ADVANCES AND PROBLEMS IN MATHEMATICAL MODELLING OF DISPERSION OF SUSPENDED PARTICLES IN RIVERS AND STREAMS

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ABSTRACT

Most expressions for dispersion of pollutants have failed to give accurate predictions in both channel and natural stream flows. This paper outlines the basic concepts on which the fundamental dispersion equations have been derived. Some of the advances on these equations are examined and their deficiencies pointed out.

The paper finally outlines the author's contribution in the establishment of a prediction method based on the latest developments on pollutant dispersion problems. Comparison of the predicted results with measured results shows good agreement.

INTRODUCTION

In a modern technological society more and more water is required as the population grows and the standard of living increases. To meet this demand, an enormous price is paid by the communities in re-purifying and indirectly reusing already polluted waters. Water pollution control has now become a major concern in almost every state of the industrialised world and often involves political decisions for long-term planning.

In many inland rivers and streams, the most serious pollutional problem arises from the fate of organic solids discharged into them. The basis of any effective pollution control strategy in these waters is a clear understanding not only of the fate and consequences of the pollutants but also the mechanisms involved in their dispersion, distribution and transportation.

Distribution of organic solid particles discharged into rivers and streams may involve the following mechanisms - dispersion by the process of turbulent diffusion, dispersion by molecular diffusion, permanent settlement at the bed and subjection to combinations of two or more of these processes acting simultaneously (the most common being joint molecular and turbulent diffusion processes). In nearly every field situation, however, turbulent diffusion is by far the most important mechanism causing not only the dispersion of suspended particles but also the distribution of sediments. If a comprehensive mathematical model is to be developed to represent this complex phenomenon an understanding of all terms and factors involved in the dispersion, mixing and transport of the organic suspended particles is essential.

The term diffusion is often used synonymously with dispersion. Briefly, diffusivity is the characteristic variable which describes the physical property of the transport activity along a certain direction in the space of diffusant movement, whereas the dispersion coefficient is the overall apparent diffusivity of a fluid mixture. Thus, both molecular and turbulent diffusion describe mixing phenomena without consideration of the velocity gradient. The dispersion term includes the mixing caused by both molecular and turbulent diffusions and that due to convective transport.

2. FUNDAMENTAL DISPERSION EQUATIONS

The problems with many dispersion models may well have originated from the basic concepts of mechanisms of mass transfer in. a hydrodynamic medium. A review of these concepts and their associated fundamental quantitative and analytical equations is therefore very necessary.

Fick's laws provide the basis for quantitative analysis of diffusion. These laws are rooted in the phenomenological concept that diffusion implies a random mixing process in which pollutants spread out from regions of higher to those of lower concentrations. The concept is purely empirical and does not give any information about the magnitude of the diffusion coefficient.

The first law, that the rate of mass' transfer of the diffusing substance through unit area of a plane is proportional to the concentration gradient measured normal to the plane, can be represented mathematically as -

$$j \,\alpha\, \frac{dc}{dx} \quad or \quad j = -\varepsilon_X \frac{dc}{dc} \qquad (1)$$

(the negative sign implying movement towards lower concentration)

where, j = flux (mass of diffusing substance passing through a reference surface of unit area per unit time) (g/s cm^2)

ε_X = coefficient of molecular diffusion (cm^2/s)

c = concentration of the diffusion substance (g/cm^3)

x = distance (cm) in x direction

An expression for the accumulation of mass in the given volume can be obtained by considering an elemental volume bounded by two unit planes at x and x + δx, with the volume of the element being ox. Thus the rate of change in accumulation of mass

$$\begin{array}{ccc} Mass\ in & - & Mass\ out \\ x & & x+\delta x \\ | & & | \\ j_{in} & & j_{out} \end{array}$$

It is easy to see that if c = diffusant concentration at the boundary x, the rate of mass diffusion away from the concentration into the control volume

$=-\varepsilon_x \frac{\partial c}{\partial x} = j_{in}$ =rate of entry into the element at point x. At the boundary x + δx, the amount of mass leaving = the amount entering at x less amount diffusion and spreading over the distance δx

i.e.

