Nigerian Journal of Technology (NIJOTECH)
Vol. 42, No. 2, June, 2023, pp.236 - 243
www.nijotech.com

Print ISSN: 0331-8443
Electronic ISSN: 2467-8821
https://doi.org/10.4314/njt.v42i2.11

LINEAR PROGRAMMING MODEL FOR SCHEDULING BUS RAPID TRANSIT IN LAGOS STATE, NIGERIA

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Article history: Received 03 October, 2022. Revised 10 January, 2023. Accepted 19 January, 2023. Published 31 July, 2023.

Abstract
The amount of time users have to wait influences their mode of transportation choice. Commuters don't like to wait in the bus station, especially when the weather is terrible and for the purpose of keeping appointments. Scheduling buses to the various bus terminals will meet the commuter's needs. The aim of the study is to reduce the waiting time of commuters at bus station and to assign buses to each route. A mathematical model called linear programming (LP) was developed to schedule buses to improve the smooth process of the Bus Rapid Transit (BRT) system. The linear programming model was applied to the Lagos Metropolitan Area Transport Authority (LAMATA) data in Lagos State. The proposed method generated three (3) shifts from 6 – 11am in the morning, 11 – 4pm in the midday and 4 – 9pm in the evening subject to two (2) shifts of fifteen (15) hours to reduce the number of hours buses operate per day and also allocated buses for each route for weekdays with five (5) hours per each shift instead of seven (7) hours buses operated per shift. A properly implemented strategy would significantly decrease passengers' long waits at bus stops, reduce the breakdown of buses, man-hours lost, and allows commuters to meet their various schedule for works, meetings, and other assignments.

Keywords: Bus Rapid Transit, Bus Scheduling, Linear Programming, Shifts, Waiting time.

1.0 INTRODUCTION

Lagos is Sub-Saharan Africa's largest city. It has experienced significant urbanization and population growth; it is regarded as the business centre of the country. Lagos, Africa's most populated metropolis, has the highest population density in Nigeria, with a population range between 19 to 22 million distributed along 3000 to 4000 square kilometers rising at a rate of 4% per year. It is the world's sixth-largest metropolis. The Lagos metropolitan region extends far beyond the state's physical boundaries. Over 70% of the country's freight is handled through its seaport and international airport. It is also Nigeria's largest manufacturing center, serving as the country's economic, commercial, and industrial hub, employing over 45 percent of the country's skilled workforce [1]–[3] . Its recent rapid growth has resulted in significant increases in demand for all modes of urban transportation. This demand, however, has not been met by an adequate supply of infrastructure and services. As a result, there is widespread traffic congestion, packed buses, and extended commuter waits [4].

Lagos is a fast-paced megacity with a population of over six million people who all require some type of transportation. As a result, the use of bus rapid transit (BRT) as a roadway system is unavoidable. It is a bus-based system in which the buses operate on a dedicated lane, in some cases, underpasses or tunnels are used to provide a dedicated route for the movement of buses and in a bid to tackle the ineffective transport system in Lagos. The Lagos State Government, in collaboration with the business sector, established the bus rapid transit (BRT) system to improve vehicular mobility [5]. BRT is a bus-based public transportation system that strives to merge rail's ability and speed with a bus's flexibility and reduced price. BRT had been implemented in 173 metropolises worldwide as of July 2020, covering 5,196 km and transporting more than 30 million people per day [6]. Brazil has the most BRT passengers per day (10,852,339 per day) and the longest cumulative BRT length in the world (778 km). Because BRT is a low-cost 'rail-like' rapid transit system, it can provide a higher level of service than traditional bus routes. Bus-only lanes, fare pre-payment systems, customer details, limited stops, and advance-ability automobiles are among the other advancements. Based on these, BRT outperforms conventional buses in terms of speed, luxury, service rate, and schedule consistency [6]. Over the last twenty years, bus rapid transit (BRT) systems have emerged as a substitute design for high-level service systems. BRT systems thrived first in Latin American metropolises, after which stretched to Asia, and eventually the rest of the world. BRT systems are emerging as a viable option to rail-bound systems, which allow for vastly greater outlay and an extended time frame. BRT is an essential component of the mass public transportation system in megacities across the whole of developing markets, especially in Asia and America [7]. In both developed and developing countries, BRT is universally acknowledged as an extremely efficient urban congestion relief mechanism. One of the most critical aspects of BRT running is the design of bus schedules [8].

