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STRUCTURAL BEHAVIOUR AND BENDING ANALYSIS OF RECTANGULAR THICK PLATE USING A NEW 3-D MODIFIED TRIGONOMETRIC DISPLACEMENT MODEL

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Abstract
In this work, a new 3-D modified trigonometric displacement model was used to study the structural behavior and bending analysis of rectangular thick plate which was clamped in one edge and other three edges simply supported. The theoretical model whose formulation is based on static elastic principle as already reported in the literature are presented herein, obviating the shear correction coefficients while considering shear deformation effect and transverse normal strain/stress in the analysis. The equilibrium equations are obtained using 3-D kinematic and constitutive relations. An exact solution of deflection and rotation are obtained from the equilibrium equation using the general variational principle. The minimization, energy equation yields the general equation which was used to obtain the 3-D trigonometric displacement model of the plate. The percentage difference between the present work and those of 2-D Refined Plate Theory (RPT) with an assumed displacement and 2-D Refined plate theory (RPT) with derived function is 1.43% and 5.15% respectively. More so, percentage difference between the present work and those using polynomial shape function is 3.29%. The result showed that the 3-D trigonometric model for the present work predicts the vertical displacement and the stresses more accurately than RPT and polynomial displacement model. It is concluded that the 3-D trigonometric model gives an exact solution unlike polynomial and can be used with confidence in the analysis of thick plate under the particular initial condition.

Keywords: Structural bending behavior, new 3-D modified theory, trigonometric displacement model, CSSS thick plate.

1.0 INTRODUCTION

A plate is basically a three-dimensional structural element with its thickness lesser than the other parallel and plane surfaces [1], whose applications spans across mechanical, marine, naval, aerospace, geotechnical and structural engineering [2], for customizing offshore and port foundations, ship hulls, retaining walls, railway structures and floor slabs [3].

Categorically, plates can be orthotropic, anisotropic, or isotropic based on their material properties and deformation nature; based on its shape, they can be triangular, square, circular, or rectangular [4-6]; as regards to support conditions, plates are either clamped, free or simply supported at their edges, and they can be thin, moderately thick, or thick according to their weight [7-9]. Considering span-to-depth ratio in [10], rectangular plates with $40 \leq a/t \leq 100$ are grouped as thin plate, $20 \leq a/t \leq 40$ as moderately thick and $a/t \leq 15$ as thick plate.

The significance desirable features of thick plates includes; extreme fatigue strength, high strength to weight, high stiffness to weight, excellent corrosion resistance, low density and improved tailor ability, have increased their demand in different industries

[11]. The behavior of thick plates can be studied through dynamic analysis, stability analysis and static analysis [12].

The deformation of thick plates at right angles to the plate surface because of the influence of forces and moments is regarded as bending [13]. Structural elements are displaced with induced stresses as loads are applied and to resist these loads, the structure becomes prone to bending. The elasticity of plates largely depends on their thickness property as transverse loads are resisted through their bending behavior [14, 15].

In order to capture the bending characteristics of thick plates, researchers have invented and advanced assorted theories; the Classical Plate Theory (CPT) which is commonly regarded a Kirchhoff Plate Theory [16], and Refined Plate Theories (RPT). RPT consists of First Order Shear Deformation Theory (FSDT), Second Order Shear Deformation Theory (SSDT), Exponential Shear Deformation Theory (ESDT) [17], Polynomial Shear Deformation Theory (PSDT) [18], Trigonometric Shear Deformation Theory (TSDT) [19] and the Higher Order Shear Deformation Theory (HSDT) [20]. The exact bending attributes of thick plates cannot be determined with CPT as it neglects the upshot of transverse shear [21]. FSDT was presented with the incorporation of correction factor [22, 23] to conquer the drawback of CPT so as to obtain a desired result.

HSDTs were developed to take into account the effect of shear deformation in the absence of correction factor [24, 25], and to predict accurately the structural bending behavior of isotropic thick plates. However, the omission of normal stress and strain along the thickness axis of the plate, makes the RPT unpredictable; hence it is considered as a 2-D theory or an incomplete 3-D theory [26]. A typical a 3D theory is required in order to obtain exact bending solutions for a 3-D structural element and this endorses the essence of this study.