$$j_{\,out} = -\varepsilon_X \frac{\partial c}{\partial x} - \left(\frac{\partial}{\partial x}\left(\varepsilon_X \frac{\partial c}{\partial x}\right)\delta x\right)$$
$$= -\varepsilon \frac{\partial c}{\partial x} - \varepsilon \frac{\partial^2 c}{\partial x^2}\delta x \qquad (2)$$

Hence the net rate of accumulation of mass in the volume element

$$= -\varepsilon_X \frac{\partial c}{\partial x} - \left(-\varepsilon \frac{\partial c}{\partial x} - \varepsilon \frac{\partial^2 c}{\partial x^2}\delta x\right)$$
$$= +\varepsilon_X \frac{\partial^2 c}{\partial x^2}\delta x \qquad (3)$$

But the rate of accumulation of mass in the volume can also be simply represented by

$$\frac{\partial(mass)}{\partial t} \times Volume = \frac{\partial(mass)}{\partial}\delta x \qquad (4)$$

Equating (3) and (4) dividing through by the volume δx, and replacing mass by conc. c x vol., then:

$$\frac{\partial c}{\partial t} = \varepsilon \frac{\partial^2 c}{\partial x^2} \qquad (5)$$

Equation (5) represents Fick's second Law which is the basic, second-order partial differential equation of diffusion in one dimension.

For the general three-dimensional case in x, y and z directions, Equation (5) is extended thus;

$$\frac{\partial c}{\partial t}=\varepsilon\frac{\partial^2 c}{\partial x^2}+\varepsilon_y\frac{\partial^2 c}{\partial y^2}+\varepsilon_z\frac{\partial^2 c}{\partial z^2} \quad (6)$$

Equations (5) and (6) are derived for an otherwise quiescent fluid medium where the only transport mechanism involved is molecular diffusion. Hence, expressing these equations with a suffix to indicate the transport mechanism involved (m suffix for molecular diffusion).

For Equation (5), $\frac{\partial c}{\partial t}=\varepsilon_{xm}\frac{\partial^2 c}{\partial x^2}$ (5a)

and for Equation (6)

$$\frac{\partial c}{\partial t}=\varepsilon_{xm}\frac{\partial^2 c}{\partial x^2}+\varepsilon_{ym}\frac{\partial^2 c}{\partial y^2}+\varepsilon_{zm}\frac{\partial^2 c}{\partial z^2} \quad (7)$$

If the elementary control volume considered is in a moving medium and moves in the direction of the flow and at the same velocity as the flowing fluid, then the above equations will be modified to take account of the rate of change due to convective transfer. Thus Equation (7) becomes;

$$\frac{\partial c}{\partial t}=\varepsilon_{xm}\frac{\partial^2 c}{\partial x^2}+\varepsilon_{ym}\frac{\partial^2 c}{\partial y^2}+\varepsilon_{zm}\frac{\partial^2 c}{\partial z^2} \quad (8)$$
$$-\frac{\partial(u_x c)}{\partial x}-\frac{\partial(u_y c)}{\partial y}-\frac{\partial(u_z c)}{\partial z}$$

This Fickian diffusion theory can be extended to dispersion in turbulent fluid motion. Such flow is defined by Hinze [1] as "an irregular condition of flow in which various quantities (for example, velocity and pressure) show a random variation with time and space, so that statistically distinct average values can be discerned". One finds that flows of engineering importance which occur in nature are predominantly turbulent. Fluctuating quantities of concentration and velocity vector involved in turbulent flows have to be combined by Reynolds rules of averages such that if a and b are fluctuating quantities and k = a constant:

$$\overline{a+b}=\bar{a}+\bar{b}$$
$$\overline{ka}=k\bar{a}$$
$$\overline{ab}=\bar{a}\bar{b}+\overline{a'b'}$$

Where $a=\bar{a}+a', b=\bar{b}+b'$ and the mean of a fluctuation is zero i.e. $\bar{a}'=\bar{b}'=0$

Equation (8) will, therefore, be valid for turbulent flows, even though it includes no turbulent diffusion terms, provided the time-averaged values of the velocities $(\bar{u}_x,\bar{u}_y,\bar{u}_z)$ and concentration $(\bar{c})$ replace their instantaneous values (u_x,u_y,u_z,c) and the transport associated with their turbulent fluctuations (u'_x,u'_y,u'_z,c') is also included [2].

