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EVALUATION OF FLEXURAL AND DISTORTIONAL ELASTIC STABILITY OF MONO-SYMMETRIC BOX GIRDERS WITH SIMPLY SUPPORTED CONDITIONS USING VLASOV THEORY AND POWER SERIES APPROACH

AUTHORS:
C. O. Ugwoke[1], M. E. Onyia[2], and O. A. Oguaghamba[3]

AFFILIATIONS:
[1,2,3]Department of Civil Engineering, University Of Nigeria, Nsukka, Enugu State, Nigeria.

***CORRESPONDING AUTHOR:**
Email: calistusobinna2013@gmail.com

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Abstract
Mono-symmetric box girders are widely used in bridge construction for their strength, durability, and design flexibility. Traditional single-variable analysis methods, such as trigonometric series under simply supported (SS) conditions, often oversimplify and converge slowly. This study addresses these limitations using Vlasov theory combined with a multi-variable power series approach. Varbanov's modified generalized displacement functions were applied to derive the governing Vlasov differential equations, simplifying strain fields through the unit displacement method at the pole and shear center. Enhanced product integrals computed critical section properties, and power, trigonometric, and Taylor-Maclaurin series shape functions solved the reduced equations, enabling detailed analysis of flexural and distortional behaviors. The results revealed significant deformation patterns and rapid convergence. In the power series, maximum deflections occurred at 5 m and 45 m, attributed to localized bending moments caused by eccentric loading. Minimum distortion points were observed away from load concentrations, with reduced cross-sectional warping. Taylor-Maclaurin series deflections peaked at mid-span, consistent with beam theory predictions, while distortional curves showed linear trends with deformation neutralization at mid-span due to opposing end constraints. Trigonometric series displayed cyclic deformation patterns, reflecting the effects of fluctuating loads, and distortional curves stabilized at mid-span. These findings emphasize the ability of mono-symmetric box girders to mitigate torsional moments and improve structural efficiency. The proposed multi-variable power series approach provides precise deformation analysis and insights into localized bending, shear forces, and distortional effects. This study validates the methodology and demonstrates its potential to enhance bridge design practices through accurate and efficient structural analysis, addressing critical performance factors and advancing the understanding of mono-symmetric girder behavior under various loading conditions.

1.0 INTRODUCTION
The behavior of mono-symmetric channel sections differs from double-symmetric ones due to the misalignment of the shear center (S) and center of gravity (C). While loading through the shear center causes pure bending, eccentric loading introduces torsion and rotation, resulting in complex deformations, as noted by [1] and [2]. Thin-walled mono-symmetric sections experience additional stresses when restrained from warping, [4] and [5]. These structures, valued for their high strength-to-weight ratio, are widely used in engineering, particularly in bridges and industrial buildings, [6].

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Mono-symmetric box girders are crucial for resisting torsional moments in bridge construction, as described by [6] and [7]. Theoretical models, such as those developed by [8] and [9], provide tools for analyzing these structures, early methods based on trigonometric series (Fourier series) as in [6] have limitations for complex geometries, alternative approaches, such as Taylor-Maclaurin series, as explored by [10] and [11], offer greater accuracy in analyzing plate stability. This study derives differential equations for mono-symmetric box girders, focusing on flexural-distortional behavior under simple support conditions using Vlasov's theory and power series.

2.0 METHODOLOGY

The displacements in longitudinal ($U(x,s)$) and transverse ($V(x,s)$) directions for thin-walled closed structures under external torque are expressed as :

$$U(x,s) = \sum_{i=1}^{m} U_i(x)\varphi_i(s); \quad V(x,s) = \sum_{k=1}^{n} V_K(x)\psi_K(s) \tag{1}$$

The elastic direct and shear strain on and between the two planes, x and s are obtained as:

Direct Strain along the longitudinal direction: $\varepsilon_x(x,s)$:

$$\varepsilon_x(x,s) = \frac{\partial U(x,s)}{\partial x} = \sum_{i=1}^{m} U_i{}'(x)\varphi_i(s) \tag{2}$$

Direct Strain along the transverse direction, $\varepsilon_s(x,s)$:

$$\varepsilon_s(x,s) = \frac{\partial V(x,s)}{\partial s} = \sum_{k=1}^{n} V_K(x)\psi_K{}'(s) \tag{3}$$

Shear Strain between the transverse and longitudinal directions, $\gamma(x,s)$:

$$\gamma(x,s) = \frac{\partial U(x,s)}{\partial s} + \frac{\partial V(x,s)}{\partial x} = \sum_{i=1}^{m} U_i(x)\varphi_i'(s) + \sum_{k=1}^{n} V_k'(x)\Psi_k(s) \tag{4}$$

In similar approach, the direct elastic longitudinal and shear stresses associated with these strains are obtained as follows, $\sigma(x,s)$:

$$\sigma(x,s) = E\varepsilon_x(x,s) = E\frac{\partial U(x,s)}{\partial x} = E\sum_{i=1}^{m} U_i{}'(x)\varphi_i(s) \tag{5}$$

$$\tau(x,s) = G\gamma(x,s) = G\left(\sum_{i=1}^{m} U_i(x)\varphi_i'(s) + \sum_{k=1}^{n} V_k'(x)\Psi_k(s)\right) \tag{6}$$

The strain energy U is derived using

$$\mathrm{U} = \frac{1}{2}\left[\int_L\int_s\left(\left(\frac{\sigma^2_{(x,s)}}{E} + \frac{\tau^2_{(x,s)}}{G}\right)t(s) + \frac{M^2(x,s)}{EI_s}\right)dxds\right] \tag{7}$$

Expanding and simplifying $\sigma^2(x,s)$ and $\tau^2_{(x,s)}$ using summation properties:

$$\sigma^2(x,s) = E^2\sum_{i=1}^{m} U'_{(x)}U_j'(x)\cdot\sum_{j=1}^{n}\varphi_i(s)\varphi_j(s) \tag{8}$$

$$\tau^2_{(x,s)} = \mathrm{G}^2\Big(\sum_{i=1}^{m} U_i(x)\,U_j(x)\cdot\sum_{i=1}^{m}\varphi_i'(s)\varphi_j'(s) + \sum_{i=1}^{m} U_i(x)V_h'(x)\cdot\sum_{k=1}^{n}\varphi_i'(s)\Psi_h(s) + \sum_{k=1}^{n}\mathrm{U_j(x)}\,V_k'(x)\cdot\sum_{i=1}^{m}\varphi_j'(s)\,\Psi_k(s) + \sum_{k=1}^{n} V_h'(x)\,V_k'(\mathrm{x})\cdot\sum_{k=1}^{n}\Psi_h(s)\Psi_k(\mathrm{s})\Big) \tag{9}$$