The bending behavior of thick plates can be investigated using different methods such as numerical methods (approximate approach) and the analytical methods (closed-form approach) [27]. In [28, 29, 6], analytical method was employed to analyze the bending of plates with varying loads and boundary conditions. This method which includes integral transform method, Eigen expansion methods, Navier and Levy series; satisfies the governing equations of the plate at the edges of the plate and at every position on the plate surface [21].

The authors in [30, 31, 32, 33], employed numerical approach which gives approximate solutions and they are challenging to obtain in complicated bending problems. This study applied a close form approach to energy method to obtain its exact solution for the bending problem of thick isotropic plates elastically restrained along one edge and other three edges simply supported (CSSS).

1.1 LITERATURE REVIEW

The bending behavior of rectangular Kirchhoff plates was analyzed by Ike [34] and Nwoji et al. [35] using Kantorovich-Galerkin method and Ritz method respectively. With one term Kantorovich-Galerkin solution, the deflection and bending moment coefficients for deflection at the center of the plates under uniform load were derived in [34] while the authors in [35] obtained exact solutions using exact shape functions and had similar outcomes when compared with Navier double Fourier sine series method. Both authors could not address thick plates as their assumption is restricted to CPT which cannot give a good outcome for a relatively thick plate. They also did not consider plates with CSSS support conditions. The authors did not apply the Three Dimensional (3-D) plate theory with energy approach and trigonometric function.

Hyperbolic shear deformation theory and Fourier series method were employed by Ike [36] to ascertain the blending solution for thick beams without the application of FSDT's shear correction factors. Using hyperbolic sine and cosine functions, transverse shear stress free conditions at the upper and lower surfaces of the beam were achieved in the study. CSSS plate boundary conditions were not covered in the study. The author did not employ trigonometric functions and three-dimensional plate theory. With the application of virtual work principle, HSDT was analytically used by Ghugal and Gajbhiye [37] to solve the bending problem of simply supported plates. They evaluated the effect of strain and shear deformation in their study neglecting the use of shear correction factor associated with FSDTs. The 3-D plate theory was not addressed in their study and plates with CSSS edge conditions were not taken into account.

RPT with exponential functions was used by Sayyada and Ghugal [17] to analyze displacement and stresses for simply supported thick plates. The solution obtains in their work was sufficiently good when juxtaposed with other refined plate theories. The authors did not cover CSSS plates and the trigonometric displacement function was not applied. The 3-D plate theory was not considered. Onyeka et al. [3] and [38] evaluated the lateral critical imposed load of isotropic plates using polynomial shear deformation theory. In [38], they studied the effect of aspect ratio, shear and deflection on the critical lateral load of the plates using the direct variational method. Both authors failed to employ 3-D trigonometric plate theory and plates with CSSS support conditions were not covered.

The 2-D plate theories with trigonometric and polynomial displacement function was applied to analyze the bending of thick plates by Mantari et al. [39] and Onyeka and Okeke [40], respectively. In [40], the trigonometric displacement model was not presented. The authors in [39, 40] did not address the stresses and strain in the thickness axis and could not cover plates with CSSS edge conditions. The in-plane displacements, deflections, moments, shear force, with the deformation rotations at the plate with two opposite edges clamped and simply supported (CSCS) arbitrary points was investigated by Onyeka et al. [41] using Refined Plate Theory (RPT) with polynomial function. Their study is not valid for a typical three-dimensional equilibrium equation as an incomplete 3-D elasticity theory was applied. The CSSS thick plates were not taken into consideration.

Ibearugbulem et al. [42], Onyeka and Mama [26], analyzed the bending of thick plates with simply supported edges. In [26], direct variational energy with trigonometric shape functions was used. In [42], exact polynomial displacement functions were derived from the governing equation with an analytical technique. Although both authors employed 3-D plate's theory, their analysis did not cover plates with CSSS boundary conditions. Hadi et al [43] employed three-dimensional elasticity theory with a numerical approach to examine the bending of rectangular plates made of functionally graded material with the variable exponential properties. They investigated the influence of graded material properties on the plate's behavior. The authors did not consider trigonometric functions and CSSS plates.

