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# *Technical Note:* PROBABILISTIC FAILURE ANALYSIS OF A SOLID TIMBER COLUMN

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**Abstract**

***In this paper, a reliability evaluation of a solid timber column of square cross section subjected to axial and lateral loading in accordance with the design requirements of Eurocode 5 is reported. The First Order Reliability Method (FORM 5) which was written in FORTRAN language was used in the reliability estimation. The results obtained showed that both load and slenderness ratio have effects on the reliability of a solid timber column. It was also shown that the safety of such a column can be enhanced if adequate and suitable dimensions are chosen to have a lower slenderness ratio.***

**Keywords:** reliability evaluation, solid timber column, Eurocode, axial loading, slenderness ratio

## 1. Introduction

The quality of an engineering structure is reflected by the extent to which the structure can withstand or resist various load conditions to which it is exposed. The main aim of structural design is with due regard to reliability, safety and economy, which guarantee any of the limit states being reached or exceeded during the expected life of the structure (1-6). The violation of limit states may cause risks and damages most especially to human lives and also results in economic losses (7-8). For this reason, it is usually verified that the limit states (states at which the structure no longer performs the intended purpose) are not reached when design values for loads, material properties and geometrical data are used in the design equations. However, attaining a limit state might prove impossible and difficult due to the stochastic nature of the design parameters such as material properties and geometrical data. Structural reliability study hence becomes very essential. The task of the structural engineer is to design and maintain the structure so that failed state is differed and in this task, he faces the problem inherent in the variability of engineering materials. However, structural reliability study often basically provides rational means of dealing with such uncertainties that are inherent in structures by the use of statistical approach. Although the use of probabilistic concept may not answer all issues of unknowns, it has helped in no small measure in the reliability evaluation of many engineering structures (9-10).

The objective of this paper is to evaluate the reliability of a solid timber column of square cross section subjected to both axial and lateral loading according to the design requirements of Eurocode 5 using First Order Reliability Method.

## 2. Formulation of Performance Functions

The performance function is derived based on Eurocode 5 requirements for timber columns (11). The timber column considered is two hinged with square cross-section subjected to axial and lateral loading.

### *2.1. Compressive stress in column*

The design compressive stress in parallel to the grain is given by:

$$\sigma_{c,d} = \frac{Q_1}{A} = \frac{Q_1}{b^2} \tag{1}$$

Where, $Q_1$ = design load, $A$ = cross-sectional area.

The design value for compressive strength parallel to the grain is given by:

$$f_{c,d} = \frac{K_{mod} f_{c,d}}{\gamma_m} \tag{2}$$

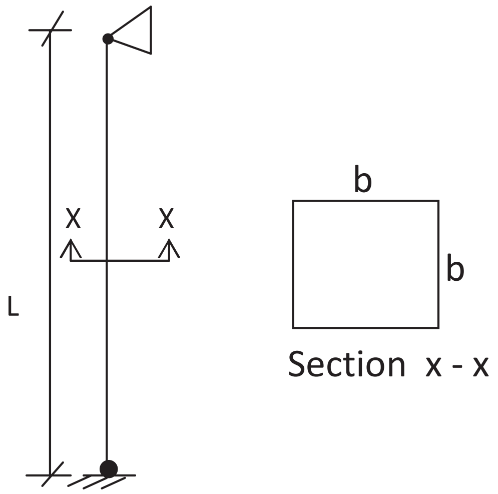


Figure 1: A two hinged column.

Where, $K_{mod}$ = modification factor taking into account the effects of the strength parameters of the duration of action and moisture contents, $\gamma_m$ = partial safety factor for the material property based on Eurocode 5, $f_{c,k}$ = characteristic value of the compressive strength.

