component operation) times vary from operation to operation and the batch sizes of individual runs also vary it would be difficult if not impossible to synchronies the process, because every machine would experience periods during which it would either still be occupied on the previous batch when the next one arrived, or be standing idle waiting for the arrival of the next batch from the machine before it. In such a situation, the duration of the throughput time (T_{tm}) will depend on the order in which the batches of components are issued to the machines for machining. The 'throughput time' here is taken to be the overall cycle time in which to complete a given number of batches.

The idle time experienced on the machine which performed the last (m^{th}) operation has close relationship with the idle times on the machines responsible for all the previous operations starting from the second (j=2) and finishing at the penultimate (j=m-l).

For p different types of components there are $(p!)^m$ different sequences of issuing them to the machines. Therefore, the problem is to find and choose the issuing sequence for machining which will minimize the individual machine idle time, in order to achieve a minimum throughput time.

3. RULES FOR DETERMINING SEQUENCING ORDER OF BATCHES.

Petrov 3 has enunciated four rules for determining the order in which batches of components following unidirectional routes and having any number of operations are selected for machining, so as to find the correct
sequence which will lead to an optimum scheduling graph with a minimum throughput time. Rule 1 suggests the setting up of orders in which components issued first for machining have positive values for the differences between sums of processing times per batch in the second and first halves respectively of the matrix ($T_{i2} - T_{i1} \geq 0$), and should be arranged in order of increasing total processing times for operations in the first half of the process, t_{i1}; while those with negative difference values ($T_{i2} - T_{i1} < 0$) are issued second, and are arranged in order of decreasing total processing times for operations in the second half of the process, t_{i2}.

Rule 2 advocates for the setting up of models which will enable the components to be machined in order of decreasing differences between the total processing times per batch for operations in the second and first halves of the matrix.

Rule 2, first of all, determines the average processing times T_{i1} or T_{i2}) for the individual batches of components for each of the two halves of the matrix from the expressions:

$$\overline{T_{i1}} = \frac{T_{i1}}{m_{i1}} \text{ and } \overline{T_{i2}} = \frac{T_{i2}}{m_{i2}} \qquad (2)$$

(2)

where m_{i1} and m_{i2} are the number of machines in the first and second halves of the matrix respectively. It then examines all the alternatives that satisfy the requirements of Rules I and 2.

Finally, Rule 4 advises on the setting up of routines in which components are machined in order of decreasing differences between the average processing times per batch for machines in the second and first halves of the matrix.

However, it is not always that the application of the above four rules results in easy solutions to the batch sequencing problem. Recently, it has been shown *[4]* that sometimes unique and indeterminate solutions occur, and the following are the criteria on which the resolution of such anomalies is based.

4.0 CRITERIA FOR RESOLVING INDETERMINATE SOLUTIONS

4.1 <u>FIRST CRITERION</u>: if the difference between the second and first halves of the processing times (ie. $T_{i2} - T_{i1}$) for individual batches of components, result in either an all positive or all negative values, the application of Rule 1 produces a 'unique solution'. Another occasion in which a unique solution will occur is when a rule is repeatedly applied in an attempt to resolve the indeterminate solutions of other rules. This situation will be made clearer later in the third criterion. In either case, batches of components are arranged in the order indicated by the signs of the differences. That is, if an all positive situation arises, batches are issued for machining in order of increasing T_{i1} values, and if an all negative, the issuing of batches is in the decreasing order of T_{i2} values.

4.2 <u>SECOND CRITERION</u>: if the values of T_{i1} or T_{i2} for several batches of components are identical, Rule 1 is said to have an 'indeterminate solution'. When this happens, batch issuing criterion becomes the difference ($T_{i2} - T_{i1}$) value, and batches are arranged in order of decreasing differences, i.e. cording to Rule 2.

4.3 <u>THIRD CRITERION:</u> if, by attempting to apply Rule 2, a situation for an indeterminate solution occurs, i.e. if several batches of components have identical values for the differences ($T_{i2} - T_{i1}$), as the spindle example has made manifest in Table 1 below, where two sets of components A, B & G and C, D & F each has a different ($T_{i2} - T_{i1}$) value of 1 and 0 respectively. In such a situation, the means of deciding the order in which batches are issued for machining becomes the arrangement according to the sign of the differences indicated by the first rule. Thus, batches of components with positive difference values are issued first, and in order of increasing values of T_{i1}; while those with negative difference values are issued second, and in decreasing order of T_{i2} values.