Users' choice of method of transportation is heavily influenced by waiting time. People dislike waiting, particularly when the weather is bad or they have important appointments to keep. Passengers would rather spend their waiting time doing something more enjoyable. As a result, they incur so-called opportunity costs, which are the worth of the passenger's best option. These can be of various types, and each passenger will place a different value on them. By putting a monetary value on the things that could be done while waiting, the passenger's time becomes valuable. To keep those costs to a minimum, the average waiting time should be kept as short as possible [9].

Over the years, due to the steady rise in technology, urban development, and human population, a lot of modeling systems and techniques have been adopted in other to effectively assign vehicles to meet the demands of passengers and customers. Stephan [9] suggested a multi-objective system for creating bus timetables with varied vehicle sizes that accomplishes two goals at the same time. To begin, reduce the variation between the determined headways and the required even headway. Second, reduce the deviation of perceived passenger loads from the vehicles' targeted even-load level at the peak point. Hafezi et al [10] observed that peak-hour passenger transportation demand is higher, and buses are overcrowded. The duration of peak-hour traffic disorganization of bus schedules is higher than non-peak-hour traffic, and it has been suggested that establishing an exclusive bus lane for bus operations can lessen disorganization. Amiruddin and Mohammad [11] observed the behavior of passengers and bus operators in mixed traffic lanes and exclusive bus lanes. Their statistical research takes into account passenger volumes, bus journey times, average speeds, and bus mission delays during morning rush hour. Their strategy also reduced total bus mission delays. Also, Hafezi et al [12] examined the various bus operation elements collected from the questionnaire for a rush and light traffic period.

Available literature shows that many researchers focused on social, human comfort, and socio-economic aspects/ conditions of BRT in Lagos State. For instance, Adebambo and Adebayo [13], Kolawole [14] Orekoya [1], Okagbue [15], Afolabi and Fashola [5], Afolabi et al [16] and Kuye et al [17] studied the passengers' attitudes and preferences regarding bus rapid transit (BRT) in Lagos State in the areas of ranges, safety, affordability, reliability, trip duration, and waiting time at BRT bus stations. The impact of bus rapid transit services on cost-cutting potential, income, and passenger satisfaction with bus rapid transit to passenger movement in Lagos State was studied, and it was discovered that the rate of accidents among bus rapid transit buses is infinitesimally low, as is the rate of sensitivity of bus rapid transit commuters to highway congestion accidents. Both were 0.000008 and 0.000006, and the issues encountered by passengers plying BRT in Lagos in terms of bus security, lowering commuting time, the attitude of the employees and drivers, currently existing routes, and the rates paid. Ticketing issues, deterioration of the bus rapid transit corridor, staff misdeeds, prevalent harm to bus rapid transit huts, decay of infrastructure at the bus rapid transit station, change management, poor maintenance of the transit body, infringements on the use of the bus rapid transit corridor, and safety issues are all examples of bus rapid transit (BRT) challenges and mitigation actions.

They recommended that the number of bus rapid transit bus stops be increased, that operational techniques be developed, and that the use of the Intelligent Transportation System (ITS) be promoted through periodic assessments of bus rapid transit drivers, buses, stops, and safety facilities to evaluate the drivers' conformance with safety policies and procedures and the buses' performance is disrupted by the lack of safety features and other security devices. Improvements in bus rapid transit services through the use of technology that allows commuters to pay with their phones, improve bus rapid transit vehicle maintenance to avert or decrease the likelihood of accidents, and ensure quick travel times. Amiegbebhor and Dickson [18] reviewed the influence of the BRT system on commuter fulfilment in Lagos State. The majority of Lagos state BRT commuters were pleased with the service in terms of protection, pace, personality and appearance, ticket price arrangement, convenience, boarding/boarding device, journey time, and ability, while some were dissatisfied with reliability and waiting time. They came to the conclusion that bus schedules should guarantee that no customer needs to wait more than 5-10 minutes at a bus station before boarding a bus rapid transit bus. Ulkhaq et al [19] deduced that it is imperative to enhance the service quality of BRT in order to ensure customer satisfaction. They used the well-known SERVQUAL instrument to examine the quality of bus rapid transit, which has five dimensions: tangibles, reliability, responsiveness, assurance, and empathy. Ishaq and Cats [7] analyzed the operational performance of a new BRT system in Israel employing vast analytical data. They asserted that the emergence of the BRT system has significantly improved service delivery in several key areas when compared to the prior traditional public transport, but that they are still unable to meet the Ministry of Transport's intended level of service reliability. Suguiy et al [20] developed an approach for directing political stances to maintain the preferences of the three major decision makers in public transit systems for value, passenger satisfaction, and organizational performance indicators, in order to guarantee the viability of the experimental by service suppliers. Because of its robust analytical techniques and ability to deal with problems involving multiple measures, they chose data envelopment analysis (DEA) to classify all such indices.

