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AN ASSESSMENT OF THE IMPLICATIONS OF GRAVITY REDUCTION METHODS ON LOCAL GEOID MODELLING OVER ADO EKITI

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Abstract
There are several gravity reduction approaches that are available for use in geodetic applications. More so, with the rapid rate at which gravimetric geoid models are being incorporated into Global Navigation Satellite System (GNSS) solutions, the need for precise regional models is becoming increasingly relevant. The two chosen gravity reduction approaches (Bouguer and Residual Terrain Model (RTM)) were used to reduce gravity anomalies over Ado town. The reduced anomalies were then used to compute a local geoid for the study area using the conventional Stokes integral in the Remove Compute Restore (RCR) technique. Comparison of the computed geoid by both methods with GNSS Levelling data at selected locations show that the RTM reduced anomalies produce a better geoid model than the Bouguer reduced anomalies over the study area with a RMSE of 83.2cm and 83.5cm respectively.

Keywords: Long wavelength Geoid, Stokes Integral, Remove Compute Restore, Potential Field

1.0 INTRODUCTION

The need for precise local and regional geoid models has continued to increase in recent times as the GNSS-Levelling method for determination of Orthometric heights is gaining increased popularity [1, 2]. Several research works have been carried out on methods of local geoid modelling with different researchers having their own preferred choice based on data availability, terrain conditions and computational preference [3]. Some past works in local / regional geoid modelling are presented in Table 1. Amongst several techniques, the gravimetric method stands out as one of the most preferred methods because the geoid is actually an equipotential surface of the Earth's gravity field [4]. A major requirement in gravimetric geoid modelling is the need for a dense and accurate gravity network of points [5] and the appropriate reduction of such gravity data.

Several researchers had earlier looked at the effect of gravity data availability on regional geoid modelling. Some of such studies include Odumosu and Nnam (2020) [2] who investigated on the effects of data spacing and observational accuracy on regional geoid modelling across Nigeria. Goli et al. (2019) [6], investigated the effects of data noise, spatial distribution and interpolation of ground gravity data on uncertainties of estimated geoid heights using the Stokes-Helmert approach. Farahani et al. (2017) [7], assessed the surface gravity data requirements for a 5-mm quasi-geoid model considering the omission and commission errors over the Netherlands.

However, previous studies have not emphasized the effects of the choice of gravimetric reduction method and its implications on regional geoid modelling. Therefore, specific objectives of this study are as follows; (i) collection of gravity data from various sources (ii) implementation of the Bouguer and RTM gravity reduction schemes and (iii) computation of local geoid from both schemes. This paper presents the results of empirical investigation carried out to determine the effects of gravimetric reduction method on regional geoid modelling within Ado Ekiti township of Ekiti state. Over the years, the Bouguer reduction approach has been the standard reduction method for gravity data reduction in geoid modelling. As is well known, the theoretical assumption of the

Bouguer reduction is that the entire topographic masses is represented by a uniform plate of thickness. However, in reality, the topographic masses are not represented by a uniform plate of uniform thickness but varies significantly from location to location depending on the actual gravity field of that area; thus, a need for a more realistic representation of the topography of the Earth for which the RTM seems a preferable choice.

Table 1: Regional geoid model of some nations /continents around the world [8]

S/N	Continent	Country	Adopted Geoid	Development Method	Source
1	Africa (East Africa)	Ethiopia	Ethiopian Geoid	Gravimetric method	Hunegnaw, 2001 [9]
2	Africa (West Africa)	Ghana	Ghanaian Geoid	Gravimetric method	Klu, 2015 [10]
3	Africa (North Africa)	Egypt	EGY-HGM 2016	Gravimetric method	El-Ashquer et al, 2017 [11]
4	Africa (South Africa)	South Africa	SAGeoid 2010	Gravimetric method	Chandler and Merry, 2010 [3]
5	Europe	Across Europe	European Gravimetric Geoid Model (EGG07)	Gravimetric method	Denker *et al*, 2009 [12]
6	United States of America	U.S.A	United States Gravimetric Geoid (USGG09)	Gravimetric method	Wang *et al*, 2012 [13]
9	Asia	Japan	Japan Geoid	Gravimetric method	Matsuo *et al*, 2016 [14]
10	Australia	New Zealand	NZGeoid 2009	Gravimetric method	Amos, 2007 [15]

1.1 GRAVITY REDUCTION METHODS

We examine two reduction approaches in this study. The approaches examined are; the Bouguer reduction and the Residual Terrain Model.