5.0 NUMERICAL EXAMPLE

This method of finding the optimum sequence of machining time can best be illustrated by an example, as shown in Table 1. Taking the spindle as an example, the left-hand side of Table 1 records the basic processing time matrix for seven different sizes of the spindle which follow the same route through six machines. For Rules 1 and 2, the parameters, T_{i1} T_{i2} and ($T_{i2} - T_{i1}$) in the first (X) and second (Y) halves of the matrix have been found by direct calculation and are shown in the next three columns of Table 1,

Thus, for a 4-inch spindle:

Let 4-inch = A

$\therefore T_{A1} = 54 + 18 + 12 = 84$

and $T_{A2} = 41 + 26 + 18 = 85$

hence, $T_{A2} - T_{A1} = 85 - 84 = +1$.

For a 5-inch spindle:

Let 5-in = B

$\therefore T_{B1} = 45 + 15 + 10 = 70$

And $T_{B1} = 34 + 22 + 15 = 71$

Hence, $T_{B2} - T_{B1} = 71 - 70 = +1$, and so on.

Continuing with Table 1, and considering Rules 1 and 2 only, all the sizes of the spindle, except the 10-inch size (ie. component E), have positive difference values. This means that they must be machined first, and in order of increasing T_{i1} values, eg.

Component G = 38
Component B = 70
Component A = 84
Component F = 102
Component D = 107
Component C = 119

The 16-inch spindle (ie. component G) is issued first, followed by components B, A, F, D and C.

As stated earlier, the 10 -inch spindle (ie. component E) has a negative difference value of -1, and since it is the only component with a non-positive value, it is issued last. Had there been more than one component with negative difference values, the sequencing of batches of components for machining would have been in order of decreasing values of Ti2' Hence Rule 1 shows that the seven different sizes of the spindle must be machined in the order, G,B, A, F, D, C, E, as recorded in column I of Table 1.

Having established the order in which batches of components must be issued for machining under Rule 1, the next step is to consider that of Rule 2, using the same Table. Rule 2 says that the batches must be issued for machining according to the decreasing values of the difference between T_{i2} and T_{i1} Because there are two groups of components with identical values for the differences, ($T_{i2} - T_{i1}$),it iS necessary to apply one of the criteria for resolving indeterminate solutions. The first group contains components A,B and G having a difference

COMPONENT TYPE AND SIZE PINION	MACHINE NUMBER						CALCULATED VALUES						SEQUENCES			
	1	2	3	4	5	6	T_{i1}	T_{i2}	$(T_{i2}-T_{i1})$	$\overline{T_{i1}}$	$\overline{T_{i2}}$	$(T_{i2}-T_{i1})$	I	II	III	IV
	BATCH MACHINING TIMES (HOURS) X										Type e					
4" = A	54	18	12	41	26	18	84	85	+1	28	28.3	+ 0.3	G	G	G	B
5" = B	45	15	10	34	22	15	70	71	+ I	23.3	23.7	+0.4	B	B	B	G
6": C	76	26	17	58	36	25	119	119	0	39.7	39.7	0	A	A	A	A
8" = D	69	23	15	52	33	22	107	107	0	35.7	35.7	0	F	F	F	F
I0" = E	66	22	15	50	31	21	103	102	-I	34.3	34.0	-0.3	D	D	D	D
12": F	65	22	15	50	31	21	102	102	0	34.0	34.0	0	C	C	C	C
16^{11}= G	24	8	6	19	12	8	38	39	+1	12.7	13	+0.3	E	E	E	E

Y

TABLE 1: MACHINE TIME MATRIX

value of 1 each; while the second consists of components C,D and F, each with a zero difference value. By applying the third criterion, the 7 different sizes of the spindle must be issued for machining in order, G,B,A,F,D,C,E, as listed in column II of Table 1.

By applying Rules 3 and 4, and using the same example as for Rules 1 and 2, the average processing times for each machine can be found from equation (2) as follows:

For the 4-inch spindle (ie. component A),

$\overline{T_{A1}}$ = 84/3 = 28

And $\overline{T_{A2}}$ = 85/3 = 28.3

Hence $\overline{T_{A1}}$ - $\overline{T_{A2}}$ = 28.3 - 28 = 0.3

Similarly, for the 5-inch spindle (ie. component B),

$\overline{T_{B1}}$ = 70/3 = 23.2

$\overline{T_{B2}}$ = 71/3 = 23.7

Hence T_{B2} - T_{B1} = 23.1 - 23.3 = 0.4, and so forth, for the remaining 5 sizes.

