ANALYTIC METHOD FOR PRESSURE RECOVERY IN TRUNCATED DIFFUSERS

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ABSTRACT

A prediction method is presented for the static pressure recovery in subsonic axisymmetric truncated conical diffusers. In the analysis, a turbulent boundary layer is assumed at the diffuser inlet and a potential core exists throughout the flow. When flow separation occurs, this approach cannot be used to predict the maximum pressure location. The analysis can easily be modified to accommodate two-dimensional diffuser and conical diffusers fitted with tailpipes.

The predicted overall static pressure recovery compares favorably with available experimental results and agreement is generally within ±5 per cent.

NOTATION

A	Cross-sectional area
AR	Diffuser area ratio
C_F	Local skin friction coefficient
C_P	Statistic pressure recovery coefficient defined as $\left(\bar{p}_2 - \bar{p}_1\right) \quad \left(\bar{p}_{01} - \bar{p}_1\right)$
H	Boundary layer shape parameter (= δ^* /θ)
$H_1, \bar{H}$	Shape parameters defined in the text
M	Mach number
m	mass flow rate
P_o, P	Pressures: total and static
R	Local radius
R*	Throat Radius
R_e	Reynolds number base on pipe diameter
R_θ	Reynolds number base on momentum thickness
R_O	Gas constant
r	Recovery factor = $(T_r - T_e \;/\; (T_o - T_e)$
X	Axial distance
T_o, T, T_r	Temperatures: total, static and recovery.
u	Velocity
δ	Boundary Layer thickness
δ^*, θ	Boundary layer displacement and momentum thickness
ρ	Local density of fluid
α	Diffuser cone angle
γ	Ratio of specific heats
μ	Dynamic viscosity

Subscripts

e	Free stream condition
s	Value at separation point
t	Tailpipe value
1,2	Inlet and outlet

Bar indicates mean values over cross-section

1. INTRODUCTION

A diffuser which terminates in a sudden enlargement into a parallel section is termed a truncated diffuser. In Fig 1, ABCD defines a full diffuser fitted with a tailpipe, ABED defines a truncated differs and AFEC represents a pipe flow with sudden enlargement.

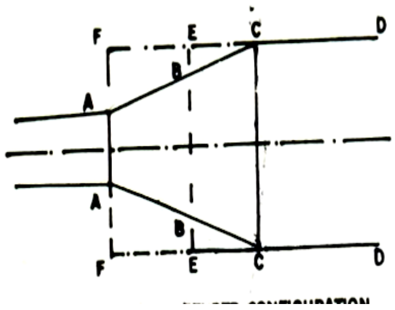

FIG I: DIFFUSER CONFIGUATION

Fig I: Diffuser configuration

In dealing with certain practical design situations where diffuser length is restricted by space consideration, it might be preferable to use a truncated diffuser of low cone angle rather than a wide angle diffuser. The former will give a more stable and symmetric flow than the latter, although at the expense of some loss in the static pressure recovery

A systematic survey of literature on this topic revealed that only in [1] and [2] has performance prediction of truncated diffusers been considered. In [2] the loss in a truncated diffuser was given by the sum of the loss in the remaining conical portion treated as a full diffuser discharging into a parallel tailpipe and the loss at the sudden enlargement. In [1] the author postulated that the loss of a truncated diffuser would be greater than that of a full diffuser by the amount of the sudden enlargement loss and presuppose these theories given knowledge of the full diffuser and sudden elargement losses for the particular diffuser geometry and inlet conditions.

In this paper, the authors have developed a prediction method which will give the overall static pressure recovery coefficient for truncated diffusers when the inlet conditions are specified. The pressure rise within the diffuser cone is limited by the growth of the boundary layer on the diffuser wall, hence a reliable turbulent boundary layers prediction method has been invoked in the analysis.

2. THE ANALYSIS

The flow model is treated as three distinct regions, namely,

i. a region of attached flow,
ii. a separated region, and
iii. a region of flow re-attachment and development.

For the three regions the following assumptions are applicable, viz,

a) The fluid is a perfect gas and the recovery factor is constant.
b) The flow is axi-symmetric.
c) A potential core exists throughout the flow region.
d) Adiabatic flow, i.e. heat transfer to or from the diffuser is neglected.
e) Static pressure is a function of axial distance only.
f) Radial velocities are negligible.

For the attached flow the only additional assumption is that the flow will separate if $H = 3.1$ or $d\theta/dx = 0.012$

For the separated flow it is also assumed that an axial zone of constant pressure extends into the parallel pipe. Skin friction term is neglected.

For reattaching flow which takes place in the parallel pipe the skin friction is neglected [3].