Zheng et al [6] studied variations in performance and different service attributes after implementing BRT for customer perception in three scenarios: previous to BRT implementation, yet another year after BRT deployment, and four years succeeding BRT operation. The outcomes of the study can assist decision-makers to adjust performance indicators for various kinds of services. Sheth and Sarkar [21] study concentrates on the cost-benefit analysis for social services of Ahmedabad's electric bus rapid transit system (e-BRTS). They figured out and quantified many elements of advantages and expenses to traffic participants and agencies as a flourishing investment for the city. Basso et al [22] proposed a dynamic traffic concept that constitutively models waiting in line on the highway as well as at BRT stations. According to their findings, instituting a BRT is always proficient because it lowers total social cost; if potential is not excellently divisible, a BRT is efficient throughout many cases. Moreno et al [23] developed a constructive heuristic algorithm with two phases to help solve the problem of generating work shifts in the Central Western Metropolitan Area (AMCO) Bus Rapid Transit System. The algorithm generates work shifts for Integra SA's drivers. Morales et al [24] developed a stochastic model to examine the impact of introducing a bus at a single station on waiting times.

The study indicates an injection criterion based on the length of the headway preceding each bus. Ren et al [8] developed an hybrid approach for optimizing the BRT bus route and censor priority control at the same time to enhance the BRT's operational effectiveness. The model increases the operational efficiency of BRT vehicles by 10% by reducing travel time. Filabadi et al [25] studied the perspective of a BRT controller and developed a stochastic mixed-integer nonlinear programming (MINLP) model for BRT scheduling of commuter waiting periods and commuter journey times in order to reduce total related time of commuters. Sevim et al [26] explored the scheduling of Istanbul's Metrobus fleet BRT system. It was proposed to use a multiple depot vehicle scheduling problem (MDVSP) model with a developed heuristic called trips merger (TM).

Berhan et al [27] and Nyor et al [28] and Abdullah and Masadeh [29] applied the Linear Programming model (LP) for allocation or scheduling buses to routes in a public bus – transportation service in Addis Ababa, in and around the city in Ethiopia, Niger state for Intra and interstates routes in Nigeria and City of Aqada in Jordan. At present, mathematical programming model for scheduling (BRT) in Lagos state is scanty in literature. This mathematical programming will also improve the quality of service to commuters using or plying the (BRT) in Lagos state. This research work developed a mathematical programming model for bus schedules in other to ensure the smooth operation of the bus rapid transit (BRT) system, determine the number of operating buses per shift, reduce the waiting time of commuters at the bus terminals and determine the number of buses that will be assigned to each route.

2.0 MODEL DEVELOPMENT

A. Brief Description of Problem

The bus schedule is among the activity-planning processes in a public transportation company that is responsible for properly assigning buses to satisfy the anticipated travel demands.

However, the bus assignment decision-making process is an exchange between customer satisfaction and expenditures for bus operating firms. The reason is that utilizing several buses increases the company's operating costs, while using fewer available buses reduces the service quality level [27]. As a result, a mathematical programming model for the Lagos state bus rapid transit system was developed.