By applying the same technique to the moment expression:

$$M^2(x,s) = \sum_{k=1}^{n} M_k(s)M_h(s)\cdot\sum_{h=1}^{n} V_k(x)\,V_h(x) \tag{10}$$

Using the total work done by external loads, $\mathrm{W_E} = -q\int_L\int_S V(x,s)dxds$, the total potential energy becomes:

$$\Pi = \mathrm{U} + \mathrm{W_E} = \frac{1}{2}\left[\int_L\int_s\left(\left(\frac{\sigma^2_{(x,S)}}{E} + \frac{\tau^2_{(x,S)}}{G}\right) + \frac{M^2(x,s)}{EI_s}\right)t(s)dxds\right] - q\int_L\int_S V(x,s)dxds \tag{11}$$

In simplifying form, substituting expansions for σ^2, τ^2, M^2, and strain energy contributions, Π becomes:

$$\Pi = \frac{1}{2}\Big[\Big(E\sum_{i=1}^{m} U'_{(X)}U_j'(x)\cdot\sum_{j=1}^{n}\varphi_i(s)\varphi_j(s)\Big)t(s)ds + \Big(\mathrm{G}\Big(\sum_{i=1}^{m} U_i(x)\,U_j(x)\cdot\sum_{i=1}^{m}\varphi_i'(s)\varphi_j'(s) + \sum_{i=1}^{m} U_i(x)V_h'(x)\cdot\sum_{k=1}^{n}\varphi_i'(s)\Psi_h(s) + \sum_{k=1}^{n}\mathrm{U_j(x)}\,V_k'(x)\cdot\sum_{i=1}^{m}\varphi_j'(s)\,\Psi_k(\mathrm{s}) + \sum_{k=1}^{n} V_h'(x)\,V_k'(\mathrm{x})\cdot\sum_{k=1}^{n}\Psi_h(s)\Psi_k(\mathrm{s})\Big)\Big)t(s)ds + \frac{1}{EI_s}\sum_{k=1}^{m} M_k(s)M_h(s)\cdot\sum_{h=1}^{n} V_k(x)\,V_h(x)ds - \sum q_hV_h(x,s)\Big]dx \tag{12}$$

Where, $t(s)ds = dA$

Taking the limits, i, j, k and h as an integers 1,2,3,4 representing the modes of interaction, we have:

$$\sum_{j=1}^{n}\varphi_i(s)\varphi_j(s)\,dA = \int\varphi_i(s)\varphi_j(s)dA = a_{ij};\ \sum_{i=1}^{m}\varphi_i'(s)\varphi_j'(s)\,dA = \int\varphi_i'(s)\varphi_j'(s)dA = b_{ij};\ \sum_{k=1}^{n}\varphi_i'(s)\Psi_h(s)dA = \int\varphi_i'(s)\psi_h(s)dA\,j = c_{hi};\ \sum_{i=1}^{m}\varphi_j'(s)\,\Psi_k(\mathrm{s})dA = \int\varphi_j'(s)\psi_k(s)dA = c_{jk};\ \sum_{k=1}^{n}\Psi_h(s)\Psi_k(s)dA = \int\psi_h(s)\psi_k(s) = r_{hk};\ \sum_{h=1}^{n}\frac{M_k(s)M_h(s)}{EI_{(s)}}dA = \frac{1}{E}\int\frac{M_k(s)M_h(s)}{EI_{(s)}} = s_{hk};\ \sum q_hV_h(x,s) = \int q\psi_h ds = q_h \tag{13}$$

Thus, equation (12) becomes:

$$\Pi = \frac{E}{2}\sum a_{ij}U_i'(x)U_j'(x)dx + \frac{G}{2}\Big[\sum b_{ij}U_i(x)U_j(x) + \sum c_{ih}U_i(x)V_h'(x)\Big]dx + \frac{G}{2}\Big[\sum c_{jk}U_j(x)V_k'(x) + \sum r_{hk}V_k'(x)V_h'(x)\Big]dx + \frac{E}{2}\sum V_k(x)V_h(x)dx - \sum q_h\,V_h\mathrm{dx} \tag{14}$$

Therefore, equation (14) shows that the total potential energy Π is a functional of the form:

$$\Pi = F\left(U_iU_jV_kV_hU_i'U_j'V_k'V_h'\right) \tag{15}$$

Here, in equation (14), φ *and* ψ represent generalized warping and distortional strain modes; M is bending moment while associated terms describe displacement functions, bending moments, and distortion effects influenced by material properties, E (modulus of elasticity), G (shear modulus), and external load, q_h.

2.1 Governing Equation of Distortional Equilibrium of Box – Girder

The governing equations of distortional equilibrium for a box-girder are derived by minimizing the functional equation using the Euler-Lagrange technique in both the longitudinal and transverse directions. In the longitudinal direction, the equilibrium equation is

$$\frac{\partial F}{\partial u} - \frac{d}{dx}\left(\frac{\partial F}{\partial u'}\right) = 0 \tag{16}$$

In the transverse direction, it is

$$\frac{\partial F}{\partial V_h} - \frac{d}{dx}\left(\frac{\partial \pi}{\partial V_h'}\right) = 0 \tag{17}$$

Here, $\mathrm{F_x}$ is the first moment of area about, $\bar{y}-axis$. F is the area of section material, Carrying out the partial differential of equation (16) with respect to U_j, and equation (17) with respect to V_h, using Euler-Lagrange, we have:

$$\sum b_{ij} U_i(x) + \sum c_{kj} V_k'(x) - \frac{E}{G}\frac{d}{dx}\sum a_{ij} U_i'(x) = 0 \quad (18)$$

Where, $k = \frac{E}{G} = 2(1+\gamma)$, we have:

$$k a_{ij}\frac{d}{dx}\sum U_i'(x) - b_{ij}\sum U_i(x) - c_{kj}\sum V_k'(x) = 0 \quad (19)$$

$$c_{ih}\sum_{i=1}^{m} U_i'(x) + r_{kh}\sum_{k=1}^{n} V_k''(x) - ks_{hk}\sum_{h=1}^{n} V_k(x) + \frac{1}{G}\sum q_h = 0 \quad (20)$$

Taking the bounds of the variables i, j and k for i and j = 1,2,3 and k = 1,2,3,4 and the limits of the variables i, h and k for i = 1,2,3 and h, k = 1,2,3,4, and extending, [6], determined certain coefficients with zero values for mono-symmetrical cross-sections, emphasized the interaction of torsional- distortional, flexural-tortional, and flexural- distortional deformations, and highlighted the importance of non-trivial coefficients associated with deformation modes 2, 3 and 4.

