Nigerian Journal of Technology (NIJOTECH)
Vol. 42, No. 1,March, 2023, pp.130 - 135
www.nijotech.com

Print ISSN: 0331-8443
Electronic ISSN: 2467-8821
https://doi.org/10.4314/njt.v42i1.16

DYNAMIC ANALYSIS OF THE LARGE STRAIN DEFORMATION OF FLEXIBLE PIPES CONVEYING TWO-PHASE FLUIDS. PART II: NONLINEAR VIBRATION ANALYSIS

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Article history: Received 15 June, 2021. Revised 12 April, 2023. Accepted 13 April, 2023

Abstract

This work presents the nonlinear analysis of the dynamics of large strain deformation of subsea flowlines and jumpers conveying two-phase fluid. Precisely, flexible pipes operating in the stated condition undergo large strain deformations. Thus, the known nonlinear deterministic model of the system is solved using method of discretized perturbation. Precisely, this study obtained the nonlinear natural frequency of simply supported flexible pipes modelled using the large strain deformation theory. Results show that both hardening and softening nonlinear behaviors are indicated for pipe undergoing large strain deformation compared with deformations modelled with small strain theory. Thus, operational insights and parameters for sustainable management of large strain deformed subsea flow lines and jumpers conveying two phase flow are made available.

Keywords: Large strain deformation, Simply supported, Hardening, Softening.

1.0 INTRODUCTION

The first part of this work considered the effects of temperature, pressure and tension on linear free vibration of a pipe conveying two-phase flow modeled using large strain deformation theory.

However, the present study investigates the effects of nonlinearities due to large strains in a straight pipe conveying two-phase flow. It considers the influence of thermal strains, pressurization and tension on the system's dynamics. By adopting the discrete form of multiple time scale perturbation method, the nonlinear equations in [1] were expanded and regrouped. This facilitates the determination of nonlinear natural frequencies and responses of pipes subjected to large strains deformation [2] [3]. It also enabled the investigation of the effects of thermal strains, pressurization and tension in the presence of large nonlinear strains.

2.0 NONLINEAR VIBRATION ANALYSIS

Recalling the dimensionless transverse and longitudinal vibration equations as derived in the first part of this work, we have;

$$\ddot{w} + 2 + \left(v_{f1}\sqrt{\beta_{f1}}\sqrt{\beta_1} + v_{f2}\sqrt{\beta_{f2}}\sqrt{\beta_2}\right)\dot{w}' + \left(\beta_{f1}{v_{f1}}^2 + \beta_{f2}{v_{f2}}^2\right)w'' + \left(\dot{v}_{f1}\sqrt{\beta_{f1}}\sqrt{\beta_1} + \dot{v}_{f2}\sqrt{\beta_{f2}}\sqrt{\beta_2}\right)w' + w^{IV} + \frac{3}{2}\left(u'''w'' + 2u''w''' + u'w^{IV}\right) - \frac{\eta}{2}\left(u''w' + u'w''\right) - \frac{\eta\delta u'w'}{2} + \frac{\eta}{2}\left(\delta\,\theta th\,w' + \theta th'w' + \theta th\,w''\right) + \frac{1}{\eta}\left(\frac{\theta th''w''}{2} - \theta th'w''' - \theta th\,w^{IV}\right) - Te\,w'' + \frac{2}{\eta}Te\,w^{IV} + \left(Pr'w' + \delta Prw' + Prw''\right) - \frac{1}{\eta}\left(Pr''w'' + 2Pr'w''' + Prw^{IV}\right) = 0 \tag{1a}$$