Hence, by substituting

$$u_x=\bar{u}_x+u'_x$$
$$u_y=\bar{u}_y+u'_y$$
$$u_z=\bar{u}_z+u'_z$$
$$c=\bar{c}+c'$$

and applying the rules of averages, equation (8) becomes

$$\frac{\partial\bar{c}}{\partial t}=\varepsilon_{xm}\frac{\partial^2\bar{c}}{\partial x}+\varepsilon_{ym}\frac{\partial^2\bar{c}}{\partial y^2}+\varepsilon_{zm}\frac{\partial^2\bar{c}}{\partial z^2}$$
$$-\frac{\partial\bar{u}_x\bar{c}}{\partial x}-\frac{\partial\bar{u}\quad\bar{c}}{\partial y}-\frac{\partial\bar{u}_z\bar{c}}{\partial z}-$$
$$\frac{\partial\overline{u'_x c'}}{\partial x}-\frac{\partial\overline{u'_y c'}}{\partial y}-\frac{\partial\overline{u'_z c'}}{\partial z} \quad (9)$$

The bars indicate the time-average of the quantity under the bar.

Elder [3] defined coefficients of turbulent diffusion (eddy diffusivity) as:

$$-\overline{u'_x c'}=\varepsilon_{xt}\frac{\partial\bar{c}}{\partial x}$$
$$-\overline{u'_y c'}=\varepsilon_{yt}\frac{\partial\bar{c}}{\partial y}$$

$$-\overline{u'_z c'} = \varepsilon_{zt}\frac{\partial \bar{c}}{\partial z}$$

(t suffix referring to turbulent process).

Substituting these in Equation (9)

$$\frac{\partial \bar{c}}{\partial t} = \varepsilon_{xm}\frac{\partial^2 \bar{c}}{\partial x^2} + \varepsilon_{ym}\frac{\partial^2 \bar{c}}{\partial y^2} + \varepsilon_{zm}\frac{\partial^2 \bar{c}}{\partial z^2}$$

$$-\frac{\partial \bar{u}_x \bar{c}}{\partial x} - \frac{\partial \bar{u}_y \bar{c}}{\partial y} - \frac{\partial \bar{u}_z \bar{c}}{\partial z} +$$

$$\frac{\partial}{\partial x}\varepsilon_{xt}\frac{\partial \bar{c}}{\partial x} + \frac{\partial}{\partial y}\varepsilon_{yt}\frac{\partial \bar{c}}{\partial y} + \frac{\partial}{\partial z} - \varepsilon_{zt}\frac{\partial \bar{c}}{\partial z} \quad (10)$$

The processes of molecular and turbulent diffusion are independent and therefore additive so that combined diffusivity ε_{mt} is given by

$\varepsilon_{xmt} = \varepsilon xm + \varepsilon_{xt}$

$\varepsilon_{ymt} = \varepsilon_{ym} + \varepsilon_{yt}$

$\varepsilon_{xmt} = \varepsilon_{zm} + \varepsilon_{zt}$

Substituting in Equation (10) and assuming steady flow conditions in which

$\frac{\partial \bar{u}_x}{\partial x} = \frac{\partial \bar{u}_y}{\partial y} = \frac{\partial \bar{u}_z}{\partial z} = 0$ and secondary convective currents $\bar{u}_y\frac{\partial \bar{c}}{\partial y}$ and $\bar{u}_z\frac{\partial \bar{c}}{\partial z}$ are reduced to zero, then Equation (10) reduces to

$$\frac{\partial \bar{c}}{\partial t} = \varepsilon_{xmt}\frac{\partial^2 \bar{c}}{\partial x^2} + \varepsilon_{ymt}\frac{\partial^2 \bar{c}}{\partial y^2} + \varepsilon_{zmt}\frac{\partial^2 \bar{c}}{\partial z^2}$$

$$-\bar{u}_x\frac{\partial \bar{c}}{\partial x} \quad (11)$$

Subject to appropriate boundary conditions and knowledge of $\bar{u}_x$ and ε_{imt}, the above three-dimensional equation can, in principle, be solved to determine the spatial and temporal distribution of the time-mean concentration $\bar{c}$. However, in attempts to obtain complete solution of the above equation using either the finite difference method [4] or the so-called Monte Carlo technique [5], researchers have come up against computational complexities which are prohibitively expensive in terms of both human and computer time and labour.