Onyeka et al. [46] studied the bending analysis of thick plate while Ibearugbulem et al. [47] studied bending analysis of all clamped (CCCC) rectangular thick plate using higher order shear deformation theory based on Ritz energy method and the displacement function based on polynomial function. They formulated the total potential energy equation of the plate and ensured that the transverse shear stress from constitutive relation that satisfied zero shear stress condition on the top and bottom surfaces of the plate. They did not derive the displacement function from the elasticity theory rather an assumed shape function was used which could not produce a close form solution thereby mathematically unreliable for the thick plate analysis. More so, they could not apply a complete 3-D theory of elasticity which is capable of analysing all the stress element in the plate material, rather 2-D Ritz theory was used. The work in [47] could not study the bending analysis of rectangular thick plate which was clamped in one edge and other three edges simply supported (CSSS) rather their work was limited to the plate that is clamped at all the edges.

In contrast to previous studies, the uniqueness of this study stems from its analytical approach, application of 3-D elasticity theory, the use of trigonometric functions and the boundary condition of the thick plate. The studies reviewed shows that most researchers considered numerical approach and polynomial functions or exponential function [40, 41, 42, 43, 45, 46]. Approximate solutions produced by this approach coupled with the impossibility in the determination of displacements at any point in the plate; is a gap worth filling.

The outcome of polynomial functions tends to infinity whereas the application of trigonometric function employed in this study, yields closed-form solution. Also, the need for analyzing a typical three-dimensional plate structure with a 3-D plate theory validates the relevance of this study. The aim of this work is to investigate the structural behavior and bending of rectangular plates elastically restrained along one edge and other three edges simply supported (CSSS), using a modified 3-D trigonometric displacement model to consider the effect of deflection and stresses on the plate.

2.0 METHODOLOGY

Using the thick plate assumption (see [46]), the in-plane displacements along x-axis and y axis u and v are given as:

$$u = ts.\theta_{sx} \tag{1}$$

$$v = ts.\theta_{sy} \tag{2}$$

Thereafter, the total potential energy which is the summation of the dot product of stress, strain and external work done in the plate is mathematically expressed as ([46]):

$$\Pi = \frac{D^*ab}{2a^2}\int_0^1\int_0^1\left[(1-\mu)\left(\frac{\partial\theta_{sx}}{\partial R}\right)^2 + \frac{1}{\beta}\frac{\partial\theta_{sx}}{\partial R}.\frac{\partial\theta_{sy}}{\partial Q} + \frac{(1-\mu)}{\beta^2}\left(\frac{\partial\theta_{sy}}{\partial Q}\right)^2 + \frac{(1-2\mu)}{2\beta^2}\left(\frac{\partial\theta_{sx}}{\partial Q}\right)^2 + \frac{(1-2\mu)}{2}\left(\frac{\partial\theta_{sy}}{\partial R}\right)^2 + \frac{6(1-2\mu)}{t^2}\left(a^2{\theta_{sx}}^2 + a^2{\theta_{sy}}^2 + \left(\frac{\partial w}{\partial R}\right)^2 + \frac{1}{\beta^2}\left(\frac{\partial w}{\partial Q}\right)^2 + 2a.\theta_{sx}\frac{\partial w}{\partial R} + \frac{2a.\theta_{sy}}{\beta}\frac{\partial w}{\partial Q}\right) + \frac{(1-\mu)a^2}{t^4}\left(\frac{\partial w}{\partial S}\right)^2\right]dR\,dQ - \int_0^1\int_0^1 abqhA_1\partial R\partial Q \quad (3)$$

Let:

$$D^* = \frac{Et^3}{12(1+\mu)(1-2\mu)} \quad (4)$$

Where:
D^*, E and μ are the Rigidity, modulus of elasticity and Poisson's ratio
θ_{sx} and θs_y are the shear deformation rotation along x axis and y axis