B. Assumptions of Model

The following assumptions are set for the model

1. Numbers of commuters are the same for morning and evening shift
2. The number of daily commuters and the pattern of commuting is the same for every working day
3. All buses in the fleet have the same capacity
4. The buses have exclusive lanes and there is no barrier on the lane
5. All buses are available and in good working condition hence no breakdown during operation
6. The number of commuters reduces to **P%** during the off-peak period
7. All busses are filled
8. Busses are allowed to run only one shift

A linear programming problem can be stated in the following way:
The objective function should be maximized or minimized.

$$Z = C_1Y_1 + C_2Y_2 + C_nY_n \tag{1}$$

Subject to the constraints

$$a_{11}y_1 + a_{12}y_2 + ...+a_{1n}y_n \le d_1 \tag{2}$$
$$a_{21}y_1 + a_{22}y_2 + ...+a_{2n}y_n \le d_2 \tag{3}$$
$$a_{m1}y_1 + a_{m1}y_2 + ...+a_{mn}y_n \le d_m \tag{4}$$

$y_1, y_2,........., y_n \ge 0$ (Non-negative restriction)

Where;
y_i =Decision variables (They should completely describe the decision to be made).
ci = Coefficient variable.
a_1 =Constant or structural coefficient.
d_1= is a column vector of m constants [28] and [30].

C. Model Formulation

The model was formulated based on the objective function:
Maximize the number of buses for all routes.

$$Max\ Z = \sum_{m\epsilon M}\sum_{r\epsilon R} x_{ar} + \sum_{a\epsilon A}\sum_{r\epsilon R} x_{mr} + \sum_{e\epsilon E}\sum_{r\epsilon R} x_{er} \tag{5}$$

Constraints of the model
1. It is required that a minimum of K buses should be released to ply the routes per day

$$\sum_{m\epsilon M}\sum_{r\epsilon R} x_{mr} + \sum_{a\epsilon A}\sum_{r\epsilon R} x_{ar} + \sum_{e\epsilon E}\sum_{r\epsilon R} x_{er} \ge k \tag{6}$$

2. The total number of busses available is q with equation (6). The total number of busses released cannot be more than those available

$$\sum_{m\epsilon M}\sum_{r\epsilon R} x_{mr} + \sum_{a\epsilon A}\sum_{r\epsilon R} x_{ar} + \sum_{e\epsilon E}\sum_{r\epsilon R} x_{er} \le q \ and\ k < q \tag{7}$$

3. The number of commuters during off-peak hours drop to P% of the peak period value. The off-peak period is usually the afternoon shift.

$$\frac{\sum_{m\epsilon M}\sum_{r\epsilon R} x_{mr}}{\sum\sum x_{mr}+\sum\sum x_{ar}+\sum\sum x_{er}} = o.p \tag{8}$$

$$\sum\sum x_{mr} - O.P\left(\sum\sum x_{mr} + \sum\sum x_{ar} + \sum\sum x_{er}\right) = 0 \tag{9}$$

4. The number of commuters who leave for morning work and business is equal to the number coming home in the evening.

$$\sum\sum x_{mr} = \sum\sum x_{er}$$

That is,

$$\sum_{m\epsilon M}\sum_{r\epsilon R} x_{mr} - \sum_{e\epsilon M}\sum_{r\epsilon R} x_{er} = 0 \tag{10}$$

Notations

M: Set of busses assigned to the morning shift
A: Set of busses assigned to the afternoon shift
E: Set of busses assigned to the evening shift
R: Set of routes for the buses
x_{mr}= Busses on morning shift assigned to $r^{th}\ routes$
x_{ar}= Busses on afternoon shift assigned to $r^{th}\ routes$
x_{er}= Busses on evening shift assigned to $r^{th}\ routes$
k = The current number of buses in operation
q = The total number of buses available

3.0 ANALYSIS, RESULTS AND DISCUSSION

Data were collected at the Lagos Metropolitan Area Transport Authority (LAMATA) which was empowered to plan an integrated transport system for the Lagos State Government with a specific focus on implementing and regulating mass transit systems, the amount of buses accessible from operating and nonoperational, the potential of each vehicle, the hours each bus operates per day, and the number of shifts operated per day were all collected. The current situation was 2 shifts with equal distribution of 50%.

A. Model Application for Number of Buses for Each Shift

From the data collected, the following information were used for the scheduling of the bus allocation:

- The total number of buses available per day is 390.
- The carrying capacity of each bus is 80.
- The Total trip made per bus per day is 7.
- The Total number of passengers taking all routes per day is 157000.