$$\ddot{u} + \left(\dot{v}_{f1}\sqrt{\beta_{f1}}\sqrt{\beta_1} + \dot{v}_{f2}\sqrt{\beta_{f2}}\sqrt{\beta_2}\right) + \left(\dot{v}_{f1}\sqrt{\beta_{f1}}\sqrt{\beta_1} + \dot{v}_{f2}\sqrt{\beta_{f2}}\sqrt{\beta_2}\right)u' + 2\left(v_{f1}\sqrt{\beta_{f1}}\sqrt{\beta_1} + v_{f2}\sqrt{\beta_{f2}}\sqrt{\beta_2}\right)\dot{u}' + \left(\beta_{f1}{v_{f1}}^2 + \beta_{f2}{v_{f2}}^2\right)u'' - \frac{3}{2}w''w''' - \eta\left(\delta u' + \eta u''\right) - \frac{3}{4}\eta\left(\delta u'^2 + 2u'u''\right) - \frac{\eta}{4}\left(2w'w'' + \delta w'^2\right) + \frac{1}{2}\left(\delta\,\theta th + \theta th'\right) + \frac{1}{2}\left(\delta\,\theta th\,u' + \theta th'u' + \theta th\,u''\right) + \left(-\eta\,Te\,u''\right) + \left(\frac{1}{2}Pr' + \frac{1}{2}\delta.Pr\right) + \left(Pr'u' + \delta Pr\,u' + Pr\,u''\right) = 0 \tag{1b}$$

In Equations (1a) and (1b), $u(x,t)\ and\ w(x,t)$ are the dimensionless displacements in the longitudinal and transverse directions respectively, (vfj) is the flow velocities of the constituent phases/components used in the analysis of the dynamics of the system, (βj) is the mass ratio which relates the mass of a fluid phase to the total mass of the fluids and the pipe as derived in [1] for a single phase fluid, $(\ \beta fj\ )$ is the mass ratio which relates the mass of a fluid phase to the total fluid mass, and η index the pipe flexibility, T_e is the dimensionless Tension, while Pr is dimensionless pressurization.

The complementary boundary conditions are;

$$w(0) = \frac{d^2w}{dx^2}(0) = w(1) = \frac{d^2w}{dx^2}(1) = u(0) = u(1) = 0 \quad (1c)$$

For nonlinear analysis, approximate solutions of the coupled nonlinear problem can be obtained using a three-time scale discretized perturbation technique [4] by assuming basis solution of the form:

$$w(x,t) = \sum_{n=1}^{\infty} q_n(t)\phi_n(x) \quad (2a)$$

$$u(x,t) = \sum_{n=1}^{\infty} p_n(t)\psi_n(x) \quad (2b)$$

Here, $q_n(t)$and $p_n(t)$are the generalized coordinates, $\phi_n(x)$ and $\psi_n(x)$ are the eigenfunctions of the linear vibration of a simply supported pipe conveying fluid, [5].

Substituting Equations (2a) and (2b) into the coupled governing nonlinear equations (1a) and (1b). Minimizing the resultant residual equations require that the integrand of the convoluted weighing functions should vanish, such that:

$$\int_0^1 R_w \cdot \phi_{mo}(x)dx = 0 \quad (3a)$$

$$\int_0^1 R_u \cdot \psi_{mo}(x)dx = 0 \quad (3b)$$

Where R_w and R_u are residual equations along the transverse and longitudinal directions respectively. This results in a system of simultaneous equations. Thus, based on finite mode analysis, the following system of differential equations is obtained:

$$aw11\, q_n(t) + aw12\, p_n(t)q_n(t) + aw13\, \dot{q}_n(t) + aw14\, \ddot{q}_n(t) = 0 \quad (4a)$$

$$au11 p_n(t) + au12 p_n(t)^2 + au13 q_n(t)^2 + au14\, \dot{p}_n(t) + au15\, \ddot{p}_n(t) = 0 \quad (4b)$$

Where,
$aw11, aw12, aw13,\ aw14, au11, au12, au13$ and $au14$ are specifically defined below.

$$aw11 = Pr\int_0^1 \phi_{mo}(x)\phi_n''(x)\,dx - Te\int_0^1 \phi_{mo}(x)\phi_n''(x)\,dx + \left(\int_0^1 \phi_{mo}(x)\phi_n''(x)\,dx\right)v_{f1}^2\beta_{f1} + \left(\int_0^1 \phi_{mo}(x)\phi_n''(x)\,dx\right)v_{f2}^2\beta_{f2} + \frac{1}{2}\theta th\int_0^1 \phi_{mo}(x)\phi_n''(x)\,dx + \int_0^1 \phi_{mo}(x)\phi_n^{(4)}(x)\,dx - \frac{Pr\int_0^1 \phi_{mo}(x)\phi_n^{(4)}(x)\,dx}{\eta} + \frac{2Te\int_0^1 \phi_{mo}(x)\phi_n^{(4)}(x)\,dx}{\eta} - \frac{\theta th\int_0^1 \phi_{mo}(x)\phi_n^{(4)}(x)\,dx}{\eta}$$