2.1 GOVERNING EQUATION

The solution of the governing equation is presented as the result of energy functional minimization with respect to deflection to give the exact plate's shape function (see [44]):

$$h = [1 \;\; R \;\; Cos\,(c_1R) \;\; Sin\,(c_1R)]\begin{bmatrix}a_0\\a_1\\a_2\\a_3\end{bmatrix}. [1 \;\; Q \;\; Cos\,(c_1Q) \;\; Sin\,(c_1Q)]\begin{bmatrix}b_0\\b_1\\b_2\\b_3\end{bmatrix}/A_1 \quad (5)$$

Let:

$$\cup = A_1.h \quad (6)$$

$$\theta_{sx} = \frac{A_2}{a}.\frac{\partial h}{\partial R} \quad (7)$$

$$\theta_{sy} = \frac{A_3}{a\beta}.\frac{\partial h}{\partial Q} \quad (8)$$

Where; A_2 and A_3 are the coefficient of shear deformation along x axis and coefficient of shear deformation along y axis respectively.

Substituting Equation (6), (7) and (8) into (3), gives:

$$\Pi = \frac{D^*ab}{2a^4}\left[(1-\mu){A_2}^2k_x + \frac{1}{\beta^2}\left[A_2.A_3 + \frac{(1-2\mu){A_2}^2}{2} + \frac{(1-2\mu){A_3}^2}{2}\right]k_{xy} + \frac{(1-\mu){A_3}^2}{\beta^4}k_y + 6(1-2\mu)\left(\frac{a}{t}\right)^2\left(\left[{A_2}^2 + {A_1}^2 + 2A_1A_2\right].k_z + \frac{1}{\beta^2}.\left[{A_3}^2 + {A_1}^2 + 2A_1A_3\right].k_{2z}\right) - \frac{2qa^4k_hA_1}{D^*}\right] \quad (9)$$

$Where$: $k_z = \int_0^1\int_0^1\left(\frac{\partial h}{\partial R}\right)^2 dRdQ$; $k_{2z} = \int_0^1\int_0^1\left(\frac{\partial h}{\partial Q}\right)^2 dRdQ$;

$k_x = \int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial R^2}\right)^2 dRdQ$;

$k_{xy} = \int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial R\partial Q}\right)^2 dRdQ$; $k_y = \int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 dRdQ$;

$k_h = \int_0^1\int_0^1 h\,.\,dRdQ$;

Minimizing Equation (9) with respect to A_2 and A_3 and simplifying the outcome gives:

$$A_2 = MA_1 \quad (10)$$

$$A_3 = NA_1 \quad (11)$$

Let:

$$M = \frac{(r_{12}r_{23} - r_{13}r_{22})}{(r_{12}r_{12} - r_{11}r_{22})} \quad (12)$$

$$N = \frac{(r_{12}r_{13} - r_{11}r_{23})}{(r_{12}r_{12} - r_{11}r_{22})} \quad (13)$$

Where:

$$r_{11} = (1-\mu)k_x + \frac{1}{2\beta^2}(1-2\mu)k_{xy} + 6(1-2\mu)\left(\frac{a}{t}\right)^2 k_z \quad (14)$$

$$r_{22} = \frac{(1-\mu)}{\beta^4}k_y + \frac{1}{2\beta^2}(1-2\mu)k_{xy} + \frac{6}{\beta^2}(1-2\mu)\left(\frac{a}{t}\right)^2 k_{2z} \quad (15)$$

$$r_{12} = r_{21} = \frac{1}{2\beta^2}k_{xy};\; r_{13} = -6(1-2\mu)\left(\frac{a}{t}\right)^2 k_z;\; r_{23} = r_{32} = -\frac{6}{\beta^2}(1-2\mu)\left(\frac{a}{t}\right)^2 k_{2z} \quad (16)$$

Minimizing Equation (9) with respect to A_1 simplifying gives:

$$A_1 = \frac{qa^4}{D^*}\left(\frac{k_h}{T}\right) \quad (17)$$

$$Where: T = 6(1-2\mu)\left(\frac{a}{t}\right)^2 * \left([1+U].k_z + \frac{1}{\beta^2}.[1+V].k_{2z}\right) \quad (18)$$

2.2 INITIAL/BOUNDARY CONDITION

A rectangular thick plate with CSSS boundary conditions whose Poisson's ratio is 0.3 and carrying uniformly distributed load (including self-weight) shown in the Figure 1 is presented as problem of this study.