In situations where one or two characteristic lengths (e.g. in the vertical and/or lateral directions) may be much smaller than others (e.g. in the horizontal direction), the procedure conventionally adopted to bring the solution of Equation (II) down to manageable size is to average over one or more of the lesser of the space dimensions. For instance, averaging vertically, i.e. with respect to y (depth), yields a two-dimensional model for wide vertically mixed estuaries and bays and by averaging transversely, i.e. with respect to z, a two-dimensional model suitable for the investigation of narrow stratified estuaries will be obtained.

For channels, rivers and streams, a one-dimensional model of Equation (II) - Longitudinal Dispersion Model - has proved to be a useful and adequate tool for predicting their dispersion processes. The model is achieved by averaging the concentration and the longitudinal velocity transversely over the entire cross-sectional area of the flow. Thus, referring to Fig.1 and using capital letters U and C for the cross-sectional mean velocity of flow and mean concentration of the dispersant, respectively

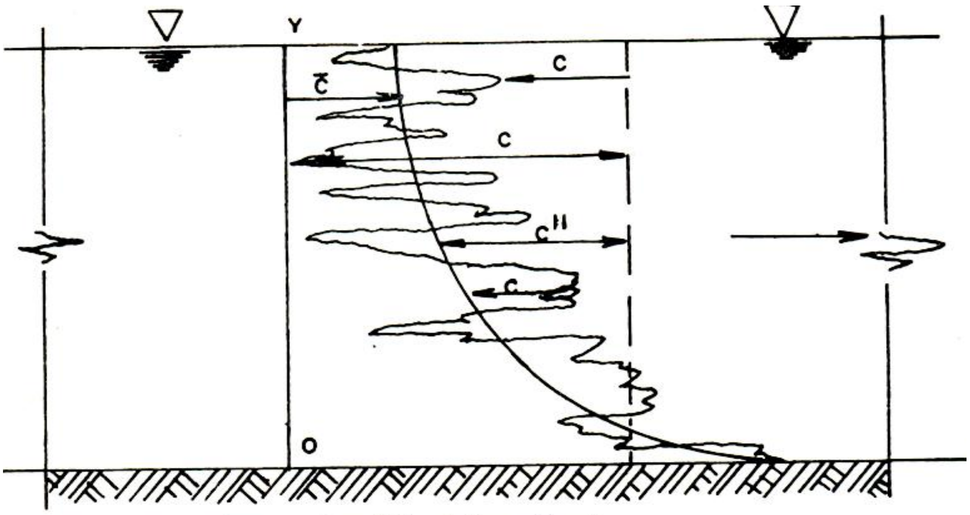
$$\bar{u}_x = U + u''$$

$$\text{and} \quad \bar{c} = C + c''$$

Where, $\bar{u}_x$ and $\bar{c}$ are as previously defined and u″ and c″ are fluctuations of the temporal means from the cross-sectional means.

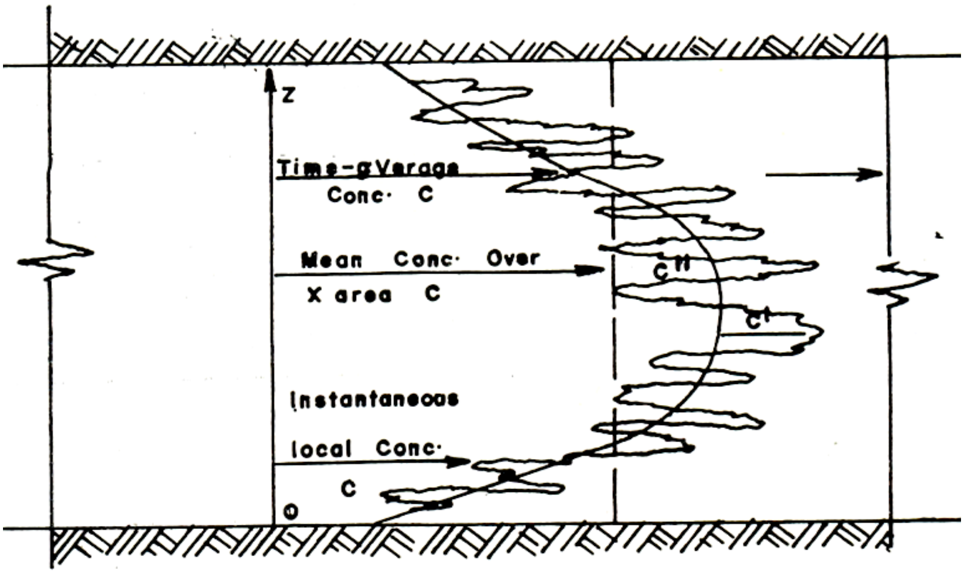
Substituting the component values of $\bar{u}$ and $\bar{c}$ in Equation (II) and apply the rule of averages for the last term:

