ANGULAR SPACE – TIME RELATIONS IN SOLAR RADIATION

by

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ABSTRACT

The engineer relies greatly on Meteorological data for solar energy applications. In most case the available equipments indicate only the hourly or daily total irradiance on a flat horizontal surface. However, a more basic or fundamental information may also be necessary especially when application call for a knowledge of the apparent motion of the sun relative to an observer on earth. In such cases, the controlling equations are often stated without proof.

The present submission synthesizes this aspect of spherical geometry in heavenly kinematics in form concise enough for engineering and meteorological applications. The final expressions will be familiar to workers in meteorology and to engineers involved in solar energy instrumentation and utilization on ground. The analyses are educational adaptations of engineering mechanics to this growing field of heliotechnoloy.

NOTATION [1]

α = solar altitude angle

β = surface tilt angle, towards Equator $+\beta$, away from Equator $-\beta$

γ = solar azimuth angle, clockwise from North

δ = solar declination angle

θ, i = incidence angle (on surface)

ϕ = latitude

ω = solar hour angle, solar noon = 0.00, afternoon +ve

L = longitude

t = time (hours)

INTRODUCTION

In solar energy measurements and applications some convenient reference co – ordinates are necessary for locating the Sun both in space and in time. The following study first generates compact spatial inter-relation between such relevant parameters as the solar declination, altitude, azimuth and hour angles as related to the observer's latitude [2,3]. Next, true Solar time is deduced in terms of the longitudes of the observer and the observer's time zone with a complementary modifier called the "Equation of Time" [2,4]. Insolation data must of necessity be reported on the basis of the True Solar Time often termed the Local Apparent Time.

This paper considers what is often taken as the necessary first step towards equipment installation and data reporting. In the case of the North-South or "Equatorially" mounted solar equipment, latitude, declination and azimuth settings are usually necessary if the apparent motion of the Sum is to be accurately tracked. Complexity then arises since the device must be geared to solar time using a clock-type actuating subsystem. In the more common utilitarian flat – plate collector assembly, the azimuth is fixed North-South or at the local solar noon position and the declination follower is the only gear that maybe varied with time (perhaps weekly and sometimes annually) since a location's latitude is constant for a rigidly fixed equipment, it is important that the optimum angle of tilt be employed, and this, in turn depends on the particular solar energy application being maximised.

General relations with particular reference to Nigeria are also considered.

Samples of optimum tilt angles are computed for maximised solar cooling in comfort room air-conditioning for some Nigerian cities.

2. THEORY

2.1 CO-ORDINATES ON THE LOCAL HORIZON PLANE: THE SOLAR AZIMUTH ANGLE (γ^0) AND ALTITUDE ANGLE (α^0)

On the local horizon plane, the Sun can be located in terms of the azimuth angle (γ) and the altitude angle (α). The azimuth is measured in the observer's horizon plane either:

i. from the direction of True North (N) through 360^0clockwise towards the East (E), or
ii. From the direction of True South (So) towards the West (positive) or East (negative), the empirical maximum value being 180^0.

In either case the angle (γ) is bounded by the plane that contains both the Sun (S) and the local zenith or directly overhead direction (θ_0). The altitude angle (α) is measured in the θ_0 - S plane from the horizon upwards to the sun.

It follows from Fig. 1 that if the point 0 represents the observer and

0S = 1.00

$$e_1 = \text{Cos}(\alpha) . \text{Sin}(\gamma)$$

$$n = \text{Cos}(\alpha) . \text{Cos}(\gamma) \qquad (1)$$

$$q = \text{Sin}(\alpha)$$

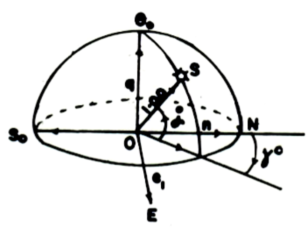


Fig. 1: Altitude and Azimuth Angles.

In effect, the scalar quantities e_1, n and q represent the decomposition values of the unit vector $\bar{S}$ $(= \vec{O}S)$ along the corresponding Cartesian co-ordinates. A more general vector expression takes the form

$$\bar{S} = [(\cos(\alpha) . \sin(\gamma)]\bar{E} + [\cos(\alpha) . \cos(\gamma)]\bar{N} + [\sin(\alpha)]\bar{\theta} \qquad (2)$$

Where $\bar{E}$, $\overline{N}$, and $\overline{\theta o}$ are unit vectors in the orthogonal system.