2.1 ATTACHED FLOW

In this region the problem reduces to the calculation of compressible turbulent boundary layers. Green's method [4] is employed in the analysis. Following Green, Morkovim's hypothesis is invoked to extend to the compressible flow the existing relations for entrainment function. The boundary layer momentum integral equation is solved simultaneously with the diffuser overall continuity equation and an auxiliary equation derived from consideration of the rate of entrainment of fluid into the boundary layer. The basic equations are listed below. The momentum-integral equation:

$$\left(1-\frac{\theta}{R}\right)\frac{d\theta}{dx}+\theta\left[\frac{1}{u_e}\frac{du_e}{dx}(2+H)-\frac{\theta}{2R}(H^2+2)\right]+\frac{1}{\rho_e}\frac{d\rho_e}{dx}\left(1-\frac{\theta}{2R}\right)+\frac{1}{R}\frac{dR}{dx}=\frac{C_f}{2}$$, or

$$\frac{d\theta}{dx}+\theta\left[(2+H-M_e^2)\frac{d(1nu_e)}{dx}+\frac{d(1nR)}{dx}\right]=\frac{C_f}{2} \qquad (1)$$

The skin-friction equation: Modified Ludwig and Tillman correlation.

$$C_f=0.246\frac{T_e}{T_m}exp(-1.561\bar{H})\bar{R}_\theta \qquad (2)$$, where

$\bar{R}_\theta=\frac{\mu_e}{\mu_m}R_\theta$, μ_m = viscosity at temperature T_m

$$\frac{T_m}{T_e}=1+1.44\gamma M_e^2$$

$$H=\int_0^\delta \frac{\rho}{\rho_e}\left(1-\frac{u}{u_e}\right)$$

The continuity equation $\rho_e u_e(R-\delta^*)^2=constant$, or

$$\frac{R-H\theta}{2}(1-M_e^2)\frac{d(lnu_e)}{dx}+\frac{dR}{dx}-\frac{Hd\theta}{dx}-\theta\frac{dH}{dx}=0 \qquad (3)$$

In the entrainment equation for an axisymmetric flow with a thin boundary, the entrainment function is given as:

$$F=\frac{1}{\rho_e u_e R}\frac{d}{dx}\left[R\int_0^\delta \rho u dy]\right] \qquad (4)$$

Using the definition of mass flow thickness,

$\Delta=\int_0^\delta \frac{\rho u}{\rho_e u_e}dy=\delta-\delta^*$, equation (4) becomes

$$H_1\frac{d\theta}{dx}+\theta\frac{dH_1}{dx}=F-H_1\theta\left[\left(\frac{d(lnR)}{dx}+(1-M_e^2)\frac{(lnu_e)}{dx}\right)\right] \qquad (5)$$

Correlation between H_1 and H is obtained as:

$$\frac{dH_1}{dx}=\emptyset_1\frac{dH}{dx}+\emptyset_2\frac{d(lnu_e)}{dx} \qquad (6)$$

$$\emptyset_1=\frac{-1.2(\bar{H}-1)^{-7/3}}{1+0.2rM_e^2} \qquad (6a)$$

$$\emptyset_2=-0.4r\emptyset_1 M_e^2(1+0.2M_e^2)(H+1) \qquad (6b)$$

Re-writing (1), (3), and (5) in a non-dimensional form using the definitions

$$\theta'=\frac{\theta}{R_1}, x'=\frac{x}{R_1}$$

$$R'=\frac{R}{R_1}, \theta''=\frac{d\theta'}{dx}, H''=\frac{dH}{dx}$$

$$R''=\frac{dR'}{dx}, u_e'=\frac{d(lnu_e)}{dx}$$

and eliminating u'_e from equations (3) and (5) using equation (1), the system reduces to two simultaneous differential equations in the unknowns H and θ

$$aH'' + b\theta'' = C \quad (7)$$

$$eH'' + f\theta'' = g \quad (8)$$

Where

$$a = \theta'\emptyset_1 , b = H_1 - A$$

$$c = F - \frac{AC_f}{2} - (H_1 - A)\theta' \frac{R''}{R'}$$

$$e = \theta' , f = H + B$$

$$g = R''\left(1 - B\frac{\theta'}{R'}\right) + B\frac{C_f}{2}$$

$$A = \frac{\emptyset_2 + H_1(1 - M_e^2)}{2 + H - M_e^2}$$

$$\boldsymbol{B} = \frac{(\mathrm{R} - \mathrm{H}\theta)(1 - \mathrm{M_e^2})}{2\theta'(2 + \mathrm{H} - \mathrm{M_e^2})}$$

The solution is obtained by a simultaneous step by step integration of equations (7) and (8) starting from prescribed inlet conditions using a fourth-order Runge-Kutta principle.