$$aw12 = -\frac{1}{2}\eta\int_0^1 \phi_{mo}(x)\psi_n'(x)\phi_n''(x)\,dx - \frac{1}{2}\eta\int_0^1 \phi_{mo}(x)\phi_n'(x)(x)\,dx - 3\int_0^1 \phi_{mo}(x)\phi_n''(x)\psi_n''(x)\,dx + \frac{3}{2}\int_0^1 \phi_{mo}(x)\phi_n''(x)\psi_n^{(3)}(x)\,dx + \frac{3}{2}\int_0^1 \phi_{mo}(x)\psi_n'(x)\phi_n^{(4)}(x)\,dx$$

$$aw13 = 2\sqrt{\beta}\left(\int_0^1 \phi_{mo}(x)\phi_n'(x)\,dx\right)v_{f1}\beta_{f1} + 2\sqrt{\beta}\left(\int_0^1 \phi_{mo}(x)\phi_n'(x)\,dx\right)v_{f2}\beta_{f2}$$

$$aw14 = \int_0^1 \phi_{mo}(x)\phi_n(x)\,dx$$

$$au11 = Pr\int_0^1 \psi_{mo}(x)\psi_n''(x)\,dx + \left(\int_0^1 \psi_{mo}(x)\psi_n''(x)\,dx\right)v_{f1}^2\beta_{f1} + \left(\int_0^1 \psi_{mo}(x)\psi_n''(x)\,dx\right)v_{f2}^2\beta_{f2} - \eta\int_0^1 \psi_{mo}(x)\psi_n''(x)\,dx - Te\eta\int_0^1 \psi_{mo}(x)\psi_n''(x)\,dx + \frac{1}{2}\theta th\int_0^1 \psi_{mo}(x)\psi_n''(x)\,dx$$

$$au12 = -\frac{3}{2}\eta\int_0^1 \psi_{mo}(x)\psi_n'(x)\psi_n''(x)\,dx$$

$$au13 = -\frac{1}{2}\eta\int_0^1 \psi_{mo}(x)\phi_n'(x)\phi_n''(x)\,dx - \frac{3}{2}\int_0^1 \psi_{mo}(x)\phi_n''(x)\phi_n^{(3)}(x)\,dx$$

$$au14 = 2\sqrt{\beta}\left(\int_0^1 \psi_{mo}(x)\psi_n'(x)\,dx\right)v_{f1}\beta_{f1} + 2\sqrt{\beta}\left(\int_0^1 \psi_{mo}(x)\psi_n'(x)\,dx\right)v_{f2}\beta_{f2}$$

$$au15 = \int_0^1 \psi_{mo}(x)\psi_n(x)\,dx$$

Subsequently, a three time-scale perturbation technique is applied to analyze the differential equations by assuming time scale expansion of the form;

$$q_n(T_0,T_1,T_2) = \epsilon q_{n,1}(T_0,T_1,T_2) + \epsilon^2 q_{n,2}(T_0,T_1,T_2) + \epsilon^3 q_{n,3}(T_0,T_1,T_2) \quad (5a)$$

$$p_n(T_0,T_1,T_2) = \epsilon p_{n,1}(T_0,T_1,T_2) + \epsilon^2 p_{n,2}(T_0,T_1,T_2) + \epsilon^3 p_{n,3}(T_0,T_1,T_2) \quad (5b)$$

Here, the time derivative operators take the forms;

$$\left.\begin{aligned}\frac{d}{dt} &= D_0 + \epsilon D_1 + \cdots \\ \frac{d^2}{dt^2} &= D_0{}^2 + 2\epsilon D_0 D_1 + \cdots\end{aligned}\right\} \quad (5c)$$