The events of sunrise and sunset occur when α = 0.00. Sunrise occurs in the local horizon plane on scent. Sunset is in the Easterly direction as the sun crosses the Westerly direction in which case the sun just goes below the horizon on ascent. When atmospheric refraction or optical aberration in neglected, the exact times for the two events are determinate in terms of the observer's latitude (ϕ^0). For this purpose the position of the sun has to be located in some more convenient co-ordinates with the Earth's centre as origin.

2.2. THE SOLAR DECLINATION (δ^0) AND THE LOCAL HOUR ANGLE (ω^0)

With the set of co-ordinates in which the Earth's Centre (C) is the origin, two Cartesian axes are located in the equatorial plane. The first is the original Easterly direction (E). The second is represented by the line joining C to the interception of the observer's longitude with the equatorial plane it is therefore a vector in the direction of the observer's zenith as if the observer were translated along his local longitude to the Equator. This is denoted as the meridian direction (M). The third co-ordinate is the North Pole direction (P) from the Earth's centre. It may be noted that while P denote the North Pole, N (used earlier) is a Northerly direction on the surface of the globe.

In this second triad the Declination of the Sun (δ) is measured from the equatorial plane to the Sun in the plane normal to the Equator and containing the Sun. The Solar Hour Angle (ω) is measured in the equatorial plane starting from the meridian direction towards the West. By implication the solar hour angle is zero at the moment of the Sun's transit over the meridian, positive after solar noon (when the sun is in the Westerly direction) and negative before solar noon.

Fig. 2 (a) and 2 (b) represent the triad and with CS = 1.00, the orthogonal components are:

$$e_2 = -\cos(\delta) . \sin(\omega)$$

$$m = \cos(\delta) . \cos(\omega) \qquad (3)$$

and $$p = \sin(\delta)$$

or

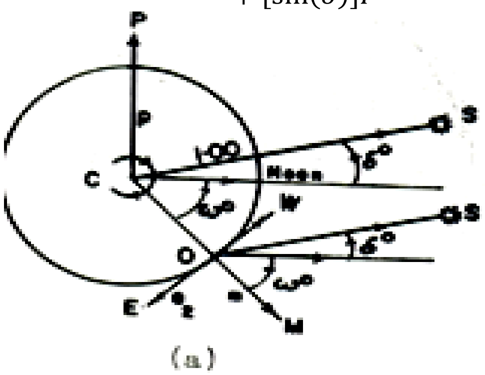
$$\bar{S} = [-\cos(\delta).\sin(\omega)]\bar{E} + [\cos(\delta).\cos(\omega)]\bar{M} + [\sin(\delta)]\bar{P}$$

(4)

In this case, the unit vector $\vec{C}S$ is decomposed in the direction CE, CM and CP.

In Fig. 2 (a) the equatorial plane is drawn with the observer at 0 having been displaced ω^0 from local solar noon. Both the Sun (S) and the North pole (P) are at a plane normal to the Equator, and from the sketch, the horizontal component of the solar beam must be due West.

Equations (3) and (4) may be deduced from Fig. 2 (a), but, Fig. 2 (b) gives the same results more directly except for a minor deficiency in visualization: it is



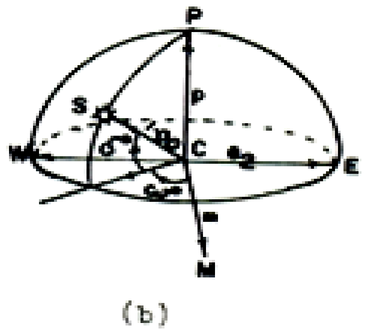


Fig.2: Declination and Hour Angles.

not as immediately obvious as in Fig. 2 (a) that, in the equatorial plane, the co-ordinates are Cartesian and that ω measured positive from solar noon.