$$\frac{\partial C}{\partial t} = \varepsilon_{xmt}\frac{\partial^2 C}{\partial x^2} - \frac{u\partial C}{\partial x} + \frac{\partial(\overline{-u''c'''})}{\partial z^2} \qquad (12)$$



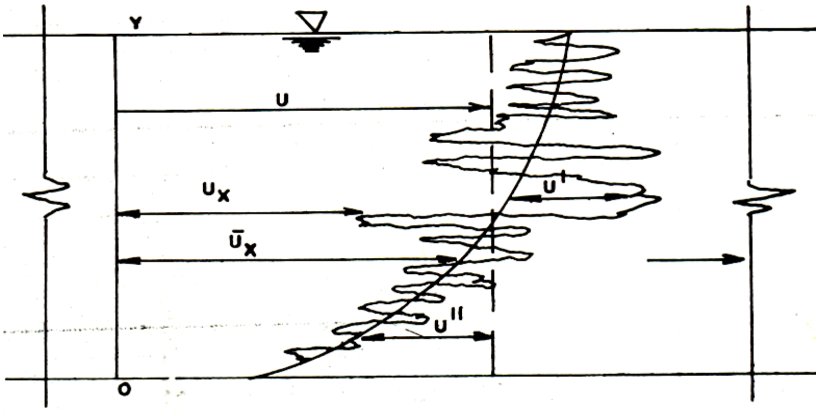
Conc Profile (elevation)

(a)



Conc. Profile (Plan)

(b)



Long Velocity, Profile (Elevation)

(C)

Fig. 1. One-Dimensional Dispersing of a Pollutant Slug in a two-dimensional flow

Since $\frac{\partial C}{\partial y} = \frac{\partial C}{\partial z} = 0$ (mean value of concentration over the cross-sectional area assumed).

The double overbar refers to cross- sectional averaging. The product -u"c" defines convection due to the difference between $\overline{u}$ and U. Thus for the local mass transport -rate may be expressed as:

$$\int_A (-u''c'')dA = AD_x \frac{\partial C}{\partial x} \quad (13)$$

which is equivalent to using the product of mean cross-sectional values $\overline{-u''c''}$

In general, for such Fickian formulations,

$$-u''c'' = FD_x \frac{\partial C}{\partial x}$$

Where, F = cross-sectional area A (or F could be depth, d, if averaging vertically over depth has been carried out).

Substituting Equation (13) in Equation (12),

$$\frac{\partial C}{\partial t} = \varepsilon_{xmt}\frac{\partial^2 C}{\partial x^2} - \frac{U\partial C}{\partial x} + \frac{1}{A}\frac{\partial}{\partial x}\left\{AD_X \frac{\partial C}{\partial x}\right\}$$

$$\text{i.e.}\ \frac{\partial C}{\partial t} + \frac{U\partial C}{\partial x} = \left\{\varepsilon_{xmt} + D_X\right\}\frac{\partial^2 C}{\partial x^2} \quad (14)$$

Replacing $\{\varepsilon_{xmt} + D_x\}$ by D_L we get

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - U\frac{\partial C}{\partial x} \quad (15)$$

which is a one-dimensional longitudinal dispersion equation in which the dispersion coefficient, D_L, can be seen from Equations (13) and (14) to be a function of velocity distribution and longitudinal molecular and turbulent diffusion. It has been said that diffusion contributes only about one per cent or less of the total dispersion in turbulent flow [6]. This means that D_L - D_X. Elder [7] pointed out that DL is known theoretically only in simple cases, otherwise it must be estimated from measurements.