To determine the current number of buses in operation

$\frac{80\times7}{157000} = 280$

Hence, the current number of buses in operation is 280.

Conditions:

1) Assign a 40% bus allocation during peak times (morning and evening shifts). This condition is based on the busy morning and evening traffic as a result of high number of commuters during these times.
2) Assign a 20% bus allocation during off-peak times (afternoon shift). This condition is based on the idle afternoon traffic as a result of few commuters during this time. Rearrang the equation to account for the amount of buses and assign each routing path according to the percentage of passengers using it. Table 1 proposes shift for the BRT operation and Table 2 shows the distribution of passengers for each route.

Objective:

$Max\ Z = 0.4x_{mr} + 0.2x_{ar} + 0.4x_{er}$

Subject to

1. $0.4x_{mr} + 0.2x_{ar} + 0.4x_{er} \geq 280$
2. $0.4x_{mr} + 0.2x_{ar} + 0.4x_{er} \leq 390$
3. $0.4x_{mr} - 0.2(0.4x_{mr} + 0.2x_{ar} + 0.4x_{er}) = 0$
4. $0.4x_{mr} - 0.4x_{er} = 0$

Table 1: Proposed Shift for BRT Operation

Proposed Buses For Each Shift		
6am – 11am (Morning Peak)	**11am – 4pm (Afternoon Peak)**	**4pm – 9pm (Evening Peak)**
156	78	156

B. Routes Allocation

The bus routes are as follows:

i. Ikorodu to Mile 12 = (x_{1r})
ii. Ikorodu to CMS = (x_{2r})
iii. Mile 12 to CMS = (x_{3r})
iv. Ikorodu to Fadeyi = (x_{4r})
v. Mile 12 to Fadeyi = (x_{5r})
vi. Fadeyi to CMS = (x_{6r})

Table 2: Number of people taken the bus per day at each of the bus routes and the percentage of passengers.

Routes	Passengers	Percentage (%)
Ikorodu - Mile12	21980	14
Ikorodu - CMS	17270	11
Mile12 - CMS	23550	15
Ikorodu - Fadeyi	58090	37
Mile12 - Fadeyi	21980	14
Fadeyi - CMS	14130	9
Total	**157000**	**100**

For routes allocation, we applied equations 1, 2, 3, and 4 based on table 2 above, and individual routes were calculated based on the percentage allocated to each shift in table 3 for current and new buses.

Table 3: Bus Route Allocation

Total Allocation			Individual Allocation		
Routes	**Current**	**New**	**6am – 11am**	**11am – 4pm**	**4pm – 9pm**
Ikorodu – Mile12	39	55	22	11	22
Ikorodu – CMS	31	43	17	9	17
Mile12 – CMS	42	58	23	12	23
Ikorodu – Fadeyi	104	144	58	28	58
Mile12 – Fadeyi	39	55	22	11	22
Fadeyi – CMS	25	35	14	7	14
TOTAL	280	390	156	78	156

3.1 Discussion

The formulated mathematical problem was solved using the LINGO 17.0 Software package for solving Linear Programming developed.

A total of fifteen (15) hours within 3 shifts developed for the BRT operation from 6 – 11am in the morning, 11am – 4pm in the midday and 4 – 9pm in the evening was proposed, instead of two (2) shifts in the same fifteen (15) hours operated by the organization to enable the efficient running of the BRT. The proposed shift will ensure optimization of route timing for the commuters, eliminate or reduce drivers' idle time, assist drivers start and end shifts to minimize travel time, reduce passengers waiting time at the bus station, reduce the total commuters travel time, improve patronage or daily passenger comfort, improve commuters' satisfaction by reducing their complaints.This result is similar to Abdullah and Ma-sadeh's [29] study with 15 routes, 89 operational buse s, and 4 shifts, in which the number of buses in the n-ew shift scheme was increased while the number of buses in the old shift scheme was reduced to meet commuter demand for the proposed routes for economic benefits.

Furthermore, Berhan et al [27] identified that during peak periods, the number of buses required was higher than during offpeak periods, with 2.1% bus usage above the existing model.