$$\begin{bmatrix} a_{11} = 0;\ a_{12} = a_{21} = 0; a_{13} = a_{31} = 0 \\ b_{11} = 0;\ b_{12} = b_{21} = 0; b_{13} = b_{31} = 0 \\ c_{11} = 0;\ c_{12} = c_{21} = 0;\ c_{13} = c_{31} = 0 \\ r_{11} = 0;\ r_{12} = r_{21} = 0;\ r_{13} = r_{31} = 0 \\ s_{11} = 0;\ s_{12} = s_{21} = 0; s_{22} = 0;\ s_{13} = s_{31} = 0;\ s_{23} = s_{32} = 0 \end{bmatrix} \quad (21)$$

According to [6], the relative coefficients for bending-deformation equilibrium are the coefficients for deformation modes 2 and 4. By replacing the irrelevant non-coefficients in the matrix equations obtained after the expansion of equations (19) and (20), while retaining the relative coefficient in equation (21), the governing differential equations (22a) and (22b) were obtained as follows:

$$V_4^{11} = K_1 \quad (22a)$$

$$\epsilon_1 V_2^{IV} + \epsilon_2 V_4^{IV} - \beta_1 V_4^{11} = K_2 \quad (22b)$$

Where, $\epsilon_1 = Ka_{22}c_{42};\ \epsilon_2 = Ka_{22}r_{44}; \beta_1 = (b_{22}r_{44} - c_{24}c_{42})$;

$$K_1 = \left(\frac{c_{22}}{r_{24}c_{42} - c_{22}r_{44}}\right)\frac{q_4}{G} - \left(\frac{c_{42}}{r_{24}c_{42} - c_{22}r_{44}}\right)\frac{q_2}{G}\ ;\ K_2 = b_{22}\frac{q_4}{G} \quad (23)$$

2.2 Non-dimensional Differential Equilibrium Equations

They are derived for deformation system (flexural-distortional),by expressing the longitudinal coordinate as a non-dimensional parameter within the structure's limits,

$$X = LR : 0 \le R \le 1, \quad (24)$$

Where, X is the directional coordinate of the thin-walled structure along the span, $L; R$ is the corresponding non-dimensional surface or longitu-dinal dimension of the structure in the limits 0 to 1, [12].

Recall: $V_2^{iv}(x) = \frac{d^4V_2(x)}{dx^4};\ V_2''(x) = \frac{d^2V_2(x)}{dx^2}; V_4''(x) = \frac{d^2V_4(x)}{dx^2};\ V_4^{iv}(x) = \frac{d^4V_4(x)}{dx^3}$ (25)

From Equation (24),

$$X = \mathrm{LR};\ \mathrm{dx} = \mathrm{LdR};\ \mathrm{dx}^2 = (\mathrm{LdR})^2 = \mathrm{L}^2\mathrm{dR}^2;\ \mathrm{dx}^4 = (\mathrm{LdR})^4 = \mathrm{L}^4\mathrm{dR}^4 \quad (26)$$

Substituting equation (26) into equations (22a) and (22b), we have:

$$\frac{d^2V_4(R)}{\mathrm{L}^2\mathrm{dR}^4} = K_1 \quad (27)$$

$$\epsilon_1\frac{d^4V_2(R)}{\mathrm{L}^4\mathrm{dR}^4} + \epsilon_2\frac{d^4V_4(R)}{\mathrm{L}^4\mathrm{dR}^4} - \beta_1\frac{d^2V_4(R)}{\mathrm{L}^2\mathrm{dR}^2} = K_2 \quad (28)$$

The solution to Vlasov's flexural-distortional equilibrium equations for a mono-symmetric box girder involves power series displacement functions. It emphasizes transverse deformation and its energy contribution through general solutions and boundary conditions.

2.3 Power Series General Solution for Displacement Functions:

The power series is a mathematical technique for solving differential equations by representing a function as an infinite sum of terms involving powers of a variable, [13]. It is particularly useful for linear ordinary differential equations, ODEs, allowing solutions to be expressed as power series expansions like:

$$w = w(x) = \sum_{m=0}^{\infty}\delta_m(x-x_0)^m = \delta_0 + \delta_1(x-x_0) + \delta_2(x-x_0)^2 + \delta_3(x-x_0)^3 + \delta_4(x-x_0)^4 + \delta_5(x-x_0)^5 + \delta_6(x-x_0)^6 + \delta_7(x-x_0)^7 + \delta_8(x-x_0)^8 + \ldots \quad (29)$$

The function $w(x)$ is expressed as a power series centered at x_0, with coefficients δ_m representing real or complex constants. If $x_0 = 0$, the series simplifies to a power series in powers of x, equation (30) and differentiation of this series up to the seventh and eighth times is discussed.

$$w = w(x) = \sum_{m=0}^{8}\delta_m x^m = (\delta_0 + \delta_1 x + \delta_2 x^2 + \delta_3 x^3 + \delta_4 x^4 + \delta_5 x^5 + \delta_6 x^6 + \delta_7 x^7 + \delta_8 x^8) \quad (30)$$

$$w^{V11} = 5040\delta_7 + 40320\,\delta_8 x + \ldots\sum_{m=7}^{\infty} m\,(m-6)\,\delta_m x^{m-7} \quad (31)$$

$$w^{V111} = 40320\delta_8 + \ldots \sum_{m=8}^{\infty} m\ \ (m-7)\delta_m x^{m-8} \quad (32)$$

Into the ordinary equation

$$(\delta_1 + 2\delta_2 x + 3\delta_3 x^2 + \cdots + 8\delta_8 x^7) - (\delta_0 + \delta_1 x + \delta_2 x^2 + \cdots + \delta_7 x^7) = 0 \quad (33)$$

Then we collect like powers of x, finding:

$$(\delta_1 - \delta_0) + (2\delta_2 - \delta_1)x + (3\delta_3 - \delta_2)x^2 + \ldots + (8\delta_8 - \delta_7)x^7 = 0 \quad (34)$$

Equating the coefficient of each power of x to zero, we have:

$$\delta_1 - \delta_0 = 0, 2\delta_2 - \delta_1 = 0, 3\delta_3 - \delta_2 = 0, \ldots 8\delta_8 - \delta_7 = 0 \quad (35)$$