1.1.1 Bouguer Reduction

The Bouguer reduction is one of the most common gravimetric reduction schemes used both in geodesy and geophysics. This reduction removes all the masses above the geoid using a Bouguer plate. Thereafter, Terrain Correction (TC), which represents the effect of the topography deviating from the Bouguer plate is then considered to remove rigorously all topographic masses above the geoid surface [16]. Figure 1 shows the Bouguer reduction. The Bouguer plate of thickness hp, which is equal to the height of a point P removes all the topographical masses above the geoid except TC. Obviously, the Bouguer reduction is implemented using equation 1.

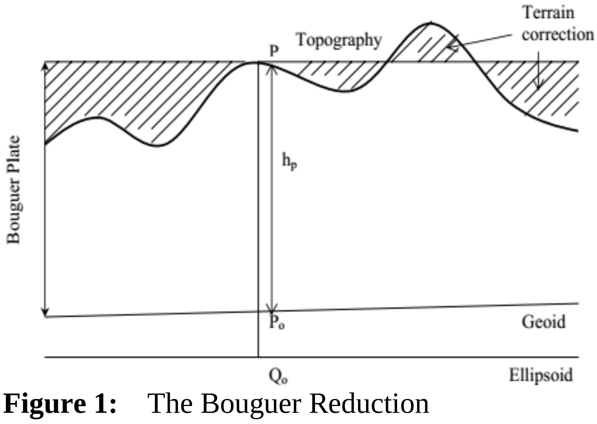

Figure 1: The Bouguer Reduction

$$\Delta g_{bouguer} = g_r - \gamma + FA - 2\pi G\rho h_p + c \qquad (1)$$

Where:
g_r = Measure gravity on the Earth surface
γ = Normal gravity
FA = Free-air anomaly
$2\pi G\rho h_p$ =Topographic attraction due to the Bouguer plate
G = Gravitational constant
ρ = Density
h_p = Height of point P above the geoid
c = Terrain Correction

The TC (c) is a key auxiliary quantity in gravity reductions, which are used in solving the geodetic boundary value problem of physical geodesy and in geophysics. It contains the high frequency part of the gravity signal representing the irregular part of the topography which deviates from the Bouguer plate [17] .

The formular for TC is given as equation (2)

$$c_p = G\iint_E \int_{h_p}^{h} \frac{\rho(x,y,z)(h_p - z)}{s^3(x_p - x, y_p - y, h_p - z)} dx\, dydz \qquad (2)$$

Where:
$\rho(x,y,z)$ is the topographical density at the running point,
h_p, h are the heights at the running and computation points, respectively,
E denotes the integration area, and
s is the distance between the points.

1.1.2 The Residual Terrain Model (RTM)

The Residual Terrain Model (RTM) is another common terrain reduction methods used in geoid determination. This reduction scheme was introduced by [18]. A reference surface (a mean elevation surface), which is defined by low pass filtering of local terrain heights, is used in this terrain reduction. The topographical masses above this reference surface are removed and masses are filled up below this surface. The RTM reduction is illustrated by Figure 2. A quasigeoid is obtained using this mass reduction model. The RTM model is implemented using equation 3a and 3b.