The declination (δ) varies rather slowly over the period of one day but quite significantly within the year. The maximum and minimum values of δ occur at the solstices (about 21st June and 21st December). The variations are virtually sinusoidal with an amplitude (δ max.) of about 23^0 27' (= 23.45^0). this value represents what is normally referred to as the "Obliquity of the Ecliiptic" which is the angle at which the Earth's axis of rotation is inclined to its celestial orbit around the Sun. At the time of the Equinoxes (about 21st March and 21st September) δ = 0.00. One aspect of deduction from theory considers some acceptable working data base for solar declination angles. More precise values for any day within a year are reported annually in some Almanacs.

2.3 Rotated Co-Ordinates

A relationship is found between the first two sets of equations by a rotation of the co-ordinate systems around the direction E through the observer latitude angle (ϕ^0). Since the minimum Sun – Earth distance is about 1.445 x 10^8 km and the maximum planetary radius (Earth's Equator) is some 6.378 x 10^3 km, it follows that the Sun's distance from the Earth is well over 20,000 times large than the planetary radius. Such a rotation thus makes the observer position (0) and the Earth's centre (c) virtually coincident.

This is analogous to orthogonal vectors in triads for which

$$OS = \overrightarrow{CS} = \bar{S}$$

$$= e_2\bar{E} + m\bar{m} + p\bar{p}$$

$$= e_2\bar{E} + m\bar{M} + p\bar{P}$$

Since this rotation is about the direction

E,

$e_1 = e_2$ (5)

and the remaining components are such that $\bar{N}, \bar{\theta}_0$ and $\overline{m}, \bar{p}$ are pairs of perpendicular unit vectors in a plane. From the co – planner reproduction of Fig. 3 it is observed that

q = mCos (ϕ) + pSin (ϕ) and

n = mSin (ϕ) + pCos (ϕ) (6)

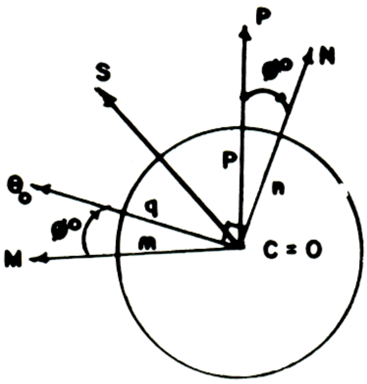


Fig. 3: Rotated Co-ordinates.

From equations (1) to (4)

e_1 = Cos (α) .Sin (γ)

n = Cos (α) .Sin (γ)

q = Sin (α)

e_2 = -Cos (δ) .Sin (ω)

m = Cos (δ) .Cos (ω)

and p = Sin (δ)

Using these in (5) and (6) the following transcendental equations are obtained:-

Sin (α) = Cos (ϕ) .Cos (δ) .Cos (ω) + Sin (ϕ) .Sin (δ) (7)

Cos (α) .Cos(γ) = -Sin (ϕ) .Cos (δ) .Cos (ω)+ Cos (ϕ) .Sin (δ) (8)

Cos(α) .Sin (γ) = -Cos (δ) .Sin (ω) (9)

In the above relations the azimuth angle (γ) is over 360^0 starting from true North. When measured from true South with East negative (i.e before solar noon) and West positive, (8) and (9) transform into:

Cos (α) .Cos(γ) = Sin (ϕ) .Cos (δ) .Cos (ω)-Cos (ϕ) Sin (δ) (10)

since $\text{Cos}(180 \pm \gamma)^0 = -\text{Cos}(\gamma)^0$.

and Cos(α) .Sin (γ) = ±Cos (δ).Sin (ω) (11)

since $\text{Sin}(180 \pm \gamma)^0 = \pm \text{Sin}(\gamma)$

Equations (7) to (11) from the basis for many kinematics relations necessary in solar irradiation measurements, collector tilts and general data reporting.