In the case of Rules 3 and 4, indeterminate situations are resolved by the same criteria as for those of Rules 1 and 2, using the parameters $\overline{T_{i1}}$, $\overline{T_{i2}}$ and ($\overline{T_{i2}}$- $\overline{T_{i1}}$),

Hence, Rules 3 and 4 show that the 7 sizes of the spindle are issued for machining in the order G,B,A,F,D,C,E and B,G,A,F,D,C,E as shown in columns III and IV of Table 1 respectively.

6.0 ALGORITHM FOR COMPUTING THE THROUGHPUT TIMES WITH SERIES FLOW.

Case 1:

Having established the sequencing order in which batches of components are issued to the machine shop for machining, the next step is to consider the algorithm *[3]* for computing the throughput times for the various alternative sequences. The matrix arrangement is best to use in computing the throughput times and arriving at the alternative sequence giving the minimum throughput time value when drawing up schedule plans for workloads on machines arranged in the group flowline system. The rows in the matrix relate to the component numbers (i), and the columns to the machine numbers (j) in that particular machining order. Each position in the matrix contains two numbers, the first relates to the processing time, t_{ij}, for that machine and the second to the cumulative time, T_{tmij}, on the machine in hours. This cumulative time for each component and machine is determined by successfully adding the machining times, t_{ij}, of components with maximum values of the machine occupation time. This 'machine occupation time' is made up of two integral parts. One part is the processing time for the same batch of components on the preceding machine ($T_{i,j-1}$ on the same row),and the other is the processing time for the same machine on the preceding batch of components ($T_{i-1,j}$ up the Column). The mathematical expression for this is as follows:

$$T_{tmij} = t_{ij} + \max\begin{Bmatrix} T_{i,j-1} \\ or \\ T_{i-1,j} \end{Bmatrix} \qquad (3)$$

Again, using Table 1 as an example, there are only two alternatives orders, the first order being sequences I, II and III (which are identical and will, therefore, yield the same throughput time) and the second is sequence IV. Taking sequence I (which is similar to those of sequences II and III), the throughput time matrix is as shown in Table 2.

From equation (3), the cumulative time (and hence the throughput time) is calculated as shown below. Considering the first batch of component *(G)* and machine no. 1, the cumulative machine occupation times (second numbers in each pair) are obtained by successfully adding the processing times (in hours) as follows:

(i) adding along the first row, we have

0 + 24 = 24; 24 + 8 = 32, 32 + 6 = 38; 38 + 19 = 57; 57 + 12 = 69 and 69 + 8 = 77.

(ii) adding diagonally and dawn column 1,

Component type and size	MACHINE NUMBER					
	1	2	3	4	5	6
PINON	CUMULATIVE MACHINE OCCUPATION TIMES (HOURS)					
16" = G	24/24	8/32	6/38	19/57	12/69	8/77
5" = B	45/69	15/84	10/94	34/128	22/150	15/165
4" = A	54/123	18/141	12/153	41/194	26/220	18/238
12" = F	65/188	22/210	15/225	50/275	31/306	21/327
8" = D	69/257	23/280	15/295	52/347	33/380	22/402
6" = C	76/333	26/359	17/376	58/434	36/470	25/495
10" = E	66/399	22/421	15/436	50/486	31/517	21/538

TABLE 2: OVERALL CYCLE TIME MATRIX FOR SEQUENCES I,II&III WITH SERIES FLOW.

We have, 24 + 45 = 69; 69 + 45 = 123; 123 + 65 = 188; 188 + 69 = 257; 257 + 76 = 333; 333 + 66 =399

The other points in the matrix can be obtained in a similar manner. For example, for machining component B, on machine:

No. 2, 69 32 ∴ $^{T}tmB2$ = 69 + 15 = 84; NO. 3, 84 38 ∴ $^{T}tmB3$ = 84 + 10 = 94

No. 4, 94 57 ∴ $^{T}tmB4$ = 94 + 34 + 128; no.5, 128 69 ∴ $^{T}tmB5$ =128 + 22 = 150

No. 6, 150 77 ∴ $^{T}tmB6$ = 150 + 15 = 165, and so on

Thr throughput time is arrived at in the right – hand bottom corner of the matrix. For this example ands the particular order considered, the throughput time = 538 hours.