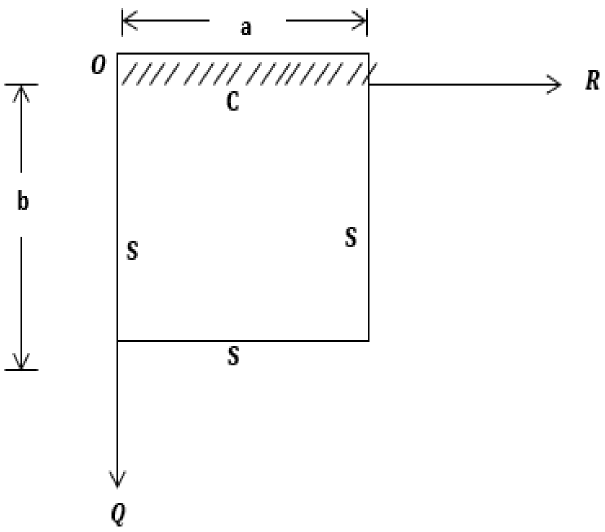

Figure 1: CSSS Rectangular Plate

The analytical particular solution of the trigonometric deflection function of the plate after satisfying the boundary conditions of the structure in Figure 1 is given as (see [44]):

$$\cup = A_1(Sin\,\pi R).(f_1 - f_1Q - f_1Cos\,f_1Q + Sin\,f_1Q) \quad (19)$$

2.3 EXACT DISPLACEMENT AND STRESS EXPRESSION

Substituting the value of A_1 in Equation (17), A_2 Equation (10) and A_3 in Equation (11) into Equation (1&2) and simplify, the in-plane displacement along x and y-axis becomes:

$$u = ts.\frac{M}{a}.\frac{qa^4}{D^*}\left(\frac{k_h}{T}\right)\frac{\partial h}{\partial R} \quad (20)$$

$$v = ts.\frac{N}{a\beta}.\frac{qa^4}{D^*}\left(\frac{k_h}{T}\right)\frac{\partial h}{\partial Q} \quad (21)$$

Substituting the value of A_1 in Equation (17) into Equation (19) and simplify, the deflection equation of the plate becomes:

$$\cup = (Sin\,\pi R).(f_1 - f_1Q - f_1Cos\,f_1Q + Sin\,f_1Q).\frac{qa^4}{D^*}\left(\frac{k_h}{T}\right) \quad (22)$$

Substituting the value of A_1, A_2 and A_3 in Equation (17), (10) and (11) into Equation (3) – (8) and simplify appropriately, the six stress elements becomes:

$$\sigma_x = \frac{E}{(1+\mu)(1-2\mu)}\left[(1-\mu)\frac{ts}{a}.\frac{\partial^2 h}{\partial R^2} + \mu\frac{ts}{a\beta}.\frac{\partial^2 h}{\partial Q^2} + \mu\frac{1}{t}.\frac{qa^4}{D^*}\left(\frac{k_h}{T}\right)\frac{\partial h}{\partial S}\right] \quad (23)$$

$$\sigma_y = \frac{E}{(1+\mu)(1-2\mu)}\left[\frac{\mu ts}{a}.\frac{\partial^2 h}{\partial R^2} + \frac{(1-\mu)ts}{a\beta}.\frac{\partial^2 h}{\partial Q^2} + \frac{\mu}{t}.\frac{qa^4}{D^*}\left(\frac{k_h}{T}\right)\frac{\partial h}{\partial S}\right] \quad (24)$$