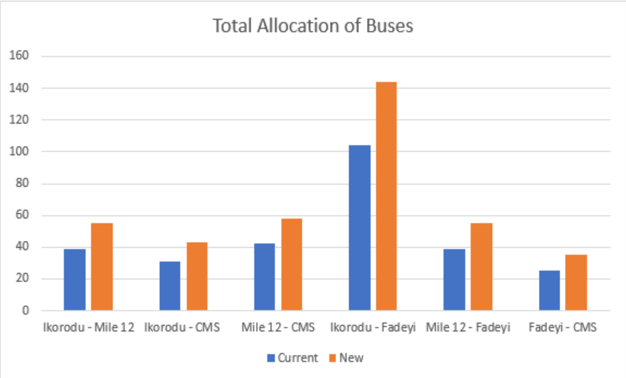

Figure 1: Total Allocation of Route Buses

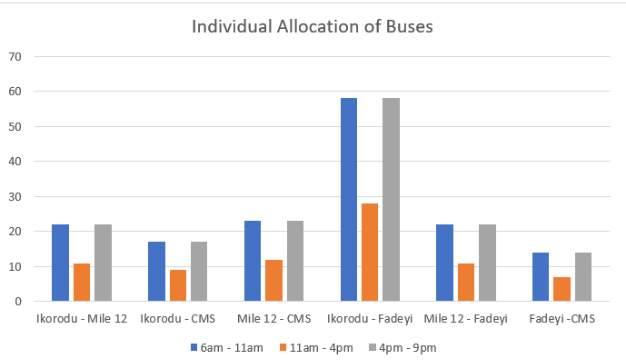

Figure 2: Individual Allocation of Route Buses

As envisaged by this study, the morning peak period is for commuters going to work, business and education; the afternoon/inter peak period is for commuters going to shops and returns from schools (education), while the evening peak period is also for commuters returning from work, business and shops. The buses available for operation is 390, but 280 of the buses were dispatched daily. Based on the proposed three shifts, it was shown that for the first shift, 156 operational buses were scheduled, 78 buses for the second shift, and 156 buses for the third shift. The allocation of buses for weekdays for the proposed shifts to each terminal based on our study considers Ikorodu – Mile (55), Ikorodu – CMS (43), Mile 12 – CMS (58), Ikorodu – Fadeyi (144), Mile 12 – Fadeyi (55) and Fadeyi – CMS (35) to utilize the 390 available buses for operation. The buses for each route on morning, afternoon and evening shifts were also allocated thus, Ikorodu – Mile 12 (22 – 11 - 22), Ikorodu – CMS (17 – 9 - 17), Mile 12 – CMS (23 – 12 - 23), Ikorodu – Fadeyi (58 – 28 - 58), Mile 12 – Fadeyi (22 – 11 - 22) and Fadeyi – CMS (14 – 7 - 14). The above bus allocation will still allow the total 7 trips made by bus per day.

The increase in the use of buses can also be found in a similar study of Ngutor et al [28], which used 49 operational buses for both intra and interstate travel, and the number of buses in the current allocation increased in some routes while others remained unchanged.

For both total and individual allocation of route buses, Figures 1 and 2, showed that, Ikorodu – Fadeyi route recorded the most number of buses followed by Mile 12 – CMS route buses, which translate to more commuters on these routes. This should allow the operator(s) to improve qualify of service for the commuters and enhance other routes.

4.0 CONCLUSION

A Linear programming model was applied in this study for bus scheduling of the Bus Rapid Transit system in Lagos. The problem was formulated mathematically and solved using the LINGO 17.0 Software package.

The proposed approach developed three (3) shifts instead of the current two (2) shifts. This reduces the number of hours the buses operate per day and also allocates buses for each route for weekdays from seven (7) hours to five (5) hours per shift. This subsequently reduces the stress on the bus drivers.

If the proposed approach is properly implemented, it will reduce the waiting time of commuters at the bus route, reduce the breakdown of buses, man-hours lost, and allows commuters to meet their various schedule for works, meetings, and other assignments. Lagos state government should invest more on rail and waterway transportation to make life easy for the citizens. Meanwhile, the proposed model was limited to BRT. However, further studies will be required through Benefit-Cost Analysis (BCA) on the need to analyze the full utilization of the buses in terms of the costs of maintenance and employing new drivers against the benefits.

It's also recommended that weekend allocation should be considered by adopting another model because of the peak and off-peak periods which might not be constant like the weekdays.

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