Solving these equations, we may express $\delta_1, \delta_2, \delta_3, \delta_4 \ldots, \delta_8$ in terms of δ_0, which remains arbitrary:

$$\delta_1 = \delta_0, \delta_2 = \frac{\delta_1}{2} = \frac{\delta_0}{2!}, \delta_3 = \frac{\delta_2}{3} = \frac{\delta_0}{3!}, \delta_4 = \frac{\delta_3}{4} = \frac{\delta_0}{4!}, \ldots, \delta_8 = \frac{\delta_7}{8} = \frac{\delta_0}{8!} \quad (36)$$

$$40320\delta_8 - 5040\delta_7 = 0\ ,\ \ \delta_8 = \frac{\delta_7}{8} \quad (37)$$

With these values of the coefficients, the series solution becomes the known general solution, viz. general solution, That is,

$$w(x) = \delta_0 + \delta_0 x + \frac{\delta_0}{2!}x^2 + \cdots \frac{\delta_0}{8!}x^8 = \delta_o\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!}\right) = \delta_0 e^x \quad (38)$$

2.4 Extension of a Single-Variable Power Series to a Multi-Variable Finite Polynomial Displacement Function Incorporating Step Functions:

The potential energy of a thin-walled box girder under flexural-distortional load is represented by the beam's transverse deformation "w," which can be expressed as a power series displacement function for simply supported (SS) conditions.

STEP 1: Homogeneous Solution. From equation (38),

$$w(x) = \delta_0 e^x \quad (39)$$

Based on the binomial coefficients and theorem at a point where m = 8, we have:

$$(\delta + 1)^m = 1.\delta_0 + 8.\delta_0 + 28.\delta_0 + 56.\delta_0 + 70.\delta_0 + 56\,\delta_0 + 28\delta_0 + 8\delta_0 + \delta_0.1 = 0 \quad (40)$$

Then general solution of the homogenous ODE is represented as a finite polynomial in x with coefficients, $\delta_1, \delta_2, \delta_3, \ldots, \delta_8$, multiplied by the series expansion of e^x as follow:

$$w_h = (\delta_1 + \delta_2 x + \delta_3 x^2 + \delta_4 x^3 + \delta_5 x^4 + \delta_6 x^5 + \delta_7 x^6 + \delta_8 x^7)e^x \quad (41)$$

Therefore, in general, the logarithmic base (e), log. $e^x$is equal to Ine^x,where $x > 0$. From the key properties of the natural logarithm, it follows that, $Ine^x{=}x$, hence, equation (41) becomes:

$$w_h = \delta_1 x + \delta_2 x^2 + \delta_3 x^3 + \delta_4 x^4 + \delta_5 x^5 + \delta_6 x^6 + \delta_7 x^7 + \delta_8 x^8 \quad (42)$$

Applying the properties of binomial expansion, we obtain;

$$(\delta + 1)^8 = \delta_1 x + \delta_2 x^2 + \delta_3 x^3 + \delta_4 x^4 + \delta_5 x^5 + \delta_6 x^6 + \delta_7 x^7 + \delta_8 x^8 \quad (43)$$

The binomial expansion of $(\delta{+}1)^8$ becomes binomial expansion coefficients similar to the sum of consecutive positive integers as follows:

$$1.\frac{\delta_1}{\delta_0} + 2.\frac{\delta_2}{\delta_1} + 3.\frac{\delta_3}{\delta_2} + 4.\frac{\delta_4}{\delta_3} + 5.\frac{\delta_5}{\delta_4} + 6.\frac{\delta_6}{\delta_5} + 7.\frac{\delta_7}{\delta_6} + 8.\frac{\delta_8}{\delta_7} \quad (44)$$

The hypothesis of equation (44) corresponds to the following expression:

$$\sum_{R=1}^{8} R.\frac{\delta_R}{\delta_{R-1}} = 1.\frac{\delta_1}{\delta_0} + 2.\frac{\delta_2}{\delta_1} + 3.\frac{\delta_3}{\delta_2} + 4.\frac{\delta_4}{\delta_3} + 5.\frac{\delta_5}{\delta_4} + 6.\frac{\delta_6}{\delta_5} + 7.\frac{\delta_7}{\delta_6} + 8.\frac{\delta_8}{\delta_7} \quad (45)$$

Where, $R = 1$

Then, from the arithmetic series formula for the sum of consecutive integers, we obtained the following sum of the first 8 positive integers as follow:

$$\frac{[8(8+1)]}{2} = 36 \quad (46)$$

STEP 2: Particular Solution: From equation (38), let,

$$w_p = \delta_0 x^8 e^x \quad (47)$$

$$\begin{bmatrix} w_{p^{1}} = \delta_o\left(8x^7 + x^8\right)e^x \\ w_{p^{11}} = \delta_o\left(56x^6 + 16x^7 + x^8\right)e^x \\ w_{p^{111}} = \delta_o\left(336x^5 + 169x^6 + 24x^7 + x^8\right)e^x \\ w_{p^{1v}} = \delta_o\left(1680x^4 + 1350x^5 + 337x^6 + 32x^7 + x^8\right)e^x \\ w_{p^{v}} = \delta_o\left(6720x^3 + 8430x^4 + 3372x^5 + 561x^6 + 40x^7 + x^8\right)e^x \\ w_{p^{v1}} = \delta_o\left(20160x^2 + 40440x^3 + 25290x^4 + 6738x^5 + 841x^6 + 48x^7 + x^8\right)e^x \\ w_{p^{v11}} = \delta_o\left(40320x + 141480x^2 + 141600x^3 + 58980x^4 + 11784x^5 + 1177x^6 + 56x^7 + x^8\right)e^x \\ w_{p^{v111}} = \delta_o(40320 + 323280x + 566280x^2 + 377520x^3 + 117900x^4 + 18846x^5 + 1569x^6 + 64x^7 + x^8)e^x \end{bmatrix} \quad (48)$$

Here, equation (40), becomes;

$$1.\delta_0 + 8.\delta_0 + 28.\delta_0 + 56.\delta_0 + 70.\delta_0 + 56\,\delta_0 + 28\delta_0 + 8\delta_0 + \delta_0.1 = 36 \quad (49)$$

Substituting equation (48) into equation (49), gave:

$$(\delta_o(40320 + 323280x + 566280x^2 + 377520x^3 + 117900x^4 + 18846x^5 + 1569x^6 + \delta + 64x^7 + x^8) + 8_o(40320x + 141480x^2 + 141600x^3 + 58980x^4 + 11784x^5 + 6738x^5 + 841x^6 + 48x^7 + 70\delta_o(1680x^4 + 1350x^5 + 337x^6 + 32x^7 + x^8) + 56\delta_o(336x^5 + 169x^6 + 24x^7 + x^8 + 1177x^6 + x^8) + 56\delta_o(6720x^3 + 8430x^4 + 3372x^5 + 561x^6 + 40x^7 + x^8) + 56x^7 + x^8) + 28\delta_o\big(20160x^2 + 40440x^3 + 25290 + 28\delta_o(56x^6 + 16x^7 + x^8)x^4 + 8\delta_o(8x^7 + x^8) + \delta_o(x^8)\big)e^x = 36 \quad (50)$$