Considering the second alternative (ie. sequence IV), the procedure for applying equation (3) and hence, obtaining a value for the throughput time is exactly the same as for alternative I, considered above. Again, the throughput time (Table 3) is 538 hours. It must, however, be emphasized that, it is not usual to arrive at

COMPONENT SIZE AND TYPE	MACHINE NUMBER					
	1	2	3	4	5	6
PINION	CUMULATIVE MACHINE OCCUPATION TIMES (HOURS)					
5" = B	45/45	15/60	10/70	34/104	22/126	15/141
16" = G	24/69	8/77	6/83	19/123	12/138	8/149
4" = A	54/123	18/141	12/153	41/194	26/220	12-/238
12" = F	65/188	22/210	15/225	50/275	31/306	21/327
8" = D	69/257	23/280	15/295	52/347	33/380	22/402

6" = C	76/333	26/359	17/376	58/434	36/470	25/495
10" = E	66/399	22/421	15/436	50/486	31/517	21/538

Operation on the ith batch of

TABLE 2: OVERALL CYCLE TIME MATRIX FOR SEQUENCE I, II & III WITH SERIES FLOW.

the same value of throughput time for all the alternatives, this only happens when a unique situation occurs. In a normal situation the smallest time is taken as the minimum throughput time. Thus, in this example, the value of 538 hours obtained for the throughput time is the minimum of the $(7!)^6$ possible alternative sequences for producing 7 different sizes of the spindle on 6 machines.

6.1 Algorithm for Computing the Throughput Times with Parallel- Series Flow.

Case 2

The rules established for determining the batch-machining order in the previous example are retained for the second case. However, the use of the parallel-series combinations for completing successive operations on each batch of components completely changes the algorithm for calculating the cumulative machine tool times and the overall cycle time. In fact, instead of using a single algorithm (equation 3) for calculating the cumulative times (Ttmij) on machine tools, two algorithms are now used. The structure of the difference between the algorithms depend primarily on the relation between the labour contents of successive operations on a given batch of components and the occupation time of the machines on these operations
The algorithms are formulated as follows:

1. If the preceding, (j-l)th, components is of smaller labour content than the j^{th} operation, the machines used for this adjacent pair of operations must simultaneously complete the work on these operations and the j^{th} machine must no longer be occupied on machining the preceding (i-l)th batch of components. Otherwise the cumulative cycle time must be found by adding the occupation time (Ti-l,j up the column) of the machine too1 concerned to the component machining time, t_{ij}., for that The mathematical expression for the first algorithm is:

$$T_{tmij} = \max\begin{Bmatrix} if\ T_{i,j-1} \geq t_{ij}, then \\ T_{i,j-1} \quad T_{i-1,1} + t_{ij} \quad or \end{Bmatrix} \qquad (4)$$

2. If the first, (j-l)th, operation on the ith batch of components is of a higher labour content than the jth operation, the machines used for the successive pair of operations must start work simultaneously on these operations, and the machine tool for the jth operation must no longer be occupied on the preceding (i-l)th batch of components. Otherwise the cumulative cycle time is determined as for the similar condition in the first algorithm. The mathematical expression for the second algorithm is:
If $t_{i,j-1} < t_{ij}$, then
$T_{tmij} = \max T_{i,\ j-1} - t_{i,\ j-1} + t_{ij}$ or
$T_{i-1,j} + t_{ij}$

(5)

An example of the construction of the numerical model for the schedule for a parallel-series combination of operations is given in Table 4. The matrix data are as in Table 3 and the optimum order for machining sequence IV (i.e. the second alternative) is used.

As before, the occupation time for the first machine is found by successfully adding the single-operation times in column 1.

(i) Let us consider the first component, B (i.e. the 5-inch spindle).

a) Component B is issued first, and since the labour content of the second operation (ti,j-l) is less than for the first, t_{ij} (i.e. 15 < 45), the machines must complete the work simultaneously and the occupation time for the second machine equals that of the first (i.e. $T_{tB2} = T_{tBl}$ - 45 hours).