Substituting Equations (5a), (5b) and (5c) into Equations (4a) and (4b), and making the coefficients of $\epsilon^i, i = 1,2,3.$ to vanish; the three time-scale perturbation analysis returns the following equations for $q_{n,1}, q_{n,2}, q_{n,3}, p_{n,1}, p_{n,2}$ and $p_{n,3}$;

$$\epsilon^1:\ aw13(D_1 q_{n,1}) + aw14(D_1^2 q_{n,1}) + aw11 q_{n,1} = 0 \tag{6a}$$

$$\epsilon^1:\ au14(D_1 p_{n,1}) + au15(D_1^2 p_{n,1}) + au11 p_{n,1} = 0 \tag{6b}$$

$$\epsilon^2:\ aw13(D_1 q_{n,2}) + aw14(D_1^2 q_{n,2}) + aw11 q_{n,2} = -aw13(D_2 q_{n,1}) - 2aw14(D_1 D_2 q_{n,1}) - aw12 p_{n,1} q_{n,1} \tag{6c}$$

$$\epsilon^2:\ au14(D_1 p_{n,2}) + au15(D_1^2 p_{n,2}) + au11 p_{n,2} = au14(D_2 p_{n,1}) - 2au15(D_1 D_2 p_{n,1}) - au12 p_{n,1}^2 - au13 q_{n,1}^2 \tag{6d}$$

$$\epsilon^3:\ aw13(D_1 q_{n,3}) + aw14(D_1^2 q_{n,3}) + aw11 q_{n,3} = -aw13(D_2 q_{n,2}) - 2aw14(D_1 D_2 q_{n,2}) - aw14(D_2^2 q_{n,1}) - aw13(D_3 q_{n,1}) - 2aw14(D_1 D_3 q_{n,1}) - aw12 p_{n,2} q_{n,1} - aw12 p_{n,1} q_{n,2} \tag{6e}$$

$$\epsilon^3:\ au14(D_1 p_{n,3}) + au15(D_1^2 p_{n,3}) + au11 p_{n,3} = -au14(D_2 p_{n,2}) - 2au15(D_1 D_2 p_{n,2}) - au15(D_2^2 p_{n,1}) - au14(D_3 p_{n,1}) - 2au15(D_1 D_3 p_{n,1}) - 2au12 p_{n,1} p_{n,2} - 2au13 q_{n,1} q_{n,2} \tag{6f}$$

Consequently, approximate solutions to Equations (6a) and (6b) can be presented as:

$$q_{n,1} = e^{iT_0\omega_n} A_n(T_1) + e^{-iT_0\omega_n} \bar{A}_n(T_1) \tag{7a}$$

$$p_{n,1} = e^{iT_0\alpha_n} B_n(T_1) + e^{-iT_0\alpha_n} \bar{B}_n(T_1) \tag{7b}$$

Substituting equations (7a) and (7b) into equations (6c) and (6d), the approximate solutions of Equations (6c) and (6d) can be obtained as:

$$q_{n,2} = e^{-iT_0\alpha_n - iT_0\omega_n} awF21_n(T_1,T_2) + e^{-iT_0\alpha_n + iT_0\omega_n} awF22_n(T_1,T_2) + e^{iT_0\alpha_n - iT_0\omega_n} awF23_n(T_1,T_2) + e^{iT_0\alpha_n + iT_0\omega_n} awF24_n(T_1,T_2) \tag{8a}$$

$$p_{n,2} = e^{-2iT_0\omega_n} auF21_n(T_1,T_2) + e^{-2iT_0\alpha_n} auF22_n(T_1,T_2) + e^{2iT_0\omega_n} auF23_n(T_1,T_2) + e^{2iT_0\alpha_n} auF24_n(T_1,T_2) \tag{8b}$$

Where,
$awF21_n(T_1,T_2)$, $awF22_n(T_1,T_2)$, $awF23_n(T_1,T_2)$, $awF24_n(T_1,T_2)$, $auF21_n(T_1,T_2)$, $auF22_n(T_1,T_2)$, $auF23_n(T_1,T_2)$ and $auF24_n(T_1,T_2)$ are specifically expressed below.