$$\sigma_z = \frac{E}{(1+\mu)(1-2\mu)}\left[\frac{\mu ts}{a}.\frac{\partial^2 h}{\partial R^2} + \frac{\mu ts}{a\beta}.\frac{\partial^2 h}{\partial Q^2} + \frac{(1-\mu)}{t}.\frac{qa^4}{D^*}\left(\frac{k_h}{T}\right)\frac{\partial h}{\partial S}\right] \quad (25)$$

$$\tau_{xy} = \frac{E(1-2\mu)}{(1+\mu)(1-2\mu)}.\left[\frac{ts}{2a\beta}.\frac{\partial^2\partial h}{\partial R\,\partial Q} + \frac{ts}{2a}.\frac{\partial^2\partial h}{\partial R\,\partial Q}\right] \quad (26)$$

$$\tau_{xz} = \frac{(1-2\mu)E}{(1+\mu)(1-2\mu)}.\left[\frac{1}{2}\frac{\partial h}{\partial R} + \frac{1}{2a}.\frac{qa^4}{D^*}\left(\frac{k_h}{T}\right)\frac{\partial h}{\partial R}\right] \quad (27)$$

$$\tau_{yz} = \frac{(1-2\mu)E}{(1+\mu)(1-2\mu)}.\left[\frac{1}{2}\frac{\partial h}{\partial Q} + \frac{1}{2a\beta}.\frac{qa^4}{D^*}\left(\frac{k_h}{T}\right)\frac{\partial h}{\partial Q}\right] \quad (28)$$

3.0 RESULT AND DISCUSSION

The result covered the 3-D bending and stress analysis of rectangular plate at varying length, breadth and thickness of the plate. This work presents the result of the analysis between the aspect ratio of 1, 1.5 and 2, while the span-thickness ratio considered is ranged between 4, 5, 10, 15, 20, 50, 100 and CPT, which is obviously seen to span from the thick plate, moderately thick plate and thin plate (see [11]).

The non-dimensional result in the Figure 2 and 3 shows that an increase in the aspect ratio of the plate, increases the displacements (u, v and w) and stresses $(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}$ and $\tau_{yz})$ that may occur due to the applied load on the plate section. On the other hand, the results in the Figure 2 and 3 shows that as the span-thickness ratio of the plate increase, the in-plane displacement along x and y axis (u and v) and the deflection (w) which occurs at the plate due to the uniformly distributed load decrease. Similarly, the stress perpendicular to the x, y and z axis (σ_x, σ_y & σ_z) decreases as the span-depth ratio of the plate increases. Meanwhile, the increase in the span-thickness ratio of the plate decreases the value of the shear stress (τ_{xy}, τ_{xz} and τ_{yz}) in the plate. This means that an increase in the ratio of span - depth causes a decrease in the value of the stress could arise due to shear deformation of the plate. It can be deduced that, further span increase without a commensurate increase in the thickness of the plate element results to failure in the plate structure. Hence, the plate structure deflects beyond the elastic yield stress and thus, can fail without further increase in the load.

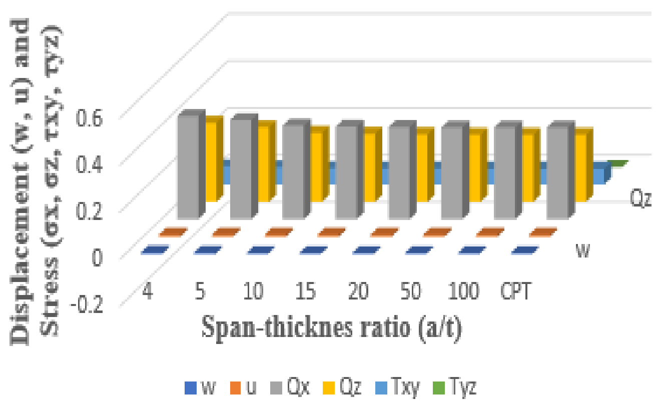

Figure 2: Displacements and stresses of a CSSS rectangular plates with aspect ratio of 1.