COMPONENT TYPE AND SIZE	MACHINE NUMBER					
SPINDLE	CUMULATIVE MACHINE OCCUPATION TIME (HOURS)					
5" = B	45/45	15/45	10/45	34/69	22/69	15/69
16" = G	24/69	8/69	6/69	19/88	12/88	8/88
4" = A	54/123	18/123	12/123	41/152	26/152	18/152
12" = F	65/188	22/188	15/188	50/223	31/223	21/223
8" = D	69/257	23/257	15/257	52/294	33/294	22/294
6" = C	76/333	26/333	17/333	58/374	36/374	25/374
10" = E	66/399	22/399	15/399	50/434	31/434	21/434

TABLE 4: OVERAL CYCLE TIME MATRIX FOR SEQUENCE IV WITH PARALLEL-SERIES FLOW.

b) The labour content of the third operation ($t_{i,j-2}$) is also less than that of the second (ie. 10 < 15), and again the machines must complete the work simultaneously and the occupation time for the third machine equals that of the second (ie. $T_{tB3} = T_{tB2}$ = 45 hours).

c) The labour content of the fourth operation is greater than that of the third (i.e $t_{i,j-3} < t_{i,j-2}$,34 > 10), and the machines must start work simultaneously and the occupation time for the fourth machine equals that of the third machine minus the labour content of the third machine plus labour content of the fourth machine. That is, $T_{tB4} = T_{tB3} - t_i,j-2 + t_i,j-3$.'. T_{tB4} = 45 - 10 + 34 = 69 hours.

d) The labour content of the fifth operation is less than that of the fourth (ie. $^{t}i.j-4 < {}^{t}i,j-3$’22 < 34), and machines must complete the work simultaneously and occupation time for the fifth machine equals that of the fourth (ie. T_{tB5} = T_{tB4} = 69 hours).

e) Lastly, since the labour content of the sixth operation is also less than that of the fifth(ie.$^{t}i,j-5<t_{i,j-4}$; 15 < 22), the machines must complete the work simultaneously and again, the occupation time of the sixth machine equals that of the fifth machine.That is $T_{tB6} = T_{tB5}$ = 69 hours).

(ii) If we now consider the second component G(i.e.the l6-inch spindle).

a) Component G is issued second, and since the labour content of the second operation ($t_{i,j-1}$) is less than that of the first, t_{ij} (ie. 8 < 24), the machines must complete the work simultaneously and the occupation time of the second machine (T_{tG2}) equals either that of the first machine (T_{tGl}) or the labour content of the second operation plus occupation time for the second machine of component B(T_{tB2}), whichever is greater. Since 69 > (8 + 45), therefore, T_{tG2} = 69 hours.

b) Similarly, since the labour content of the third operation, $t_{i,j-2}$ is less than that of the second, $^{t}i_{,j-1}$ (ie. 6

< 8), the occupation time of the third machine (T_{tG3}) is 69 hours.

c) The labour content of the fourth operation (ti,j-3) is greater than that of the third, ti,j-2 (ie, 19 > 6). Therefore, the machines must start work simultaneously,
and the occupation time of the fourth machine (T_{tG4}) equals either the occupation time for the third machine, T_{tG3}, (ie. 69) minus labour content of the third operation, $t_{i,j-2}$ (ie. 6) plus labour contents of the fourth operation, $t_{i,j-3}$(ie. 19); or labour content of the fourth operation (19) plus occupation time for the fourth machine of component B, T_{tB4}, (ie. 69), whichever is greater. That is, T_{tG4} = either (69 - 6 + 19) or (19 + 69). Since (19 + 69) > (69 - 6 + 19), the occupation time for the fourth machine (T_{tG4}) = 88 hours, and so on.

In this example, the cumulative time T_{tmij} (and hence the throughput time)= 434 hours as shown in Table 4. If the first alternative (ie. either sequences I, II or III) had been used, T_{tmij} would be 516 hours. Therefore, the smaller of the two values (434 hours) is taken. Thus, the overall cycle time has been reduced further by comparison with successive combinations of operations (Table 3). The saving in time is 538 - 434 = 104 hours or 19.3 percent.

7. CONCLUSION

The present study has shown that it is possible to manufacture small-size batches of components according to flowline production methods, and has further demonstrate that this can be economically viable only when the principles of group technology are applied and coupled with a sound system of operational planning and scheduling.

In the company studied, manufacturing costs of components have been substantially reduced by the application of group technological flowline methods. Firstly, by decreasing the throughput time and reducing the work-in-progress and secondly, by increasing productivity of manufacture. Also, by applying this new concept in manufacturing technology, a reduction by 104 hours in the overall cycle time was achieved. This represents a decrease by 19 percent in the manufacturing throughput time.

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