$$awF21_n(T_1,T_2) = -\frac{aw12\bar{A}_n(T_1,T_2)\bar{B}_n(T_1,T_2)}{aw11 - iaw13\alpha_n - aw14\alpha_n^2 - iaw13\omega_n - 2aw14\alpha_n\omega_n - aw14\omega_n^2}$$

$$awF22_n(T_1,T_2) = -\frac{aw12\bar{B}_n(T_1,T_2)A_n(T_1,T_2)}{aw11 - iaw13\alpha_n - aw14\alpha_n^2 + iaw13\omega_n + 2aw14\alpha_n\omega_n - aw14\omega_n^2}$$

$$awF23_n(T_1,T_2) = -\frac{aw12\bar{A}_n(T_1,T_2)B_n(T_1,T_2)}{aw11 + iaw13\alpha_n - aw14\alpha_n^2 - iaw13\omega_n + 2aw14\alpha_n\omega_n - aw14\omega_n^2}$$

$$awF24_n(T_1,T_2) = -\frac{aw12A_n(T_1,T_2)B_n(T_1,T_2)}{aw11 + iaw13\alpha_n - aw14\alpha_n^2 + iaw13\omega_n - 2aw14\alpha_n\omega_n - aw14\omega_n^2}$$

$$auF21_n(T_1,T_2) = -\frac{au13\bar{A}_n(T_1,T_2)^2}{au11 - 2iau14\omega_n - 4au15\omega_n^2}$$

$$auF22_n(T_1,T_2) = -\frac{au12\ \bar{B}_n(T_1,T_2)}{au11 - 2iau14\alpha_n - 4au15\alpha_n^2}$$

$$auF23_n(T_1,T_2) = -\frac{au13A_n(T_1,T_2)^2}{au11 + 2iau14\omega_n - 4au15\omega_n^2}$$

$$auF24_n(T_1,T_2) = -\frac{au12B_n(T_1,T_2)^2}{au11 + 2iau14\alpha_n - 4au15\alpha_n^2}$$

$$auF25_n(T_1,T_2) = -\frac{2(au13A_n(T_1,T_2)A_n(T_1,T_2) + au12B_n(T_1,T_2)B_n(T_1,T_2))}{au11}$$

Substituting Equations (6a) and (6b) into Equations (6e) and (6f), upon the elimination of secular terms, amplitudes solutions of the form,

$$A_n = \frac{1}{2}a_n(T_2)e^{i\beta_n(T_2)}, \quad \bar{A}_n = \frac{1}{2}a_n(T_2)e^{-i\beta_n(T_2)}, \quad B_n = \frac{1}{2}b_n(T_2)e^{I\theta_n(T_2)}, \ \bar{B}_n = \frac{1}{2}b_n(T_2)e^{-I\theta_n(T_2)} \tag{9}$$

can be obtained.

Given that steady state dynamics suggests constant amplitude and phases, it follows that $a_n'(T_2) = 0, a_n(T_2) = a_0, b_n'(T_2) = 0, \quad b_n(T_2) = b_0$. As a result, the transverse nonlinear frequency for the small strain model can be written as:

$$\omega_{nl} = \epsilon\left(-\frac{1}{4}\mathrm{Kw1I}a_0^2 - \frac{\mathrm{Kw2I}b_0^2}{4}\right) + \omega_n \tag{10a}$$

Where, Kw1I, Kw2I are specifically expressed below.

$$\mathrm{Kw1I} = \mathrm{Im}\left(\frac{2\mathrm{au13aw12}}{\mathrm{au11}(-\mathrm{aw13} - 2i\mathrm{aw14}\omega_n)} + \frac{\mathrm{au13aw12}}{(-\mathrm{aw13} - 2i\mathrm{aw14}\omega_n)(\mathrm{au11} + 2i\mathrm{au14}\omega_n - 4\mathrm{au15}\omega_n^2)}\right)$$

$$Kw2I = Im\left(\frac{2au12aw12}{au11(-aw13 - 2iaw14\omega_n)} + \frac{aw12^2}{(-aw13 - 2iaw14\omega_n)(aw11 + iaw13\alpha_n - aw14\alpha_n^2 + iaw13\omega_n - 2aw14\alpha_n\omega_n - aw14\omega_n^2)} + \frac{aw12^2}{(-aw13 - 2iaw14\omega_n)(aw11 - iaw13\alpha_n - aw14\alpha_n^2 + iaw13\omega_n + 2aw14\alpha_n\omega_n - aw14\omega_n^2)}\right)$$