3. DEDUCTIONS

3.1 SOLAR AZIMUTH ANGLE

3.1.1 THE AZIMUTH ANGLES (γ) CONTINUOUS FROM TRUE NORTH

(a) From (9), Sin (ω)

= -Cos (α) .Sin (γ)/Cos (δ) .

or Sin (γ) = -Cos (δ) .Sin (w) /Cos (α)

(12)

The rising and setting azimuth Angle (γ_0) is determined in terms of ω_0 and δ by setting α = 0.00

$\text{Sin}(\omega)_{a=0} = \text{Sin}(\omega_0) = -\text{Sin}(\gamma_0)/\text{Cos}(\delta)$

or $\text{Sin}(\gamma_0) = -\text{Cos}(\delta).\text{Sin}(\omega_0)$ (13)

(b) Dividing (9) with (8) gives the general solar azimuth equation

Tan (γ) = Sin (ω)/ [Sin (ϕ) .Cos (ω) - Cos(ϕ) .Tan (δ)] (14)

(c) (7) x Sin(ϕ) + (8) x Cos (ϕ) results, on simplification, in the relation

Sin (δ) = Sin (α) .Sin (Φ) +Cos (α) .Cos (ϕ) .Cos (γ) (15)

It would follow from the above that the rising and setting azimuth angle (γ_0) also determinate in terms of δ and Φ resulting in the relation:

$\text{Cos}(\gamma_0) = \text{Sin}(\delta)/\text{Cos}(\phi)$ (16)

3.1.2 THE AZIMUTH ANGLE (γ) FROM TRUE SOUTH (WEST:- POSITIVE, EAST:- NEGATIVE)

From (11)

(a) Sin(γ) = ±Cos (δ) .Sin(ω)/Cos (α) (17)

$\therefore \text{Sin}(\gamma_0) = \pm \text{Sin}(\omega_0).\text{Cos}(\delta)$ (18)

(b) Tan (γ) = ±Sin(ω).[Sin(ϕ) . Cos (ω) - Cos (ϕ) .Tan (δ)] (19)

(c) Sin (δ) = Sin (α) .Cos (Φ) - Cos (α).Cos (ϕ) (20)

Hence $\text{Cos}(\gamma_0) = -\text{Sin}(\delta)/\text{Cos}(\phi)$ (21)

Equations (12) to (16) are of more generalised application but workers in the Northern Hemisphere, especially beyond the Tropics prefer those from (17) to (21) since the angle γ^0 is then never up to 90^0 when $\delta^0 < 0.00$ i.e when

the Sun is south of the Equator: cold climatic periods of more current concern, demanding solar domestic heating applications.

3.2 SOLAR ALTITUDE AND SOLAR TIME EQUATIONS

3.2.1 SOLAR NOON AND MAXIMUM SOLAR ALTITUDE ANGLE

From equation (7)

Sin (α) = Cos (Φ). Cos (δ).Cos (ω)+ Sin (Φ) .Sin (δ)

This expression, with the correct use of signs, is adequate for all observed values of the declination (δ). This declination is considered positive when the sun is North of the Equatorial plane and negative when south.

At the local Solar Noon the solar hour angle (ω) is zero and the sun attains its maximum altitude in the skies

$\therefore$ Sin (α_{max}) = Cos (ϕ) .Sin (δ)+Sin (ϕ).Cos (δ)

= Cos (ϕ - δ)

= Sin[90 ± (ϕ-δ)]

Hence (α_{max} = [90 ± (ϕ - δ)] (22)

For the Φ regime bounded by $\pm\delta_{max}$ (i.e. the Tropics)

α_{max} = [90 - (ϕ - δ) when δ<ϕ (23)

or α_{max} = [90 - (δ - ϕ) when δ>ϕ (24)

In essence the maximum possible solar altitude angle is 90^0 and occur when δ =ϕfor any location. Outside the Tropics the Sun never goes directly over the zenith and α_{max} <90^0 at all times.

3.2.2 **SUN-RISE TO SUN S-RISE DURATION AND DAY – LIGHT FRACTION (F)**

Since the Earth's rotation of 15^0 represents one hour on the 24 hour cycle, then from equation (7) with α_0 = 0.00, the solar hour angle and hence time difference between sun- rise and sun – set can be found

Thus 0.00 = Cos (ϕ) .Cos (δ).Cos (ω_0)+ Sin (ϕ).Sin (δ)

$\therefore\ \omega_0^0 = (15t_0)hrs$

$= \cos^{-1}[-\sin(\delta).\sin(\phi)\ /\cos(\delta)\,.\cos\phi]$

or $\omega_0^0 = (15t_0)hrs$

$= \cos^{-1}[-\mathrm{Tan}(\delta)\,.\,\mathrm{Tan}(\phi)]$ (25)

and $2t_0 = \frac{2}{15}\cos^{-1}[-\mathrm{Tan}\,\delta\,.\,\mathrm{Tan}\,\phi]$

= Day – Light Duration (26)

In the above relations t_o is hours from solar noon to sun – rise <u>or</u> sun – set.