2.2 Separated Flow

Momentum integral equation reduces to

$$\left(1 - \frac{\theta}{R}\right)\frac{d\theta}{\theta} + \frac{1}{R}dR = 0 \quad (9)$$

Continuity equation: $\frac{d}{dx}(R - \delta^*)^2 = 0$ (10)

Integrating these equations, we obtain the displacement and momentum thickness as:

$$\delta^* = \delta_s^* + (R_1 - R_s) \quad (11)$$

$$\theta = R_t + \tfrac{1}{2}[4R_t^2 - 4\theta_s(2R_s - \theta_s)]^{\frac{1}{2}} \quad (12)$$

2.3 Re-Attaching Flow

Continuity equation:

$$\frac{2d\delta^*}{(R_t - \delta^*)} = \frac{(1 - M_e^2)dM_e}{(1 + M_e^2/5)M_e} \quad (13)$$

Integrating equation (13) we obtain

$$(R_t - \delta^*)^2 = \frac{(1 + M_e^2)}{M_e}Z$$

Where Z is a constant given by

$$Z = (R_s - \delta_s^*)^2 M_s/(1 + M_s^2/5)^3$$

$$\delta^* = R_t - \{(1 + M_e^2/5)^3/M_e\}^{\frac{1}{2}}$$

Momentum-integral equation (neglecting C_f):

$$\frac{d\theta}{dM_e} = -\frac{\theta\left[(2 + H - M_e^2) - \frac{\theta}{2R_t}(2 + H^2 - M_e^2)\right]}{M_e(1 + M_e^2/5)\left(1 - \frac{\theta}{R_t}\right)} \quad (14)$$

substituting the expression for δ* in equation (14) , an ordinary; differential equation with M_e and θ as variables is obtained. The solution is obtained by step-by-step integration of equation starting from the results already obtained for separated flow. For the prediction to be accurate, the calculations must be terminated at the maximum pressure position. (dp/dx = 0 which implies that dδ*/dx = 0).

Fig. 2 shows the growth of boundary layer parameters (H and θ) for 15 degrees diffusers with initial Mach number of 0.6 and θ_1 /R* of 0.0025.

2.4. THE OVERALL STATIC PRESSURE RECOVERY

The pressure recovery was obtained using the assumption of constant total pressure in the core flow. Momentum balance was employed in the tailpipe analysis. This is given as

$$P[R_t^2 + \gamma M^2\{(R_t - \delta^*)^2 - \theta(2R_t - \theta)\}] = constant \quad (15)$$

When this equation is applied between the tailpipe inlet and the maximum pressure position, the pressure recovery in the tailpipe is obtained. The sum of these two recoveries gives the overall pressure recovery.

In the analysis of the performance of propulsion devices, diffusers, or other systems in which there is a flow of fluid in an enclosed duct, it is desirable to use a one – dimensional representation of the fluid. Since in reality the flow properties, viz, veloicity, pressure etc. are generally

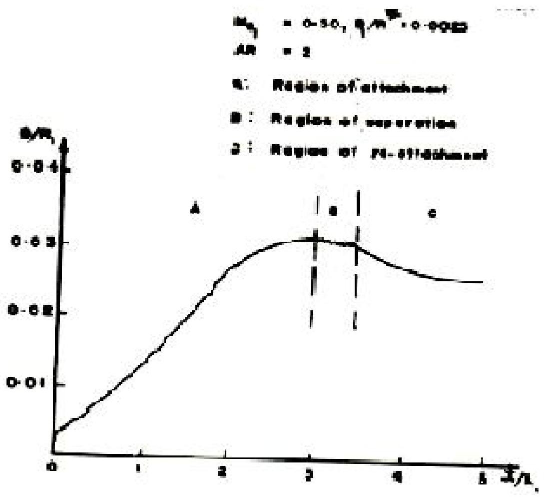

Fig. 2(a): *Growth of Boundary Layer Momentum Thickness in Truncated 15⁰ Diffuser.*