Values of D_L are essentially in stream pollution forecasting. Knowledge of D_L provides immediate information of the expected pollution concentrations at various times and locations along the downstream reach of the river flow. Distribution of organic suspended solids load, water temperature variation downstream of a source of thermal discharge, salt intrusion into tidal estuaries and the variation of the reaeration capacity of a stream are aspects of river pollution control linked to the longitudinal dispersion coefficient.

Over the past two decades or so," the Longitudinal Dispersion model has been much investigated and discussed by research workers leading to advances in the understanding, development and application of its coefficient.

3. ADVANCES IN ONE DIMENSIONAL LONGITUDINAL DISPERSION MODELLING.

3.1 Analytical Evaluation of D_L

Many investigators have attempted analytical evaluation of D_L from the basic convective-diffusion equation, equation (11) which completely describes the dispersion process. By a series of assumptions and simplifications this equation, in combination with equation (15), can be transformed to yield estimates of D_L in terms of hydraulic parameters of the stream.

A pioneer in this field, Taylor [8] was first to attempt a theoretical expression for D_L for laminar flow in a straight pipe using empirical expressions for velocity and Reymolds analogy to determine first, equation (15) and then assuming logarithmic velocity profile and using Von "Karman universal constant (0.41) obtained D_L in "terms of shear stress at the wall as

$$D_L = 10.1\, rU_* \quad (16)$$

where r = radius and

U_* = shear stress at the wall.

Following Taylor's basic derivation procedure, Thomas [9] derived an expression for D_L for a unidirectional flow in an infinitely wide channel. He assumed power 1a velocity distribution in the vertical direction:

i.e. $$\overline{U}_x = \left(\frac{y}{d}\right)^n \qquad (17)$$

where d = depth of flow and obtained,

$$D_L = -d^2 \int_0^1 u^{''} dy^{'} \int_0^y \frac{1}{\varepsilon_{ymt}} dy^{'} \int_0^1 u^{''} dy^{'} \qquad (18)$$

where y' = y/d and the values of n for varying Reynold's number can be obtained.

Elder [3] obtained simpler expression than Thomas by assuming a logarithmic velocity profile as Taylor

i.e. $$u^{''} = \frac{U_*}{\eta}(1 + \ln y^{'}) \qquad (19)$$

Where η = Von Karman constant.

D_L was obtained as equal to 5.93 dU_* (20)

A striking feature of the works of Taylor, Thomas and Elder is that lateral velocity gradients were assumed unimportant in the dispersion process. But experimentally determined values of D_L for canals and natural streams are considerably greater than the predictions of Elder and Taylor. This prompted Fischer [10] to make a comprehensive review of the three-dimensional flow equation [11]. He argued that flows will be more strongly sheared laterally across the width than in other directions. Consequently, dispersion is better based on concentrations averaged through the depth instead of laterally as his predecessors. In the light of these, Fischer derived the equation,

$$D_L = \frac{-1}{A}\int_0^b q^{'}(z)dz \int_0^z \frac{1}{\varepsilon_{zmt} d(z)} \int_0^z q^{'}(z)dz \qquad (21)$$

Where, $$q^{'}(z) = \int_0^{d(z)} u^{''}(y,z)dy \qquad (22)$$

= the depth- integrated velocity at point z

$$= \bar{u}_x - U$$

ε_{zmt} = Elder's lateral turbulent mixing coefficient = $0.23RU_*$

d = depth of flow

b = width of channel

In order to use Equation (21), a cross-sectional velocity profile is required and then the dispersion coefficient calculated by dividing the cross-section into vertical slices and forming a table of columns for, calculation of various terms in the equation.

A disadvantage of Equation (21) is that it involves a knowledge of the lateral distribution of the velocity and this is often difficult to determine. Nevertheless, even though the equation looks formidable in application, since it was developed from a logical simplification of the basic dispersion theory and flow conditions in a natural environment, it has been hailed as the most superior of theoretical derivations of D_L [11].