Theoretically this difference in time between sun – rise and sun – set would be $2t_o$ hours since ω = 0.00 and hence t = 0.00 at solar noon. Allowance for the earth's curvature and atmospheric diffraction or aberration may be made using the empirical corrective relation [5]

$\alpha_o = -[0.833 + (0.0388)\ h^{0.5}]^o$ (27)

where h = local height above standard sea level in meters. Thus the day – light duration is slightly larger than the result obtainable from (26) especially at high ground elevations.

Many meteorological observatories report the sun – rise and sun – set times and duration using the upper edge of the Sun as the transit base across the horizon [6]. Where these are not available, and defining the "Day – light Fraction (F)" as the proportion of the 24-hour period for which the sun is above the horizon plane,

F = $2t_o/24 = t_o/12 = \omega_0^0/180^0$

$\therefore F = \frac{1}{180}\cos^{-1}[-\mathrm{Tan}(\delta)\,.\,\mathrm{Tan}(\phi)]$

or $F = \frac{1}{2} + \frac{1}{180}\sin^{-1}[-\mathrm{Tan}(\delta)\,.\,\mathrm{Tan}(\phi)]$ (28)

Since Cos (90 + A) = - Sin (A)

Day – light Duration ($2t_o$ hours) or Day – light Fraction (F) are important in the estimation of cumulative total irradiance received in the day.

The precise times or the events of sun – rise, noon and sun – set are more logically considered under Solar Time Relations.

3.2.3 **TRUE SOLAR TIME CONSIDERATIONS**

In an earlier paper [7], Solar Time Relations were presented. It is shown that the 24 – hour day does not reflect the actual duration of the Solar day. The later, called the sidereal day,

represents the exact period for one complete rotation of the planet Earth around the Sun. the average value of the sidereal day is 23.93447 hrs. [8]. It is therefore vital to know the corrections that must be imposed on the 24 – hour clock time in order to establish the more correct reference for Solar Time on which solar data must necessarily be based. This correction parameter (in minutes) is termed the “Equation of Time (E.T)”: an obvious misnomer since it is not an equation by any imagination. It is a time – scale modifier such that the duration of the true solar day on any date (for which the Equation of Time is known) becomes

Solar Day = 24 hours + (E.T) mins. (29)

As may be expected, the sidereal day itself is not exactly constant and the Equation of Time for any calendar date will vary slightly between successive years. This inherent variability would limit any deductions from data to approximate solution in the nearest minute only. Reference [4] gives such values for each day of the calendar year.

With Nigeria’s adapted Standard Meridian being 15^0E, reference [7] gives the following equations for any location within Nigeria

L. A. T = [(L.S.T - I) + L/15 + E.T/60] hrs. (30)

and $(L.S.T)_{noon}$ = 13, 00 - (L/15 + E.T/60) hrs. (31)

Where L.S.T. is the Nigerian Time as given by the clock and L^o is the exact longitude of the observer’s location. It is clear from equation (30) that True Solar Time (L.A.T) and the clock time (L.S.T) may not necessarily coincide in any location.

Furthermore, this time modifier affects the Day – Light fraction (F) of equation (28) in the sense that the sidereal day is not 24 hours but more nearly (24 + E.T/60) hours. However, this correction is often neglected in computing the fraction (F) mainly because the theory precludes atmospheric diffraction which has a grater influence on the observed day – light duration.