It then follows that the longitudinal nonlinear frequency for the large strain model can be written as:

$$\alpha_{nl} = \epsilon\left(-\frac{1}{4}Ku1Ia_0^2 - \frac{Ku2Ib_0^2}{4}\right) + \alpha_n \tag{10b}$$

Also, Ku1I and Ku2I are specifically shown below.

$$\mathrm{Ku1I} = \mathrm{Im}\left(\frac{4\mathrm{au12au13}}{\mathrm{au11}(-\mathrm{au14}-2i\mathrm{au15}\alpha_n)} + \frac{2\mathrm{au13aw12}}{(-\mathrm{au14}-2i\mathrm{au15}\alpha_n)(\mathrm{aw11}+i\mathrm{aw13}\alpha_n-\mathrm{aw14}\alpha_n^2+i\mathrm{aw13}\omega_n-2\mathrm{aw14}\alpha_n\omega_n-\mathrm{aw14}\omega_n^2)} + \frac{2\mathrm{au13aw12}}{(-\mathrm{au14}-2i\mathrm{au15}\alpha_n)(\mathrm{aw11}+i\mathrm{aw13}\alpha_n-\mathrm{aw14}\alpha_n^2-i\mathrm{aw13}\omega_n+2\mathrm{aw14}\alpha_n\omega_n-\mathrm{aw14}\omega_n^2)}\right)$$

$$\mathrm{K2UI} = \mathrm{Im}\left(\frac{4\mathrm{au12}^2}{\mathrm{au11}(-\mathrm{au14}-2i\mathrm{au15}\alpha_n)} + \frac{2\mathrm{au12}^2}{(-\mathrm{au14}-2i\mathrm{au15}\alpha_n)(\mathrm{au11}+2i\mathrm{au14}\alpha_n-4\mathrm{au15}\alpha_n^2)}\right)$$

Following identical procedure, approximate solutions of the large strain model up to the third order in the transverse and longitudinal steady state conditions are also obtained as:

$$w(t) = \epsilon\, Cos\left(\frac{1}{4}Kw1IT_2a_0^2 + \frac{1}{4}Kw2IT_2b_0^2 - \beta_0 - t\omega_n\right) a_0\phi_n(x) \qquad (11a)$$

$$u(t) = \epsilon\, Cos\left(\frac{1}{4}T_2(-Ku1Ia_0^2 - Ku2Ib_0^2) + t\alpha_n + \theta_0\right) b_0\psi_n(x) \qquad (11b)$$

3.0 RESULTS OF NONLINEAR VIBRATION ANALYSIS OF A PIPE WITH LARGE STRAIN

The linear vibration problem has been solved and the results analyzed in earlier part of this work. We now present the numerical simulation of the nonlinear order 2 and order 3 dynamics as expressed in equations (6c) – (6f).

3.1 Effect of Pressure on Transverse and Axial Vibration

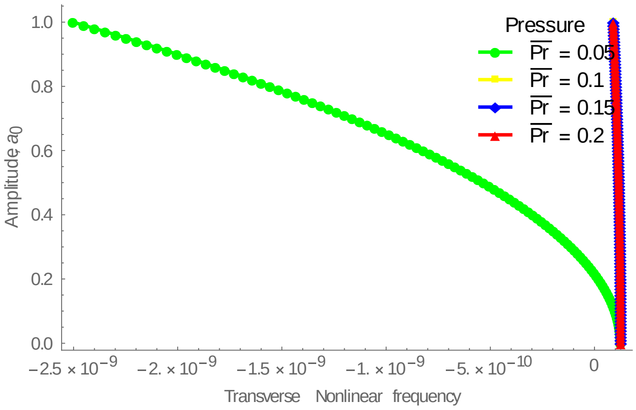

Figure 1a: Transverse amplitude-frequency response at void fraction=0.3 , mixture quality = 0.00037088; θb=1.2; Teb=0.1;Pr = 0.05; 0.10; 0.15; 0.20;

The effect of pressure on the large strain model for the transverse and longitudinal vibrations are shown in Figures 1a and 1b respectively. Clearly, changes in pressurization significantly alter the transverse nonlinear response but has negligible effect on the longitudinal response. A change from softening to hardening nonlinear behavior is observed in the transverse response whereas the longitudinal response maintained hardening nonlinear profile [6].