It is required that

$$\sigma_{c,d} \leq f_{c,d} \tag{3}$$

Therefore, the performance function $G(x)$ is given by:

$$G(x) = f_{c,d} - \sigma_{c,d} \tag{4}$$

Substituting for $f_{c,d}$ and $\sigma_{c,d}$ using equations (1) and (2) transforms equation (4) to:

$$G(x) = \frac{K_{mod} f_{c,k}}{\gamma_m} - \frac{Q_1}{b^2} \tag{5}$$

### 2.2. Bending stress in column

The design bending stress parallel to grain is given by:

$$\sigma_{m,d} = \frac{M}{Z} \tag{6}$$

Where

$$M = \frac{Q_2 L^2}{8} \tag{7}$$

and

$$Z = \frac{b^3}{6} \tag{8}$$

Substituting for $M$ and $Z$ in equation (6) using equation (7) and (8) gives:

$$\sigma_{m,d} = \frac{0.75 Q_2 L^2}{b^3} \tag{9}$$

Where $Q_2$ = short term load.

The design value for bending strength parallel to grain is given by:

$$f_{m,d} = \frac{K_{mod} f_{m,k}}{\gamma_m} \tag{10}$$

Where $f_{m,k}$ = characteristic value of the bending strength.

It is required that:

$$\sigma_{m,d} \leq f_{m,d} \tag{11}$$

Therefore, the performance function is given by:

$$G(x) = f_{m,d} - \sigma_{m,d} \tag{12}$$

Substituting for $f_{m,d}$ and $\sigma_{m,d}$ using equations (9) and (10) gives:

$$G(x) = \frac{K_{mod} f_{m,k}}{\gamma_m} - \frac{0.75 Q_2 L^2}{b^3} \tag{13}$$

## 3. Materials and Methods

The First Order Reliability Method provides appropriate computation of general failure probability which gives approximate solution to a state or system with variables some of which are uncertain. These uncertain variables are random. The random variables $X = (X_1, X_2, \ldots, X_n)$ are called basic variables with joint probability function

$$F_x(x) = P\left(n_{i=1}^{n} \{X_i \leq x_i\}\right)$$

$F_x(x)$ is at least locally differentiable. That is, the probability density exists. The performance function $G(x)$ of a structure at a limit state is usually modeled interms of certain finite uncertain basic variables which are random in nature. It is defined mathematically such that: $G(x) > 0$ represents safe, intact, acceptable domain; $G(x) = 0$ represents limiting state of failure boundary, unacceptable and adverse; $G(x) < 0$ represents failure, unacceptable adverse domain. Therefore, a first order approximation to probability of failure is given by:

$$P_f = P(x \in F) = P(G(x)) \leq 0 = \int dF_x(x) G(x) \leq 0 \tag{14}$$

A relationship exists between reliability index and probability of failure [5]. The probability of failure is estimated by:

$$P_f \approx \phi(-\beta) \tag{15}$$

Where, $G(\cdot)$ = standard normal integral and $\beta$ = geometric or reliability index, defined as

$$\beta = \min\{\| x \|\} \text{ for } \{X : G(x) < 0\} \tag{16}$$

It is shown to be the minimum distance between the origin of dimensional co-ordinate system of the basic variables and the failure surface.

Table 1: Statistics of basic variables for a solid timber column under vertical loading.

| Variables | Probability | Mean | Standard deviation | COV |
|---|---|---|---|---|
| $P_1$ | Gumbel | 65,000N | 19500N | 0.30 |
| $K_{mod}$ | Lognormal | 0.90 | 0.135 | 0.15 |
| $f_{c,k}$ | Lognormal | 21N/mm$^2$ | 3.15N/mm$^2$ | 0.15 |
| $b$ | Normal | 300mm | 3mm | 0.01 |
| $\gamma_m$ | Lognormal | 1.30 | 0.195 | 0.15 |

Table 2: Statistics of basic variables for a solid timber column under lateral loading.

| Variables | Probability | Mean | Standard deviation | COV |
|---|---|---|---|---|
| $P_2$ | Gumbel | 3.25N/mm | 0.975N/mm | 0.30 |
| $K_{mod}$ | Lognormal | 0.90 | 0.135 | 0.15 |
| $l$ | Normal | 3000mm | 30mm | 0.01 |
| $b$ | Normal | 300mm | 3mm | 0.01 |
| $f_{m,k}$ | Lognormal | 24N/mm$^2$ | 3.6N/mm$^2$ | 0.15 |
| $\gamma_m$ | Lognormal | 1.30 | 0.195 | 0.15 |

***Numerical Example***

A solid timber column of square cross-section. Subjected to vertical and lateral loading. From equations (5) and (13), the capacity- demand relationship for a solid timber column of square cross-section subject to vertical and lateral loading given by:

$G(x)$ = Design compressive strength parallel to grain - Design compressive stress parallel to grain

and

$G(x)$ = Design bending strength parallel to grain Design bending stress parallel to grain.