Figure 2 shows that, at a span-thickness ratio between 4, 5, 10 and 15, the value of out of plane displacement varies between 0.0063, 0.0058, 0.0050 and 0.0048 respectively. These values maintain a constant value of 0.0048 at the span - thickness 15 till 100 which is the same as the CPT. Similarly, Figure 2 shows that in a span-thickness ratio between 4, 5, 10 and 15, the value of in-plane displacement varies between 0.0087, 0.0084, 0.0081 and 0.0080 respectively. These values maintain a constant value of 0.0080 at the span - thickness 15 till 100 which is the same as the CPT. Furthermore, Figure 2 shows that, at a span-thickness ratio between 4, 5, 10 and 15, the value of shear stress along x-y axis varies between 0.0077, 0.0073, 0.0069 and 0.0067 respectively. These values maintain a constant value of 0.0067 at the span - thickness 15 till 100 which is the same as the CPT thereby confirms the displacement analysis outcome. It is also discovered that the variation in deflection and stress parameter is more when the plate is thicker (between 4 and 15) and less when the plate is thinner (beyond 15) under the same loading criterion. Since deflection and stress characteristics of the plate remain constant and equal to the value of the CPT at span-thickness ratio of 15 as seen in the Figure 2, it can be said that the square plate can be categorized as thick when the span – thickness ratio is between 4 and 15. Similarly, the span – thickness ratio beyond 15 till CPT, the square plate can be categorized as thin or moderately thick plate.

Figure 3 shows that, at a span-thickness ratio between 4, 5, 10 and 15, the value of out of plane displacement varies between 0.0089, 0.0091, 0.0071 and 0.0070 respectively. These values maintain a constant value of 0.0070 at the span - thickness 15 till 100 which is the same as the CPT. Similarly, Figure 3 shows that in a span-thickness ratio between 4, 5, 10 and 15, the value of in-plane displacement varies between 0.0099, 0.0091, 0.0082 and 0.0080 respectively. These values maintain a constant value of 0.0080 at the span - thickness 15 till 100 which is the same as the CPT. Furthermore, Figure 3 shows that, at a span-thickness ratio between 4, 5, 10 and 15, the value of shear stress along x-y axis varies between 0.444, 0.426, 0.401 and 0.395 respectively. These values maintain a constant value of 0.395 at the span - thickness 15 till 100 which is the same as the CPT thereby confirms the displacement analysis outcome. It is also discovered that the variation in deflection and stress parameter is more when the plate is thicker (between 4 and 15) and less when the plate is thinner (beyond 15) under the same loading criterion. Since deflection and stress characteristics of the plate remain constant and equal to the value of the CPT at span-thickness ratio of 15 as seen in the Figure 2, it can be said that the plate with aspect ratio of 1.5 to 2.0 can be categorized as thick when the span – thickness ratio is between 4 and 15. Similarly, the span – thickness ratio beyond 15 till CPT, the square plate can be categorized as thin or moderately thick plate.

Study in the Figure 2 and 3 shows that, there are major categories of rectangular plates. The plates whose deflection and vertical shear stress do not vary much or the same as CPT are categorized as thin or moderately thick plate depending on the rate of variation with the aspect ratio of the plate.

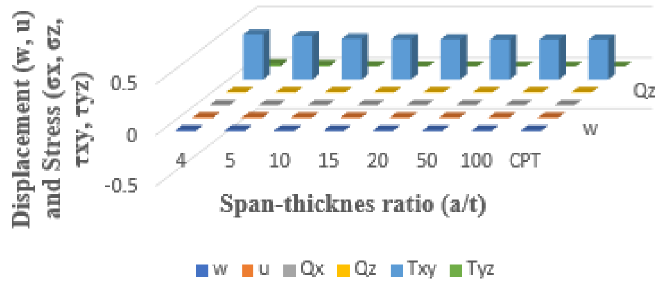

Figure 3: Displacement and stresses of 1.5 aspect ratio CSSS plate

In this study, thin or moderately thick plate is categorized as those plates whose span-thickness ratio is greater than 15 ($a/t > 15$). On the other hand, the plate whose deflection and transverse shear stress varies very much from zero is categorized as thick plates. Here, the variation between the value of deflection and stresses is much compared to those of thin plates. In this study, the results showed that a thick plate is categorized as plate whose span to thickness ratio is less than or equal to 15 ($a/t \leq 15$). The comparative analysis was performed in this study as presented in the Figure 4 to show the validity of the derived relationships and the conformity between the present study and various method/theories in the plate analysis using percentage difference evaluation techniques.