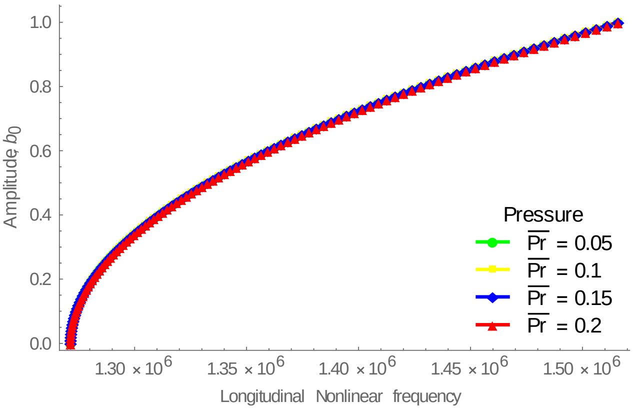

Figure 1b: Longitudinal amplitude-frequency response at void fraction=0.3 , mixture quality = 0.00037088; θb=1.2; Teb=0.1;Pr = 0.05; 0.10; 0.15; 0.20;

Meanwhile, for a pipe conveying two-phase fluid, the variation of the longitudinal frequencies with flow velocity is as shown in Figure 1b. It is evident that when the effects of temperature, pressure and tension are negligible, the dynamic response of a large strain deformed pipe is identical to that of a small strain deformed pipe. Similar results were obtained in the work of [3], however, they were obtained for cantilevered pipes.

3.2 Effect of Temperature on Transverse and Axial Vibration

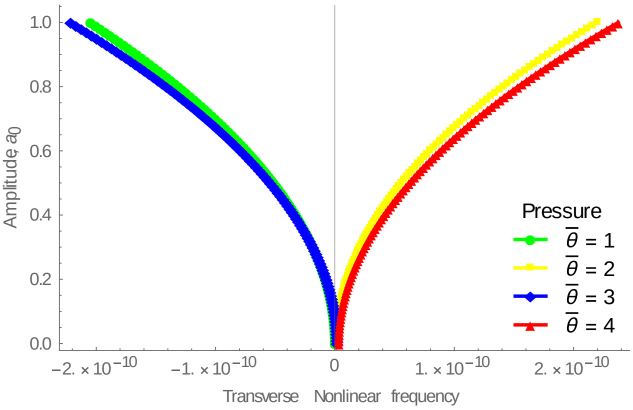

Figure 2a: Transverse amplitude-frequency response at void fraction=0.3 , mixture quality = 0.00037088; Teb=0.1; Pr = 0.05; θb=1,2,3,4;

The effect of temperature on the large strain model for the transverse and longitudinal vibrations respectively are shown in Figures 2a and 2b. It was observed that nonlinear response of the systems dynamics is very sensitive to temperature changes. It changes the transverse response from softening nonlinear to hardening nonlinear behavior. However, the longitudinal response remains a softening type, with sharp difference magnitudes of the longitudinal and transverse responses.

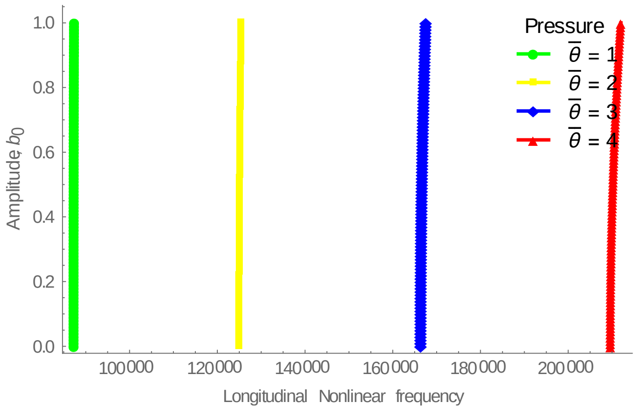

Figure 2b: Longitudinal amplitude-frequency response at void fraction=0.3 , mixture quality = 0.00037088; Teb=0.1;Pr = 0.05; θb=1,2,3,4;