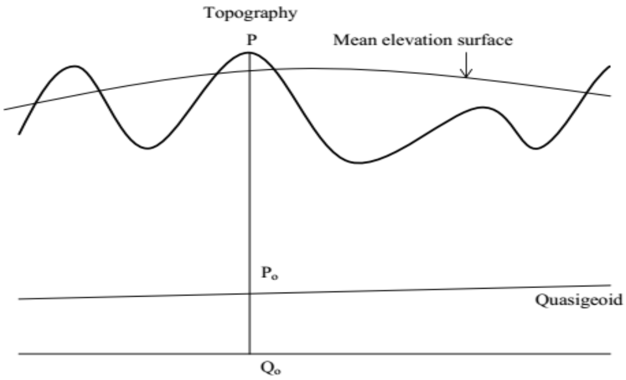

Figure 2: The Residual Terrain Model (RTM)

$$\Delta g_{RTM} = g_r - \gamma - \delta A_{RTM} \qquad (3a)$$
$$\delta A_{RTM} = 2\pi G\rho\left(h - h_{ref}\right) - c \qquad (3b)$$

where

δA_{RTM} is the topographic attraction due to the RTM

h, h_{ref} are the heights at the computation and reference points respectively

c is Terrain correction

Δg_{RTM} is Gravity anomaly due to Residual Terrain

1.2 STUDY AREA

Ado-Ekiti is located within the South Western area of Nigeria (Figure 3). The city lies between Latitude 07° 34'N and 07° 44N of the Equator and Longitude 05° 11'E and 05° 18'E of the Greenwich Meridian. It has several satellite towns around it including; Ikere-Ekiti, Ijan-Ekiti, Ilawe-Ekiti, Iworoko and Iyin-Ekiti. Ekiti State is generally an upland region with an average elevation of 455m above of the mean sea level. The topography is characterized by undulations, hills and flat lands. Owing to this elevation range, Ekiti state serves as a good test bed for examining topographic effects on local geoid modelling.

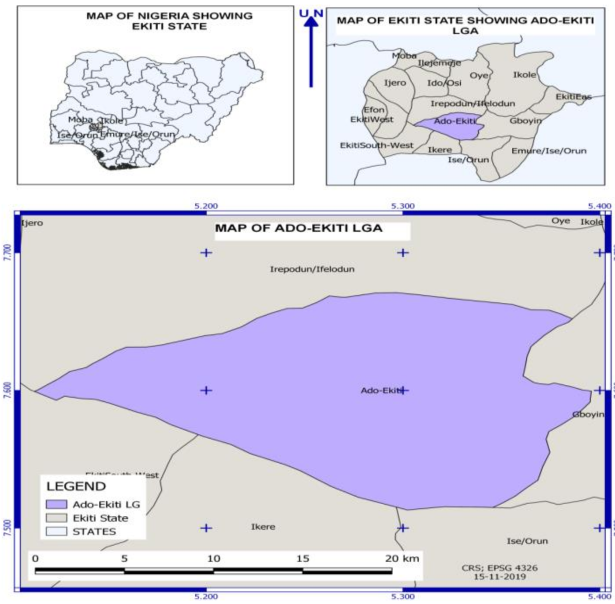

Figure 3: Study Area

2.0 MATERIALS AND METHODS

A total of 568 gravity data points distributed across Ado town were used in this study. The used data comprise of 112 terrestrial points obtained using Scrintrex CG5 gravimeter, 432 points obtained from earlier works using gravity interpolation by Kriging method [8] and 24 points from Gravity forward modelling approach [19]. Further information about the data is presented in Table 2. The spatial distribution of the data used for this study is presented in Figure 3. The quality estimates of the gravity data is presented in Table 3.

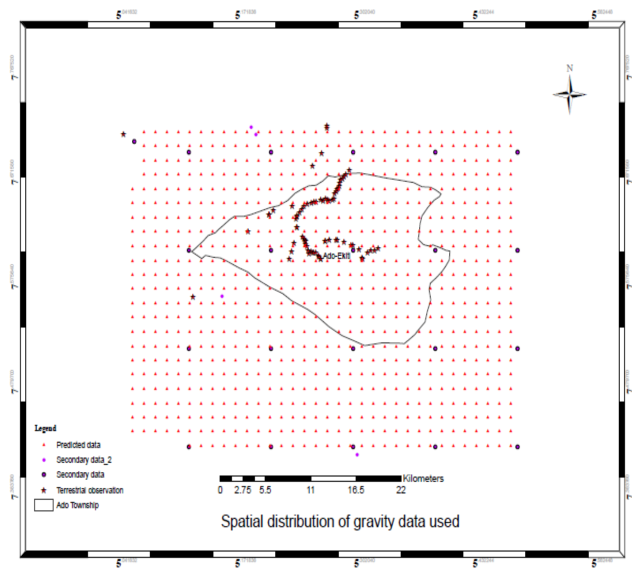

Figure 4: Spatial distribution of gravity data used for the study

Also, heights obtained from the Shuttle Radar Topographic Mission (SRTM) was used to compute the required terrain correction. The 1 arc seconds SRTM data was used in this study and shown in Figure 4.