On combining (26) and (31), the precise clock times for sun – rise and sun – set can be written yielding:

$$(L.S.T)_{rise} = 13{,}00 - [L/15 + E.T/60] - \frac{1}{15}\cos^{-1}[-\mathrm{Tan}(\delta).\mathrm{Tan}(\phi)]hrs$$

$$(L.S.T)_{set} = 13{,}00 - [L/15 + E.T/60] + \frac{1}{15}\cos^{-1}[-\mathrm{Tan}(\delta).\mathrm{Tan}(\phi)]hrs \quad (32)$$

The corresponding Solar Times are:

$$L.A.T = 12{,}00 \pm \frac{1}{15}\cos^{-1}[-\mathrm{Tan}(\delta).\mathrm{Tan}(\phi)]hrs$$

(33)

(- for sunrise, + for sunset)

3.3 SOLAR DECLINATION

It is stated that the obliquity of the Earth’s ecliptic orbit around the sun is about 23.45^0. The Earth’s axis of rotation however wobbles very gentle like that of a retarding spinning top: tracing a complete circle in about 25, 000 years [2]. This so-called “Precession of the Equinoxes” affects the obliquity of the ecliptic. In consequence, the magnitude of the maximum and minimum declination (δ_{max}) varies slightly over the years and the precise times for the solstices and the equinoxes, also oscillate slowly with time. In 1975 the quoted obliquity of the ecliptic was 23.442534^0 or 23^0 26’ 33.123” [3]. The commonly accepted mean value is 23^0 27’8.26” or 23.4522940^0 [8]. A close engineering figure is 23.45^0 which yield the approximate working formula.

$\delta^0 = 23.45^0 \mathrm{Sin}\,[\,360\,(284 + N)/365]^0$ (34)

Where N = Day count starting from January 1 or January 2. The above relation has a quoted accuracy of $\pm 0.50^0$.

Table 1 gives results based on Klein’s “average day” for each calendar month [9]. Klein’s recommendation uses equation (34) with January 1 as first count and is useful where monthly declination settings are considered sufficient.

TABLE 1 SOLAR DECLINATION IN DEGREES (MONTHLY AVERAGES) [9]

Month	Jan.	Feb.	Mar	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Date	17	16	16	15	13	11	17	16	15	15	14	10
N	17	47	75	105	135	162	198	228	258	288	318	344
δ^0	-20.92	-12.95	-2.42	+9.41	+18.97	+23.09	+21.18	+13.45	+2.22	-9.60	-18.91	-23.05

Declination values for the 21st day of each calendar month in 1964 are used in the ASHRAE correlation [10]

$\delta = 23.47^0 \text{ Sin } [360 (284 + N)/365]^0$ (35)

Choice of the 21st day of each calendar month has other implications in Sun – Earth distances and the extra – terrestrial “solar constant”. These are outside the limited objectives of this paper.

In the more expansive 5 – day values [10] the declination values are accurate for the 21st day of each month in 1977 and eqn. (35) is used in establishing other entries.

Russo’s formulation [11] is based on the ideal pure Newtonian Sun – Earth system. The slightly more elegant (and more complicated) correction takes the form:

$\delta^0 = 23.45^0 \text{ Cos } [180 (A + N + T/24)/ 186]^0$ (36)

where A = 13 (12 for leap years)

N = year Date (January 1 = 1)

T = GMT (Greenwich Mean Time)

The equation thus takes daily variations of declination into account. For average day values

T = 12

Computation with the above relation reveal that, for correction declination signs, either 23.45^0 be replaced by -23.45^0 or

$\delta^0 = 23.45^0 \text{ Cos } [180 -180(A + N + T/24)/ 186]^0$ (37)

Equation, (37) is found to give the closest fit (when compared with others) with Almanac records over the years. Maximum error observed is of the order of $\pm 0.3^0$. However, the Almanac value for any date should be used if available.

4. SAMPLE APPLICATION OF THEORY

Table 2 represents Solar Space – Time Angles and Solar Times for the City of Lagos on the 21st day of December in any year. The results are as calculated using equations (7) to (12), (17), (30) and (31).

For Lagos,

Latitude (ϕ) = 06^0, 27’N = 6.45^0N

Longitude (L) = 03^0 , 24; E = 3.40^0E

On 21st December,

Declination (δ) = -23.40^0

E.T = +2mins.