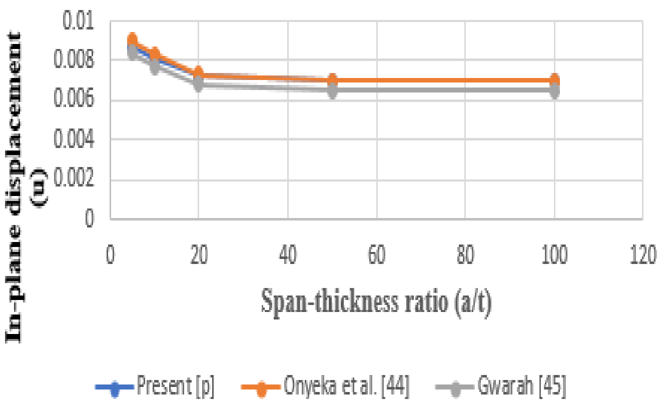

Figure 4: Comparison between different studies ([P], [44] and [45])

The present study in-plane displacement at aspect ratio of 1.5, which used a new 3-D trigonometric model is compared with the work of Onyeka *et al.* [44] and Gharah [45] that used to derive polynomial displacement and those with assumed function respectively as presented in the Figure 4. The percentage difference between the present work and those of Onyeka *et al.* [44] in a span - depth ratio of 5, 10, 20, 50, 100 and CPT are 1.354%, 1.474%, 1.683%, 1.725%, 1.741% and 1.744% respectively. On the other hand, the percentage difference between the present work and those of Gwarah [45] in a span - depth ratio of 5, 10, 20, 50, 100 and CPT are 5.574%, 5.602%, 5.870%, 6.010%, 6.083% and 6.047% respectively. It can be observed that the value of the percentage difference increase with an increase in the span-depth ratio of the plate which suggests that the credibility and the exactness of the model is higher when applied in the thick plate analysis but can also give good result in the analysis of the thin or moderately thick plate. The average percentage difference between the present work and those of Onyeka *et al.* [44] and Gharah [45] is 1.43% and 5.15%, respectively. It is found in the table that, the difference between the present work and the works of Onyeka *et al.* [44] is lower than those of Gwarah [45] percentagewise with about 3.72%, which proves that a derived displacement function produces a close-form solution when compared with 3-D elasticity trigonometric theory while an assumed deflection gives approximate solution. Hence, exact 3-D plate theory is required to achieve efficiency. The 3.29% total percentage difference obtained in this work is quite small which thereby depicts the credibility of the derived relationships.

4.0 CONCLUSION AND RECOMMENDATION

The 3-D bending and stress analysis of thick rectangular plate using 3-D elasticity theory has been investigated and following conclusion has been drawn:

i. The present theory stress prediction shows that the result of the displacement and stress of thin and moderately thick plate using the 3-D theory is the same at span-thickness ratio beyond 15% for the bending analysis of rectangular plate under the CSSS boundary condition and as such, CPT is recommended for the analysis.
ii. The 2-D derived displacement function gives close-form solution, but assumed polynomial function over-predicts loads in the relatively thick plate analysis. Thus, a derived deflection function is recommend for accurate prediction of the design load in the analysis.
iii. The 3-D exact plate model developed in this study which all the stress elements on the plate is recommended for the analysis of any category of the plate.

iv. Plate analysis required 3-D analogy for a true solution, but the 2-D shear deformation theory gives an approximate solution which is practically unrealistic.

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