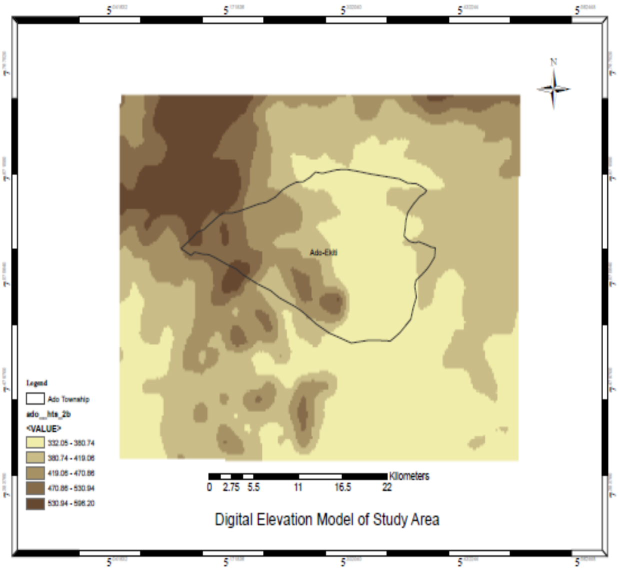

Figure 5: 1" SRTM DEM covering Ado Ekiti township

The reduction of gravity-field related quantities for topographic effect of gravity plays a very crucial role in geodetic applications especially in geoid modelling. However, terrain correction being a mathematical representation of a hypothetical geometric model that compensates for the actual deviations of the topography from the Bouguer plate is complex to model. Similarly, its implementation is difficult and has since been a subject of research in geodetic literature. In this study, TC is achieved using the Bouguer and RTM gravity reduction formulae as earlier presented in equations 1 - 3.

The Bouguer and RTM reduction models were implemented using the relevant mathematical formulae as identified in Table 2. The models were all tested within the study area with a view to identifying the effect of the choice of reduction method on geoid computation. MATLAB codes developed in this study were used to implement the Bouguer reduction using the Nagy (1966) [20] formulae, while the SRTM2Grav (a third party geodesy-based software) was used to implement the RTM model. Table 2 presents the method of implementation for each of the techniques used in the study. The flowchart for the MATLAB implementation of the Nagy prism is shown in Figure 6 below.

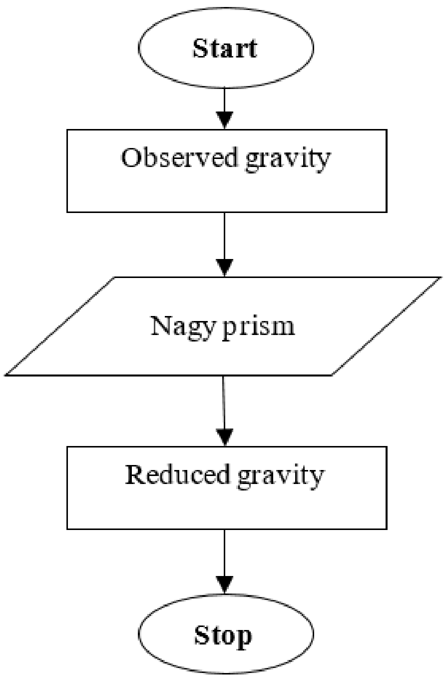

Figure 6: Flowchart for Bouguer reduction implementation in MATLAB

Table 2: Gravity Data Sets used

S/No	Gravity data type	Method of realization	Source	No of points
1	Terrestrial gravity (Primary data)	Profile method of gravity field observation using a Scrintrex CG5 gravimeter	Telford et al, 1990 [21]	112
2	Simulated gravity data (Derived data)	Kriging	Odumosu et al, 2021 [19]	773
3	Secondary data	Gravity forward modelling and data merging. Forward modelling involves using requisite mathematical correction models to convert satellite derived gravity anomaly (GGM2008) to terrestrial gravity anomaly. Then the reduced terrestrial anomalies are then merged with terrain observed data to predict more point using conventional Least square collocation (LSC)	Odumosu et al, 2021 [19]	24
	Total Number of data points			568