TABLE 2: ANGLES AND TIME DERIVATIONS FOR LAGOS ON 21ST DECEMBER

L.S.T	ω^0	α^0	$\gamma_1{}^0$	$\gamma_2{}^0$	L.A.T
6,55 (R)	-87.2	0.00	113.6	-66.4	6,11
7,00	-86.1	1.0	113.7	-66.3	6,16
8,00	-71.1	14.5	116.2	-63.8	7,16
9,00	-56.1	27.6	120.7	-59.3	8,16
10,00	-41.1	40.0	128.1	-51.9	9,16
11,00	-26.1	50.7	140.4	-39.6	10,16
12,00	-11.1	58.2	160.4	-19.6	11,16
12,44(N)	0.00	60.15	180.0	0.00	12,00
13,00	+3.9	59.9	187.2	+7.2	12,16
14,00	+18.9	54.9	211.1	+31.1	13,16
15,00	+33.9	45.4	226.8	+46.8	14,16
16,00	+48.9	33.7	236.2	+56.2	15,16
17,00	+63.9	20.9	241.9	+60.9	16,16
18,00	+78.9	7.5	245.3	+65.3	17,16
18,33(S)	+87.2	0.00	246.4	+66.4	17,49

L.S.T = Local Clock Time, L.A.T = True Solar Time, (R) = Sun–rise, (N) =Solar Noon, (S) = Sun–set

From the above

i. Clock Time and Solar Time can differ appreciably.

ii. Altitude Angle at Solar Noon

$= \underline{60.15^0} = \alpha_{max}$

As a check, $\alpha_{max} = [90 - (\phi\text{-}\delta)]^0$, (23)

$= [90 (6.45 + 23.40)]^0$

$= \underline{60.15^0}$

γ_1 = azimuth from true North

γ_2 = azimuth from true South

Although this is virtually a limiting situation, the sun does not pass daily through or even close to the zenith as our man–in–the–street may want to believe.

TABLE 3: COMPARISON WITH "AGROMET DATA"

(a) AGROMET ENTRIES

Date in 1974	Rise (a.m)	Set (p.m)	Duration Hrs.	Min
March 4	6,53	6,49	11	56
11	6,48	6,50	12	02
18	6,45	6,50	12	05
25	6,41	6,50	12	09

Other observations from results include the Day – Light duration of 11.633 hrs and the Day – Light Fraction of 0.484 each of which checks with equations (26) and (28) respectively.

Table 3 compares sun–rise, sun–set and duration data as reported in the Nigerian Agrometeorological Bulletin [6] with corresponding theoretical evaluations.

Agromet Station: Sokoto

Latitude (ϕ) = 13^0, 01'N = 13.0167^0N

Longitude (L) = 05^0, 15'E = 5.2500^0E

Elevation [12] = 350 metres

The above evaluations are based primarily on equations (25) and (26) and can be seen to under–estimate the day–light duration by about 9 minutes only. Use of the empirical correction factor given in (27) adds about 13 minutes to these theoretical values and hence slightly over–estimates the data.

5. DISCUSSIONS

The fore-going represent fairly general and concise expressions for adequately tracking the solar beam. For concentrating devices and direct insolation recorders like normal incidence pyrheliometers and cavity radiometers it would be obvious that the actuating mechanism must be geared to solar time. For the others the position of the Sun relative to the collector should be known since the amount of direct irradiance is proportional

(b) CALCULATED RESULTS

Date in 1974	E.T (mins)	N	δ^0	Rise (a.m)	L.A.T. (noon)	Set (p.m)	Duration Hr	Mins.
March 4	-12	62	-7.53	6,58	12,51	6,44	11	46
11	-10	69	-4.81	6,53	12,49	6,44	11	51
18	- 8	76	-2.02	6,49	12,47	6,45	11	56
25	- 6	83	+0.81	6,44	12,45	6,46	12	01

to the Cosine of the incidence angle (i^o)

Generally, solar energy equipments demand large area collectors and daily azimuthal adjustments would be complex and expensive. In practice, therefore most collectors are either permanently fixed or have their tilt angles slowly varied over the seasons as determined by the slowly varying solar declination and also by the desired duty expected of the installation. In the Northern Latitudes, solar space heating is a necessity and for this reason the collector tilt (β^0) is optimized for the cold winter months. A location that is 40^0 N in latitude may well have a collector tilt angle of 50^0 to 55^0 from the Northern horizon plane and facing South. The commonly recommended angle in such climates is

$\beta^0 = +$ (Latitude $+10^0$).