3.3 Effect of Tension on Transverse and Axial Vibration

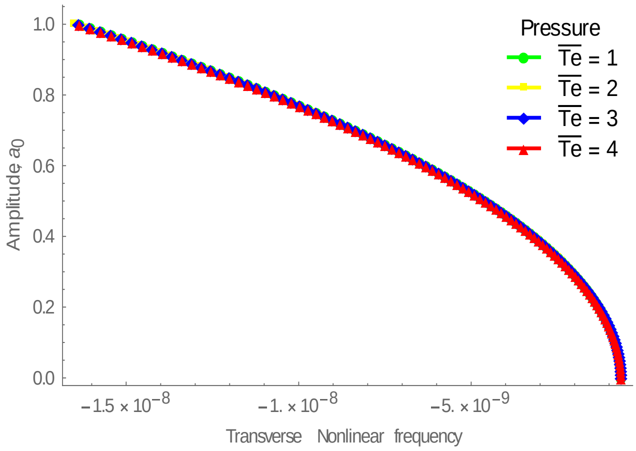

Figure 3a: Transverse amplitude-frequency response at void fraction=0.3, mixture quality = 0.00037088; θb=1.2;Prb=0.05; Teb=1,2,3,4;

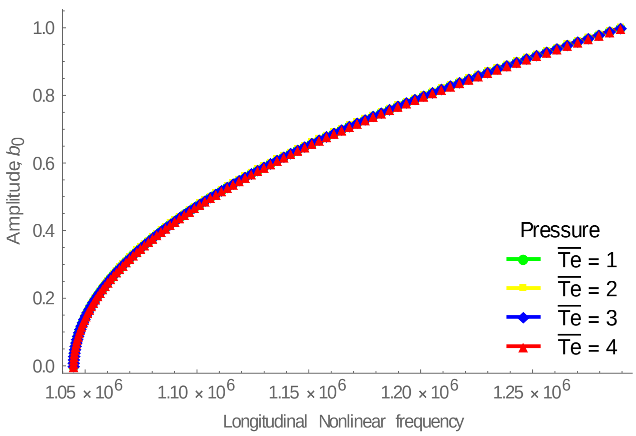

Figure 3b: Longitudinal amplitude-frequency response at void fraction=0.3, mixture quality = 0.00037088; θb=1.2;Prb=0.05; Teb=1,2,3,4;

With respect to increasing tension; in profile, the system followed a split mode response between the transverse and axial vibrations. As expected, tensioning induced a damping effect on the transverse dynamics. On the other hand, the axial dynamics indicated a strong nonlinear excitation in the frequency response under a bounded amplitude response. Clearly, Figures 3a and 3b showed the absence of a fixed point in the response of nonlinear axial and transverse vibrations of large strain deformed pipes over the range of induced tension. Hence, the range of tension considered has a linear second order control effect on the system's dynamics.

4.0 CONCLUSION

Linear theories predict an unbounded growth of the amplitude of vibration of subsea flow line conveying two phase flow with time. However, nonlinear analyses of the dynamics show a bounded response. Nonlinearities in large strain model induce hardening and softening nonlinear behaviour in the dynamics of pipes conveying two phase fluids. For the transverse vibration, changes in systems parameters have both qualitative and quantitative influence on the system's transverse response. However, in most cases, parametric changes have negligible influence on the qualitative behaviour of the longitudinal vibration.

Whereas nonlinearities in large strain deformed pipes impose a softening behavior on the pipe transverse vibration, nonlinearities due to small strain deformation cause a hardening behaviour of the pipe transverse vibration. However, converse responses were observed in the longitudinal vibration for both cases, with large strain deformation model exhibiting hardening nonlinear behaviour while the small strain deformation model shows softening nonlinear behaviour. Outcomes of these analyses provide broad spectrum information for optimum and sustainable management of subsea flowlines and jumper conveying two phase flow with attendant parametric changes in transport conditions.

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