Omitting the linear squares, the third, fourth, fiftieth, sixtieth, seventieth, and eightieth terms, and omitting the common factor, e^x, we obtain;

$$40320\delta_o = 36 \quad (51)$$

$$\delta_o = 8.9286 \times 10^{-4} \quad (52)$$

$$w_p = 8.9286 \times 10^{-4} x^8 e^x \quad (53)$$

STEP 3: Now, $w(x) = w_h + w_p$

$$w(x) = (\delta_1 + \delta_2 x + \delta_3 x^2 + \delta_4 x^3 + \delta_5 x^4 + \delta_6 x^5 + \delta_7 x^6 + \delta_8 x^7)e^x + 8.9286 \times 10^{-4} x^8 e^x \quad (54)$$

Equation (55) represents the generalized polynomial displacement function that accurately models the deformed shape of a thin-walled box girder under combined loads using Bentham's boundary conditions.

2.5 Power Series, Taylor-Maclaurin Series, and Trigonometric Series Displacement Functions for a SS Beam:

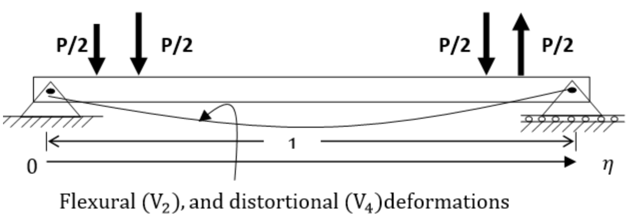

Figure 1: Simply Supported (SS) beam along R – Axis subjected to different loading Superimposed the

power series, Taylor Maclaurin series and trigonometric series Boundary Conditions along η – direction

Figure 1 shows a beam of simply supported (SS) end conditions subjected to flexural – distortional loads. The Benthem's boundary conditions of the beam are given beneath the figure [8].

2.5.1 Power series displacement function for SS beam

From equation (54):

$$w(R=0);\ w^{11R}(R=0)=0; w(R=1);\ w^{11R}(R=1)=0 \quad (55)$$

From equation (54)

$$w(R)=\delta_R(\mathrm{R}-1.1198R^3+0.1198R^8) \quad (56)$$

Here, $\delta_R=0.5\Omega$, by invoking scaled version of the original function, where each term has been multiplied by a scaling factor of 0.5., hence , equation (56), becomes;

$$w(R)=\Omega(0.5\mathrm{R}-0.6R^3+0.1R^8) \quad (57)$$

Here, Ω = constant proportionality, $V_2(R)$ and $V_4(R)$ are flexural and distortional deformations. Hence, the corresponding equation of deformations, second and fourth order derivatives, becomes:

$$V_2(R)=\Omega_2(0.5R-0.6R^2+0.1R^8); V_3(R);\ V_4(R)=\Omega_4(0.5R-0.6^3+0.1R^8) \quad (58)$$

$$V_2^{ii}(R)=\frac{d^2V_2}{dR^2}(R)=\Omega_2(-3.6R+5.6R^6); V_2^{iv}(R)=\frac{d^4V_2}{dR^4}(R)=168\Omega_2$$

$$V_4^{ii}(R)=\frac{d^2V_4}{dR^2}(R)=\Omega_4(-3.6R+5.6R^6); V_4^{iv}(R)=\frac{d^4V_4}{dR^4}(R)=168\Omega_4 \quad (59)$$

2.5.2 Taylor maclaurin's displacement function for SS beam

By equating the moment and elasticity equations of beam and integrating twice with respect to an arbitrary direction η, the displacement function is obtained as.

$$W_\eta=C_o+C_1\eta\cdot +C_2\eta^2+C_3\cdot\eta^3+\ C_4\cdot\eta^4 \quad (60)$$

Where, C_o and C_1 are constants of integration, and $C_4=\frac{q}{24D}$; $C_3=\frac{-R}{6D}$; $C_2=\frac{M_1}{2D}$. For a uniformly distributed load, the function is fourth-order, as the highest polynomial degree is 4. Thus, in the Taylor-Maclaurin series expansion for a beam strip along R, the maximum term is m = 4. The series constants along the R are denoted as A_m, [10], thus,

$$\mathrm{w(R)}=\sum_{m=1}^{\infty}A_m\mathrm{R}^m \quad (61)$$

$$\mathrm{w(R)}=\sum_{m=1}^{4}A_m\mathrm{R}^m=(\mathrm{A_0+A_1R+A_2R^2+A_3R^3+A_4R^4}) \quad (62)$$

The coefficients A_m of the series are determined from the boundary conditions at the edges of the beam.

Boundary Conditions along η – direction

$$w(R=0);\ w^{11R}(R=0)=0; w(R=1);\ w^{11R}(R=1)=0 \quad (63)$$

Using Equation (63):

$$V(R)=A_4(R-2R^3+\ R^4) \quad (64)$$

Let the constant proportionality, A_4 be taken as Ω. Equation (64) becomes:

$$V(R)=\Omega(R-2R^3+\ R^4) \quad (65)$$

Equation (65) is the shape function for simply supported ends of a mono – symmetric box girder. Hence, the corresponding equation of deformations, second and fourth order derivatives becomes:

$$V_2(R)=\Omega_2(R-2R^3+\ R^4); V_4(R)=\Omega_4(R-2R^3+\ R^4) \quad (66)$$

$$V_2''(R)=\frac{d^2V_2}{dR^2}(R)=\Omega_2(-12R+\ 12R^2)\ ;\quad V_2^{iv}(R)=\frac{d^4V_2}{dR^4}(R)=24\Omega_2$$

$$V_4''(R)=\frac{d^2V_4}{dR^2}(R)=\Omega_4(-12R+\ 12R^2);\quad V_4^{iv}(R)=\frac{d^4V_4}{dR^4}(R)=24\Omega_4 \quad (67)$$

2.5.3 Trigonometric series displacement function for SS beam

Let the approximate displacement or shape function be,

$$w=\sin\left(\frac{\pi R}{a}\right) \quad (68)$$

The corresponding second derivative of equation (68) becomes:

$$w^{11}=\frac{-\pi^2}{\mathrm{a}^2}\sin\left(\frac{\pi R}{a}\right) \quad (69)$$