Table 3: Quality estimates of the Gravity Data used

S/N	Data Source	Reference ellipsoid for gravity	Type of observation	Observational accuracy	Prediction accuracy
1	Terrestrial data	IGSN71	Profile method	±1.25mgals	Not Applicable
2	Simulated data	IGSN71	Not Applicable	Not Applicable	±4.25 mgals
3	Secondary data	IGSN71	Data merging	Not Applicable	±5.57 mgals

Table 4: Implementation method

S/No	Gravity Reduction method	Implementation tool	Source
1	Bouguer (Mass line)	MATLAB	Nagy, 1966 [20]
2	RTM (Mass Line)	SRTM2Grav	Hirt et al, 2019 [22]

3.0 RESULTS AND DISCUSSION

Extract as well as statistics of the results obtained from each reduction scheme is presented in Table 5 and 6 respectively. Also, applying the Stokes integral in the Remove Compute Restore (RCR) geoid computation technique, the Bouguer and RTM reduced gravity anomalies were used to compute the local geoid for Ado town. The obtained geoid models is presented in Figures 7a and b.

Table 5: Extract of results from 2 Gravity Reduction Schemes

Station	FA. Ano (mgals)	RTM (mgals)	Bouguer (mgals)
GPSA 47	42.068	46.3877	41.6613
GPSA148S	40.093	42.7163	39.6891
GPSA142S	42.502	43.8622	42.0982
FGPEKY081	47.247	47.3078	46.8438
GPSA118S	47.074	46.8449	46.6747
GGM06	40.353	32.1581	39.9561
GGM05	40.242	46.5569	39.8453
GPSA138S	44.000	44.9638	43.6040
G52	38.897	40.2935	38.5017

Table 6: Descriptive Statistics of the used Reduction Schemes

Parameter	*RTM (mgals)*	*Bouguer (mgals)*
Mean	42.4591	41.96883
Standard Error	0.537797	0.506586
Median	43.34412	40.5858
Mode	40.29351	38.50166
Standard Deviation	5.81716	5.479568
Sample Variance	33.83935	30.02567
Kurtosis	2.393478	-0.25161
Skewness	0.91695	0.471194
Range	36.07772	27.86214
Minimum	32.15814	33.37709
Maximum	68.23585	61.23923

The reduced gravity anomalies obtained from both reduction approaches were subjected to the Analysis of Variances (ANOVA – singe factor) statistical analysis as presented in Table 7.