3.2 Evaluation of D_L from Experimental Data Using Tracers.

The solution of Equation (15) provides the basis for various techniques used to compute D_L from concentration-time (c-t) data obtained at a fixed point or points along a channel as a tracer cloud moves downstream.

Solution of Equation (15) yields:

$$C_{(x,t)} = \frac{M}{A\sqrt{4\pi D_L t}} e^{-\frac{(x-ut)^2}{4D_L t}} \qquad (22)$$

Where, $C_{(x,t)} \equiv$ concentration of the tracer at given time t and distance x.

M ≡ Mass of tracer injected, and

A ≡ cross - sectional area of channel

Equation (22) can be treated as a concentration - distance profile if the concentration is measured at various points along the channel at a given time t. The equation predicts the normal curve for the distribution of a tracer with distance. If c-t data are obtained, at any distance x, the curve will not be normal because the denominator changes with time. Because of tailing, direct substitution of $C_{(x,t)}$,M,A, t,u and x into the solution for computation of D_L may be misleading. A computer programme for use with the solution to eliminate the tailing effect has been described [12].

Another technique for computing D_L from C-t data is the Change of Moment Method introduced by Fischer [13].

From Einstein's equation:

$$D_L = \tfrac{1}{2}\frac{d}{dt}\sigma_\xi^2 \qquad (23)$$

which applies to the tracer distribution during the diffusive period irrespective of what shape of distribution has been produced in the convective period, Fischer :deduced for uniform steady flow conditions:

$$\Delta\sigma_t^2 = U^2 \quad \Delta\sigma_\xi^2 \qquad (24)$$

Where, σ_t^2 is the variance of the time-concentration curve measured at a fixed station.

Thus when measurements are taken at various fixed points, a dispersion coefficient may be calculated by substituting Equation (24) into Equation (27).

Thus $$D_L = \tfrac{U^2}{2}.(\sigma_{t_2}^2 - \sigma_{t_1}^2) \qquad (25)$$

Where, $$\sigma_t^2 = \frac{\int_0^\infty (t-\bar{t})^2 Cdt}{\int_0^\infty Cdt} \qquad (26)$$

abd $$\bar{t} = \frac{\int_0^\infty tCdt}{\int_0^\infty Cdt} \qquad (27)$$

3.3 Empirical Determination of D_L

Many investigators have proposed expressions of D_L based on values established from experiments. Most of the expressions, no doubt, are applicable only to the given field situation and the same dynamic conditions under which the tests were performed. Some of the expressions are listed in Table 1.

4.0 OJIAKO PREDICTION METHOD

The great varieties in the analytical methods, experimental techniques and empirical formulations, give differing values of D_L which vary widely for the same flow cond1.tion. Some empirical expressions are altogether inapplicable to natural streams. The author [23] sees the reasons for inaccurate predictions from most of the expressions in the nature of equation (15) which lacks terms to account for sinks and sources of the dispersant in settlement and resuspension from stagnant places on the bed of flow. The bed roughness, bends, irregularity of the sides in the given flow regime, all combine to create pockets of stagnant pools into which some tracer elements diffuse and are temporarily (and in some cases permantly) detained, causing time lag in arriving at the sampling section. The result, as can be in Fig.2 is often a marked difference between observed and predicted dispersed profiles, shown in the shaded area. Observed distributions usually have long tails on the trailing portions and truncated on the leading edges. Equation (11) was modified by incorporating a forcing term to account for the deficiency.

Hence $$\frac{\partial c}{\partial t} = D_L\frac{\partial^2 C}{\partial x^2} - \bar{u}\frac{\partial c}{\partial x} + kc(c,t) \qquad (28)$$

The solution of this equation is

$$C(x,t) = \frac{Co}{2\sqrt{\pi D_L t}}\, e - \frac{-(x-\bar{u}t)^2}{4D_L{}^t} + kt \qquad (29)$$

k may, be determined from comparison of tracer profile and computer simmulation technique. k was studied using experimental channel runs of dye (Rhodamine B) tracer and well sieved organic particles of acrylic B and suspension polystyrene samples of various sieve sizes. Cut-bricks were used in creating roughness and dead zones on the channel bed. Results showed that k can be expressed numerically as

$$k = \frac{k_1 v^{3/2}}{\sqrt{Q}\left[Cos\,h(\mathrm{Re}_s)R_r\right]} \qquad (30)$$