Boundary Conditions along η – direction becomes:

$$w(R=0);\ w^{11R}(R=0)=0; w(R=1);\ w^{11R}(R=1)=0 \quad (70)$$

Therefore, the assumed displacement functions satisfied the boundary conditions, hence

$$w(R)=\Omega\sin\left(\frac{\pi R}{a}\right) \quad (71)$$

Thus, the corresponding equations of deformation, second and fourth order derivatives become:

$$V_2(R)=\ \Omega_2\sin\left(\frac{\pi R}{a}\right);\ V_4(R)=\Omega_4\mathrm{S}\sin\left(\frac{\pi R}{a}\right) \quad (72)$$

$$V_2^{ii}(R)=\frac{d^2V_2}{dR^2}(R)=\Omega_2\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right);\ V_2^{iv}(R)=\frac{d^4V_2}{dR^4}=\Omega_2\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)$$

$$V_4^{ii}(R)=\frac{d^2V_4}{dR^2}(R)=\Omega_4\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right);\ V_4^{iv}(R)=\frac{d^4V_4}{dR^4}=\Omega_2\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right) \quad (73)$$

2.6 Application of Power Series Shape Function to Vlasov Theory on Flexural-Distortional of Mono-Symmetric Box Girder for Simply Supported Ends

Substituting, the flexural and distortional deformations and their second and fourth order derivatives in Equations (58) and (59) into the governing differential equilibrium equation for thin – walled systems under flexural - distortional loads in

Equations (27) and (28), and solving simultaneously we have:

$$\frac{d^2 V_4(R)}{L^2 dR^4} = k_1$$

$$\epsilon_1 \frac{d^4 V_2(R)}{L^4 dR^4} + \epsilon_2 \frac{d^4 V_4(R)}{L^4 dR^4} - \beta_1 \frac{d^2 V_4(R)}{L^2 dR^2}) = k_2$$

$$\Omega_4 \frac{(-3.6R+5.6R^6)}{L^2} = k_1 \quad (74)$$

$$\Omega_4 \frac{k_1}{(-3.6R+5.6R^6)/L^2} \quad (75)$$

Substitute equation (75) into equation (28) to get Ω_2:

$$\Omega_2 \frac{168\epsilon_1}{L^2} + \Omega_4 \left(\frac{168\epsilon_2}{L^4} - \frac{\beta_1}{L^2}(-3.6R + 5.6R^6)\right) = k_2 \quad (76)$$

$$\Omega_2 = \frac{k_2 - \frac{k_1}{(-3.6R+5.6R^6)/L^2}\left(\frac{168\epsilon_2}{L^4} - \frac{\beta_1}{L^2}(-3.6R+5.6R^6)\right)}{\frac{168\epsilon_1}{L^4}} \quad (77)$$

Using the equation (58), the flexural deformation $V_2(R)$ and the distortional deformation, $V_4(R)$, for the mono-symmetric box girder gave:

$$V_2(R) = \Omega_2(0.5R - 0.6R^3 + 0.1R^8)$$

$$V_2(R) = \frac{k_2 - \frac{k_1}{(-3.6R+5.6R^6)/L^2}\left(\frac{168\epsilon_2}{L^4} - \frac{\beta_1}{L^2}(-3.6R+5.6R^6)\right)}{\frac{168\epsilon_1}{L^4}}(0.5R - 0.6R^3 + 0.1R^8) \quad (78)$$

$$V_4(R) = \frac{k_1}{(-3.6R+5.6R^6)/L^2}(0.5R - 0.6R^3 + 0.1R^8) \quad (79)$$

2.7 Application of Taylor Maclaurin Polynomial Shape Function to Vlasov Theory on Flexural-Distortional of Mono-Symmetric Box Girder for Simply Supported Ends

Substituting, the flexural and distortional deformations and their second and fourth order derivatives in Equations (66) and (67) into the governing differential equilibrium equation for thin – walled systems under flexural - ditortional loads in Equations (27) and (28) and solving simultaneously we have:

$$\Omega_4 \frac{(-12R+12R^2)}{L^2} = k_1 \quad (80)$$

$$\Omega_2 \frac{24\epsilon_1}{L^4} + \Omega_4 \left(\frac{24\epsilon_2}{L^4} - \frac{\beta_1}{L^2}(-12R + 12R^2)\right) = k_2 \quad (81)$$

Certainly! Let's isolate Ω_2 and Ω_4 from equations (80) and (81). Equation (28) involves only Ω_4, so the expression for Ω_4 is:

$$\Omega_4 = \frac{k_1}{(-12R+12R^2)/L^2} \quad (82)$$

Substitute equation (82) into equation (81) to get Ω_2:

$$\Omega_2 \frac{24\epsilon_1}{L^4} + \frac{k_1}{(-12R+12R^2)/L^2}\left(\frac{24\epsilon_2}{L^4} - \frac{\beta_1}{L^2}(-12R + 12R^2)\right) = k_2 \quad (83)$$

$$\Omega_2 = \frac{k_2 - \frac{k_1}{(-12R+12R^2)/L^2}\left(\frac{24\epsilon_2}{L^4} - \frac{\beta_1}{L^2}(-12R+12R^2)\right)}{\frac{24\epsilon_1}{L^4}} \quad (84)$$

Using the equation (66), the flexural deformation, $V_2(R)$ and the distortional deformation, $V_4(R)$, for the mono-symmetric box girder gave:

$$V_2(R) = \Omega_2(R - 2R^3 + R^4)$$

$$V_2(R) = \frac{k_2 - \frac{k_1}{(-12R+12R^2)/L^2}\left(\frac{24\epsilon_2}{L^4} - \frac{\beta_1}{L^2}(-12R+12R^2)\right)}{\frac{24\epsilon_1}{L^4}}(R - 2R^3 + R^4) \quad (85)$$

$$V_4(R) = \frac{k_1}{(-12R+12R^2)/L^2}(R - 2R^3 + R^4) \quad (86)$$

2.8 Application of Trigonometric Series Shape Function to Vlasov Theory on Flexural-Distortional of Mono-Symmetric Box Girder for Simply Supported Ends

Substituting, the flexural and distortional deformations and their second and fourth order derivatives in Equations (72) and (73) into the governing differential equilibrium equation for thin – walled systems under flexural - distortional loads in Equations (27) and (28), and solving simultaneously we have:

$$\Omega_4 \frac{1}{L^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right) = k_1 \quad (87)$$

$$\Omega_4 = \frac{k_1}{\frac{1}{L^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right)} \quad (88)$$