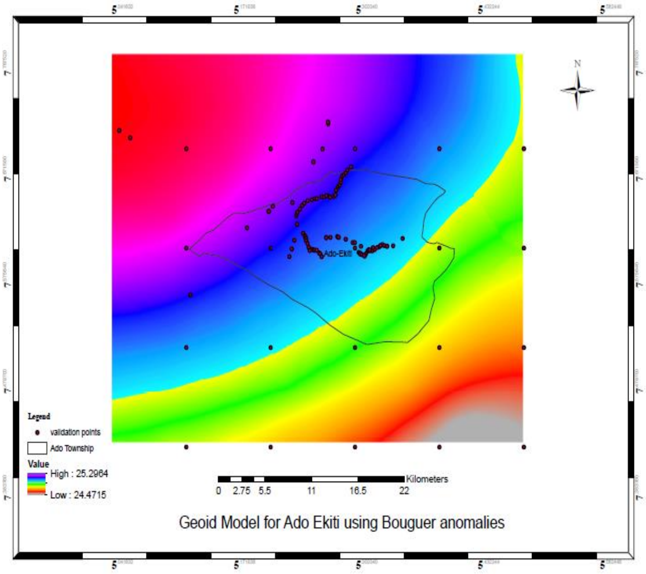

Figure 7a: Geoid from Bouguer anomaly

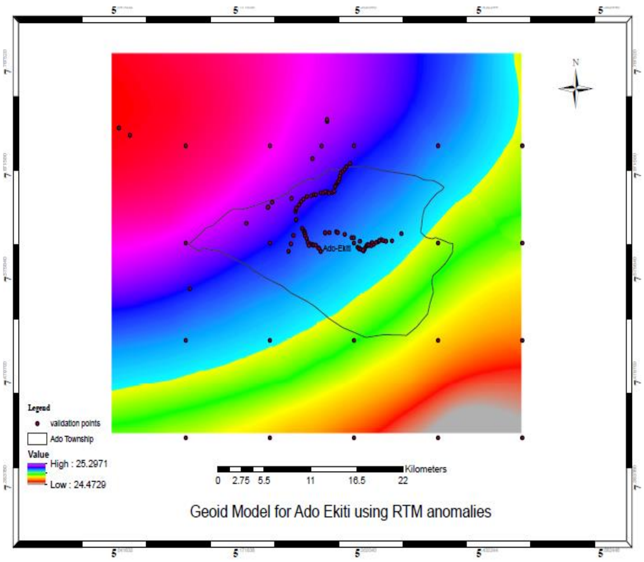

Figure 7b: Geoid from RTM anomaly

The ANOVA result below shows that there is no significant difference between the means and variances of the results obtained from both reduction techniques. Invariably, there is a very high similarity in the geoid model produced using both reduction techniques.

Although, both gravimetric geoid models look similar showing that the choice of gravimetric reduction approach has limited effect on the overall computed geoid, analysis of the result obtained by validation at GNSS-Levelling station (Table 8) done at the check points provided reveal that the RTM anomalies had a better accuracy compared to the Bouguer in geoid modelling within the study area with an overall root mean square error (RMSE) of ± 0.0832m while the Bouguer anomaly computed geoid have RMSE of ± 0.0835m. The result as obtained conform with the earlier findings by Odumosu and Nnam (2020) [2] which emphasises the need for appropriate terrain correction and dense data spacing in mountainous areas.

Table 7: ANOVA Test result

SUMMARY				
Groups	*Count*	*Sum*	*Average*	*Variance*
Bou_Ano	568	37789.32	42.4599	33.97541
RTM_Ano	568	38796.76	43.59187	49.05829

ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	570.1954	1	570.1954	13.73407	0.000217	3.846694
Within Groups	73816.95	1778	41.51685			
Total	74387.15	1779				

Table 8: Extract of validation of Gravimetric Geoid (in meters)

	Difference with GNSS-Levelling	
Sta_ID	**Bouguer (m)**	**RTM (m)**
AGST007	-0.3973	-0.3972
AGST009	0.0051	0.0051
AGST008	0.0037	0.0038
AGST010	-0.2477	-0.2477
AGST006	-0.0496	-0.0493
AGST004	0.0579	0.0583
AGST003	-0.1364	-0.1360
RMSE	0.0835	0.0832

Obviously, the RTM reduction approach does not produce same results with the Bouguer reduction approach for Geoid modelling. This is because while the Bouguer model is basically dependent on the topography of the area alone, the RTM also considers the potential of gravitational attraction within the area. Consequently, because the gravitational potential of the topographic masses (which have been modelled by the RTM) contributes significantly to the local gravity field, the RTM produced the highest accuracy in gravimetric geoid modelling for the study area [6].

4.0 CONCLUSION

The study evaluated the effects and implications of gravity reduction approach on local geoid modelling. The kriging, forward gravity modelling approach and terrestrial gravity profile observation methods have been used to obtain predicted (simulated), archived (secondary) and field (primary) data for this study. The various datasets having being ensured as homogenous were then subjected to gravity reduction using the Bouguer and RTM reduction approaches. Local geoid was thereafter computed over the study area using the conventional RCR technique and the resulting geoid models compared. Based on the outcomes of the study, it can be concluded that the RTM gravity reduction approach should be used for local geoid computation. This is because, while the common Bouguer reduction method simply relies on the Topography, the RTM also considers the gravitational potential of the topographic masses.

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