Obviously, this does not represent the optimum year – round value for maximum solar intensity that may be obtained from equation (22).

The above is an example where the season determines the equipment duty and hence the declination selection. Within the Tropics, solar energy is scarcely if ever required for space heating, and yet, as much solar energy as possible is normally required for all the known and envisaged applications. Solar grain dryers or solar domestic water heaters all demand maximal solar energy input. Solar coolers require elevated temperatures for any one of the thermodynamic cycles currently on trial, and the same goes for solar power systems. Since maximum intensity is thus desired, adjusting the collector to maximize energy harvest at solar noon is vital in all cases where azimuth tracking cannot be achieve or is undesirable. It follows that equation (22) becomes valid but the declination angle is still determined by the desired duty of the equipment. In the case of solar drying of agricultural products the harvest season may well be the deciding factor for solar cooling of buildings, the maximum demand will be during the hottest season.

Fig. 4 represents a location with latitude ϕ^0 North ($\phi^0<23.45^0$), while the solar declination is δ^0 North. At solar noon the optimum tilt angle (β^0) for this collector must be such that $\beta^0 = (\phi - \delta)^0$ from the direction of true North in the observer's horizon plane and facing South.

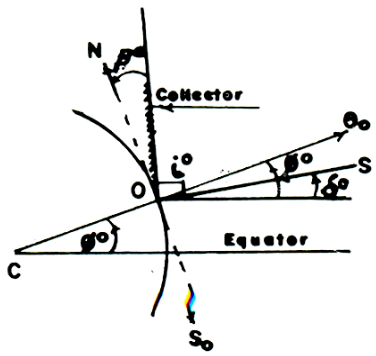


Fig. 4: Collection Tilt Angle.

Equations (22) to (24) indicate this as a condition for normal incidence. Since the latitude angle (Φ) is fixed for any location, the season of any required duty (which determines δ) would decide the angle of best tilt. Table 4 gives ranges of flat collector tilt angles as calculated for some Nigerian cities on the basis of the Nigerian society of Engineer's Code of Practice for Comfort Air-conditioning [12]. All tabulated optimum tilt angles apply only to rigidly fixed collectors for maximising solar energy harvest at local solar noon.

Design Month is on basis of maximum comfort cooling demands. Thus, if the duty is space over a limited period, declination averaging may be desirable and if over the entire year then the annual mean could be used. This averaged value for the year is represented by $\beta = \phi$

Since $\delta_{average} = 0.00$.

Finally, the response time of any energy collecting or measuring device varies with the equipment and in practice an azimuth shift or orientation of up to 10^0 past solar noon may by advisable. The choice is arbitrary and

TABLE 4: RECOMMENDED TILTS OF STATIONARY FLAT PLATE COLLECTORS FOR SOME NIGERIAN CITIES (COOLING MODE ONLY)

City	Latitude (ϕ) [6]	Cooling Design Month [12]	Declination (δ^0) [9]	Collector Tilt (β^0)
Benin	06^0,19'N	March	-2.42	+8.74±5
Calabar	04^0,58'N	March	-2.42	+7.39±5
Enugu	06^0,27'N	March	-2.42	+8.87±5
Ibadan	07^0,26'N	March	-2.42	+9.85±5
Ilorin	08^0,29'N	March	-2.42	+10.90±5
Jos	09^0,29'N	April	+9.41	+0.46±4
Kaduna	10^0,29'N	April	+9.41	+1.07±4
Kano	12^0,03'N	April	+9.41	+2.64±4
Lagos	06^0,27'N	March	-2.42	+8.87±5
Maiduguri	11^0,57'N	April	+9.41	+2.54±4
Port Harcourt	04^0,51'N	February	-12.95	+17.80±4
Sokoto	13^0,01'N	April	+9.41	+3.61±4

$\beta^0 = (\phi - \delta)^0$, + = Inclination to the North Horizontal Plane and facing South (i.e. faced towards the Equator)

- = Inclination to the South Horizontal Plane and facing North or faced towards the North pole.

$\pm 5^0$ and $\pm 4^0$ represent the limiting variations in the month. Thus $\pm 5^0$ in March 1 approximately and -5^0 is for March 31.

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