Equation (28):

$$\frac{1}{L^4}\Omega_2\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)\epsilon_1 + \frac{1}{L^4}\Omega_4\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)\epsilon_2 - \frac{\beta_1}{L^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right) = k_2$$

Substitute equation (88) into equation (28) to get Ω_2:

$$\Omega_2 \frac{1}{L^4}\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)\epsilon_1 + \frac{k_1}{\frac{1}{L^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right)}\left(\frac{1}{L^4}\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)\epsilon_2\right) - \frac{\beta_1}{L^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right) = k_2 \quad (89)$$

$$\Omega_2 = \frac{k_2 - \frac{k_1}{\frac{1}{L^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right)}\left(\frac{1}{L^4}\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)\epsilon_2\right) - \frac{\beta_1}{L^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right)}{\frac{1}{L^4}\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)\epsilon_1} \quad (90)$$

Using the equation (72), the flexural deformation $V_2(R)$, and the distortional deformation, $V_4(R)$, for the mono-symmetric box girder gave:

$$V_2(R) = \Omega_2 \sin\left(\frac{\pi R}{a}\right) \quad (91)$$

$$V_2(R) = \frac{k_2 - \frac{k_1}{\frac{1}{L^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right)}\left(\frac{1}{L^4}\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)\epsilon_2\right) - \frac{\beta_1}{L^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right)}{\frac{1}{L^4}\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)\epsilon_1}\sin\left(\frac{\pi R}{a}\right) \quad (92)$$

$$V_4(R) = \frac{k_1}{\frac{1}{L^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right)}\sin\left(\frac{\pi R}{a}\right) \quad (93)$$

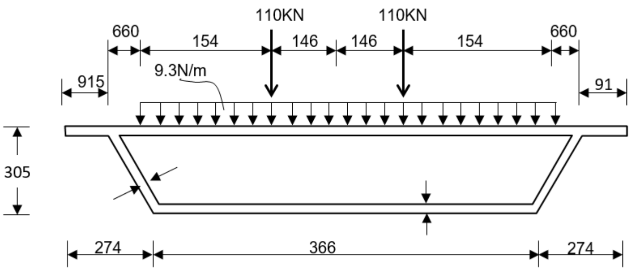

Figure 2: Mono-symmetric box girder (the bridge is span 50m between piers)

3.0 RESULTS AND DISCUSSION

3.1 Numerical Mono–Symmetric Box Girder Bridge Problem

Consider a mono – symmetric box Girder Bridge of two – ways – two – lanes carrying a live load of 9.3N/mm (HL – 93 loading according to AASHTO), [14], in addition to tandem double axle loads of 110KN each lane. The live load is uniformly distributed over the 7.32m transverse width of the bridge of two lanes – two – way. The loads are positioned at the outermost possible location to generate the maximum bending and distortional effect as shown in Figure 2.

3.2 Computation of Vlasov Coefficients

The Vlasov coefficients; a_{ij}, b_{ij}, c_{ij} and s_{kh} are obtained by multiplying, ϕ_i, ψ_i and M_k accordingly using the product integral for unit thickness (i.e; $t = 1$) as described by [6]. However, the modified product integral according to [12] is used, where the constant k value is unity, representing the thickness of the box girder,t.

3.3 Evaluation of the Flexural and Distortional Coefficients

$$\epsilon_1 = k_{a22}c_{42} = 2.5 \times 123.5117 \times 6.4170 = 1{,}981.446447 \quad (94)$$

$$\epsilon_2 = k_{a22}r_{44} = 2.5 \times 123.5117 \times 72.0033 = 22{,}233.12497 \quad (95)$$

$$\beta_1 = (b_{22}r_{44} - c_{22}c_{42}) = (14.6931 \times 72.0033 - 14.6931 \times 6.4170) = 963.6661 \quad (96)$$

$$k_1 = \left(\frac{c_{22}}{r_{24}c_{42} - c_{22}r_{44}}\right)\frac{q_4}{G} - \left(\frac{c_{42}}{r_{24}c_{42} - c_{22}r_{44}}\right)\frac{q_4}{G}\ ;\ k_1 = \left(\frac{14.6931}{6.4170\times6.4170 - 14.6931\times72.0033}\right)\frac{1.4738\times10^6}{9.6\times10^9} - \left(\frac{6.4170}{6.4170\times6.4170 - 14.6931\times72.0033}\right)\frac{1.0820\times10^6}{9.6\times10^9} = 114.4427732 \quad (97)$$

$$k_2 = b_{22}\frac{q_4}{G} = 14.6931 \times \left(\frac{1.4738\times10^6}{9.6\times10^9}\right) = 2.2556969556 \times 10^{-3} \quad (98)$$

3.4 Flexural and Distortional Deformations of Three Mathematical Tools or Series for Simply Supported Ends

From Equation (78) and (79) and determination of the associated Vlasov variables, the coefficients of the flexural and distortional deformations for the Power series are obtained as follows:

$$V_2(R) = \frac{2.2556969556\times10^{-3} - \frac{114.4427732}{(-3.6R+5.6R^6)/50^2}\left(\frac{168\times22{,}233.12497}{50^4} - \frac{963.6661}{50^2}(-3.6R+5.6R^6)\right)}{\frac{168\times1{,}981.446447}{50^4}}(0.5R - 0.6R^3 + 0.1R^8) \quad (99)$$

$$V_4(R) = -\frac{114.4427732}{(-1.44\times10^{-3}R + 2.24\times10^{-3}R^6)}(0.5R - 0.6R^3 + 0.1R^8) \quad (100)$$

From Equation (85) and (87) and determination of the associated Vlasov variables, the coefficients of the flexural and distortional deformations for the Taylor Maclaurin's series are obtained as follows:

$$V_2(R) = \frac{2.2556969556\times10^{-3} - \frac{114.4427732}{(-4.8\times10^{-3}R+4.8\times10^{-3}R^2)}\left(0.085375199 - 0.38546644(-12R+12R^2)\right)}{7.608754356\times10^{-3}} \times (R - 2R^3 + R^4) \quad (101)$$

$$V_4(R) = -\frac{114.4427732}{(-4.8\times10^{-3}R + 4.8\times10^{-3}R^2)}(R - 2R^3 + R^4) \quad (102)$$