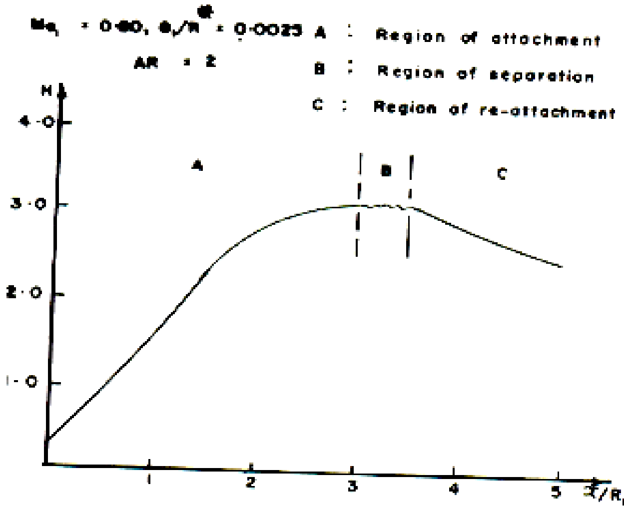

Fig. 2 (b): Growth of Boundary Layer shape Factor in Truncated 15⁰ Diffuser.

non–uniform at any station of the flow, a working knowledge is needed of the averaging procedures which are commonly employed to describe the integrated properties of the flow. The mass momentum average definition [5] is used in the present analysis and the static pressure recovery coefficient is defined as:

$$C_p = (P - \bar{P}_1)/(\bar{P}_{01} - \bar{P}_1) \qquad (16)$$

$$\bar{P}_0 = \bar{P}\left[1 + \frac{\gamma-1}{2}\bar{M}^2\right]^{\gamma/(\gamma-1)} \qquad (17)$$

$$\frac{\int_A [P(1+\gamma M^2)dA])}{(1+\gamma\bar{m}^2)A} \qquad (18)$$

$\bar{M}$ is defined by:

$$\frac{m(R_O T)^{\frac{1}{2}}}{\int_A P(1+\gamma M^2)dA} = \frac{\bar{M}\left[1+\frac{\gamma-1}{2}\bar{M}^2\right]^{\frac{1}{2}}}{(1+\gamma\bar{M}^2)}$$

3. COMPARISON WITH EXPERIMENTAL RESULTS

In Table 1 the predicted and measured static pressure recovery coefficients [6] for four diffuser configuration, viz,

i. $\alpha = 10^0$, AR = 2.0 , 1.5
ii. $\alpha = 15^0$, AR = 2.0 , 1.5

are compared. The inlet Mach number for the 10^0 diffuser was 0.7 and for 15^0 diffuser 0.2.The boundary layer at the diffuser inlet was thin in all cases and the ratio $\delta_1^*/2R_1$ is approximately 0.005.

TABLE 1: COMPARISON OF PREDICTED OVERALL STATIC PRESSURE RECOVERY COEFFICIENTS WITH EXPERIMENTAL DATA OF [6]

Diffuser Included Angle	$\frac{\delta_1^*}{2R_1}$ (approx)	AR	$\bar{M}_1$	C_P	
				Predicted	Experiment [6]
10^0	0.005	1.5	0.7	0.65	0.68
10^0	0.005	2.0	0.7	0.76	0.71
15^0	0.005	1.5	0.2	0.58	0.60
15^0	0.005	2.0	0.2	0.63	0.66

From the table it is observed that the experimental points lie within ±5 per cent of the theoretical values. There is no systematic variation of error with truncation or inlet Mach number.

In Fig. 3 the predicted and measured variations of pressure recovery are shown for 15^0 diffuser of area ratio of 1.5 and inlet Mach number of 0.2. The maximum variation is about four per cent.

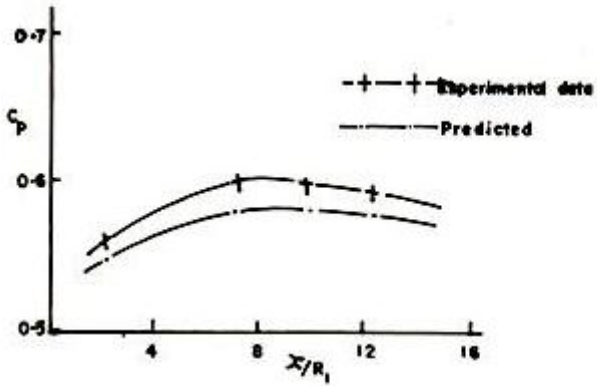

Fig. 3: predicted and Experimental; Axial pressure Distribution (15^0 Diffuser; AR 1.5, $\bar{M}_1 = 0.2$ *)*

4. CONCLUSIONS

A prediction method for the pressure recovery of truncated diffusers for adiabatic flow wth turbulent inlet boundary layers has been evolved. The predicted results compare favorably with available experimental results and the theory demonstrates that precision is attainable over a wide range of inlet Mach number and for different geometrical configurations. Mach numbers of 0.2 and 0.7 used in comparing the theoretical and measured results are within the incompressible regime and close to choking condition respectively.

It ought to be mentioned that the analysis applied to axi–symmetric flows, but it can be modified for used with two-dimensional diffusers.

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