TABLE 1 - SOME EMPIRICAL EXPRESSIONS FOR D_L

REFERENCE	FLOW FIELD	EXPRESSION FOR D_L
Krenkel and others [14]	Open Channel	$9.1U_*d$
Glover [15]	Natural Streams	$500U_*d$
Yotsukura and Fiering [4]	Open Channel	$13.0U_*d$
Patterson and Gloyna [16]	Open Channel	$0.8e^{0.34U\sqrt{A}}$
Fischer [17]	Natural Stream	$\frac{\overline{0.3u''^2\lambda^2}}{h_m U_*}$
Thackston and Krenkel [18]	Natural Streams	$7.25U_*d\left(\frac{U}{U_*}\right)^{1/4}$
Fischer [19]	Open Channel	$\frac{U^3}{2}\frac{d\sigma^2}{dx}(t)$
McQuivey and Keefer [20]	Natural Streams	$.058\frac{Q}{SeW}$
Fischer [21]	Natural Streams	$\frac{.011U^2W^2}{h_m U_*}$
Liu [22]	Natural Streams	$\frac{\beta U^2 W^3}{U_* A}$

λ = characteristic length in the cross - secton of the channel = the distance from the streamline of maximum velocity to the more distant blank.

S_e = Energy slope

$\beta = 0.18\,(\frac{U_*}{U})\,1.5$

W = Width of channel

h_m = Mean Hydraulic Radius

where v = velocity of flow

Q = discharge

R_{es} = Reynold’s number for the settling of the tracer particles

$= v_s d_s$ (31)

v_s = settling velocity of the tracer particle in quies medium

d_s = mean sieve diameter of the trace particles

v = Kinematic viscosity.

R_r = a roughness number.

$$\frac{k_2\sqrt{gRS}}{v} \qquad (32)$$

S = bed slope

R = Hydraulic mean radius

K_2 = height of equivalent sand grain roughness.

K_1 = is a constant numerically $\simeq$ 1.0

All the parameters above are measurable in any flow section of interest. It was initially difficult to estimatek_2• where, however, dye dispersion profile over the flow regime is available, k_2 is approximated from

$$k_2 = \frac{D(C_{pp} - C_{mp})}{C_{pp}} \qquad (33)$$

Where $C_{mp.}$ = measured peak concentration of the dye profile

C_{PP} = predicted peak concentration of dye profie

D = mean depth of flow

A typical dispersion result is shown in fig.3 for suspension polystreme of diameter passing sieve No.52 and retained in sieve No.72. The model data were collected from Ouseburn Stream test series.

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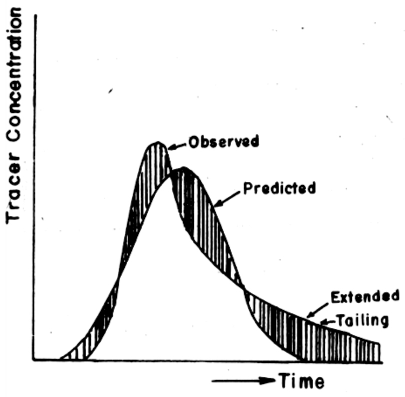

Fig.2 Typical dispersed flow corves

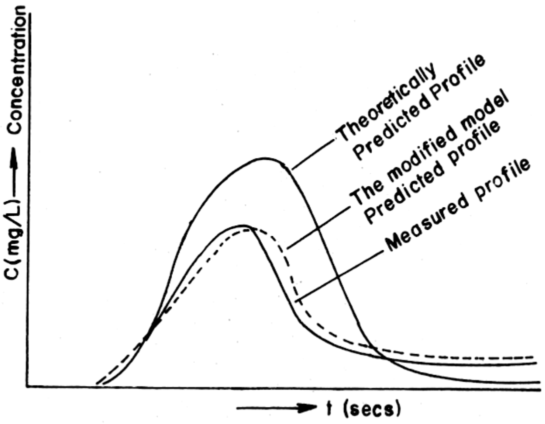

fig. 3. Modified Model, Measured and theoretical profile compared.