## 4. Discussion of Results

The reliability indices and their corresponding failure probabilities determined for different load ratio values, alpha 1 and alpha 2 for a particular slenderness ratio for the two failure modes are as shown in Tables 3, 4, 5, 6 and 7 respectively.

The results obtained from the compressive strength showed a reduction in the reliability level with increased value of alpha 1 and alpha 2. From Table 3, it can be seen that the load ratio alpha 2, has no effect on the reliability level and for an increase in length which subsequently leads to an increase in slenderness ratio, the reliability level remains constant for a particular load ratio. From Tables 4-7, it can be seen that there is decrease in reliability level as the load ratio, alpha 2, increases at constant slenderness ratio. However, the axial load ratio, alpha 1 has no marked effect on the reliability level. However, as the slenderness ratio increases, there is a significant reduction in reliability level at a constant load ratio.

## 5. Conclusions

The reliability analysis of a solid timber column of square cross-section has been presented using First

Table 3: Compressive Strength Results Slenderness Ratio = 34.50.

| $\alpha_1$ | $\beta$ | $P_f$ |
|---|---|---|
| 0.2 | 5.984 | 0.110E-8 |
| 0.4 | 5.628 | 0.912E-8 |
| 0.6 | 5.319 | 0.522E-7 |
| 0.8 | 5.046 | 0.225E-6 |
| 1.0 | 4.802 | 0.786E-6 |

Table 4: Bending Strength Results Slenderness Ratio = 34.50.

| $\alpha_1$ | $\beta$ | $P_f$ |
|---|---|---|
| 0.2 | 6.006 | 0.953E -9 |
| 0.4 | 5.652 | 0.796E -8 |
| 0.6 | 5.344 | 0.456E -7 |
| 0.8 | 5.072 | 0.198E -6 |
| 1.0 | 4.828 | 0.691E-6 |

Table 5: Bending Strength Results Slenderness Ratio = 40.20.

| $\alpha_1$ | $\beta$ | $P_f$ |
|---|---|---|
| 0.2 | 5.224 | 0.876E-7 |
| 0.4 | 4.878 | 0.537E-6 |
| 0.6 | 4.576 | 0.237E-5 |
| 0.8 | 4.309 | 0.819E-5 |
| 1.0 | 4.070 | 0.235E-4 |

Table 6: Bending Strength Results Slenderness Ratio = 46.00.

| $\alpha_1$ | $\beta$ | $P_f$ |
|---|---|---|
| 0.2 | 4.561 | 0.255E -5 |
| 0.4 | 4.220 | 0.122E -4 |
| 0.6 | 3.923 | 0.437E -4 |
| 0.8 | 3.660 | 0.126 E-3 |
| 1.0 | 3.432 | 0.310E -3 |

Table 7: Bending Strength Results Slenderness Ratio = 51.72.

| $\alpha_1$ | $\beta$ | $P_f$ |
|---|---|---|
| 0.2 | 3.984 | 0.339E -4 |
| 0.4 | 3.648 | 0.132E -3 |
| 0.6 | 3.353 | 0.339E -3 |
| 0.8 | 3.091 | 0.996E-3 |
| 1.0 | 2.885 | 0.215E -2 |

Order Reliability Method. The reliability analysis carried out showed that the Eurocode-5 requirements for timber columns are adequate. By adequate proportioning of the dimension of timber column in such a way that the safety index for the bending should be equal or even exceed that of the compressive strength, a higher reliability will be achieved. In conclusion, a low slenderness ratio gives a higher reliability value.

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