From Equation (92) and (93) and determination of the associated Vlasov variables, the coefficients of the flexural and distortional deformations for the Trigonometric series are obtained as follows:

$$V_2(R) = 2.2556969556 \times 10^{-3}\ \frac{-\frac{114.4427732}{\frac{1}{50^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right)}\left(\frac{1}{50^4}\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)\times22{,}233.12497\right) - \frac{963.6661}{50^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right)}{\frac{1}{50^4}\left(\frac{\pi^4}{a^4}\sin\left(\frac{\pi R}{a}\right)\right)\times1{,}981.446447}\sin\left(\frac{\pi R}{a}\right) \quad (103)$$

$$V_4(R) = -\frac{114.4427732}{\frac{1}{50^2}\left(-\frac{\pi^2}{a^2}\sin\left(\frac{\pi R}{a}\right)\right)}\sin\left(\frac{\pi R}{a}\right) \quad (104)$$

3.5 Discussion of Results

The power series method for multiple variables effectively derives shape functions for mono-symmetric box girder bridges, offering rapid convergence and accurate multi-dimensional represe-ntation. It adapts well to complex structures with varied boundary and loading conditions, unlike single-variable methods such as trigonometric and Taylor-Maclaurin series. Hence, this study integrates Vlasov theory with Varbanov's modified displacement functions and a power series framework to overcome limitations of slow convergence and oversimplification. The results as shown in Figure 3, revealed significant deformation patterns; Hence, in the power series, maximum deflections occurred at 5m and 45m, attributed to localized bending moments caused by eccentric loading. Minimum distortion points were observed away from load concentrations, with reduced cross-sectional warping. Taylor-Maclaurin series deflections peaked at mid-span, consistent with beam theory predictions, while distortional curves showed linear trends with deformation neutralization at mid-span due to opposing end constraints. Trigonometric series displayed cyclic deformation patterns, reflecting the effects of fluctuating loads, and distortional curves stabilized at mid-span. These findings emphasize the ability of mono-symmetric box girders to mitigate torsional moments and improve structural efficiency.

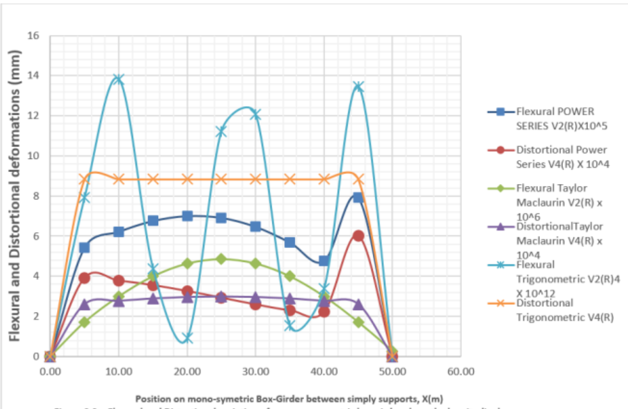

Figure 3.2.: Flexural and Distortional variation of a mono-symmetric box girder along the longitudinal apan

4.0 CONCLUSIONS

Mono-symmetric box girder sections combine strength, durability, and design flexibility, making them ideal for modern bridge construction. The multi-variable power series approach offers rapid convergence and accurately captures complex behaviors by accounting for both axial and transverse deformations under combined bending and torsional loads. This method overcomes the oversimplifications and slow convergence of single-variable techniques, providing a precise representation of structural responses. Its ability to address intricate deformations and torsional moments establishes a robust framework for designing efficient and reliable bridge girders. Comparing multi-variable and single-variable methods highlights their fundamentally different capabilities in structural analysis.

REFERENCES

[1] Chao.Z., Yi. Z., Xingu. Z.,Gang. W.,Qungu. Y., Xiang. H. "A beam –type element foranalyzing the eccentric load effect of box girder bridges", February 2022. *Structrure* 36(5):1-12. Journal homepage: www.elsevier.com/locate/structures

[2] Dahmani, L., Drizi, S., Djemai, M., Boudjemia, A., and Mechiche, M. O. "Lateral torsional buckling of an eccentrically loaded channel section beam", *International Journal of Civil and Environmental Engineering, 9*(6), 2015, 689 - 692.

[3] Rathore, A. K., and Kushwah, N. "A Review on Simulation Analysis in Lateral Torsional Buckling of Channel Section by Ansys Software", *International Journal of Trend in Scientific Research and Development (IJTSRD).* Volume 4 Issue 1, December 2019:www.ijtsrd.com e-ISSN:2456-6470.

[4] Murray, N. W. "Introduction to Thin-Walled Structures", *Oxford Engineering Science Series13*. Published 18 march, 2022.

[5] Tesfaldet, G., and Vera, V.G. "The behavior of thin-walled beam with restrained torsion", *Magazine of Civil Engineering*. 2022. 110 (2). Article N0.11009. journal homepage://engstroy.spbstu.ru/

[6] Chidolue, C. A. "Torsional - distortional analysis of thin-walled box girder bridges using Vlasov's theory", *University of Nigeria, Nsukka (PhD Thesis).* 2013.

[7] Nwokoye, O. S., Chidolue, C. A., and Ezeagu, C. A. "Effect of Cross-Sectional Dimension on Deformation of Double Spine Mono-Symmetric Box- Girder due to Torsional – distortional Loads", *Nnamdi Azikiwe University Journal of Civil Engineering (NAUJCVE),* Volume -1, Issue -1, 2023, pp.98-120, www.naujcve.com

[8] Vlasov V. Z. "Thin walled space structures" *Gosstrojizdat, Moscow*. 1958.

[9] Varbernov, C. P. "Theory of Elasticity". *Technika Press Sofia, 4th edition,* 1976, p.254-271.

[10] Ezeh, J. C., Ibearugbulem, O. M., Nwadike, A. N., and Echehum, U. T. "The Behaviour of Buckled CSFS Isotropic Rectangular Plate Using Taylor Maclaurin Polynomial Series Shape Function on Ritz Method", *International Journal of Engineering Science and Innovative Technology (IJESIT), 3*(1), 2014, January; 60 - 64.

[11] Onyechere, I. C., Ibearugbulem, O. M., Anyaogu, U. C., Awodiji, C. T. G. "Free-Vibration Study of Thick Rectangular Plates Using Taylor Maclaurin Polynomial Displacement Function", *Saudi Journal of Engineering and Technology*, 2020; pp.77-78.Journal homepage: https://saudi-journals.com/sleat.

[12] Bassetti, A., Ruggiero, S., & Candeloro, M. (2017). "Finite Element Analysis of Thin-Walled Structural Elements Using Numerical Integration". *International Journal of Structural Engineering*, 8(2), 104-118. DOI: 10.1504/IJSTRUCTE.2017.1000253.

[13] Kreyszig, E. "Advanced Engineering Mathematics", *Tenth Edition*, pg. 167-174.

[14] American Association of State Highway and Transportation Officials (AASHTO), "Interim Revisions for LRFD Steel Bridge Design Guide and material specifications for steel bridges", Washington, D.C.