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Probabilistic Modeling of Failure Times and Replacement Maintenance of Television Transmitter System

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Abstract

Assessment of the state of health and working condition of equipment in the production and service industries is a necessary condition for effective maintenance. Therefore, this work models the failure times and other operational characteristics of television transmitter system with sudden but non-constant failure rate to enhance smooth operation of the system. The failure times was shown to follow lognormal distribution with mean; $\bar{x} = 7.6$ and variance; $s^2 = 1.4$. The Probability functions, hazard, reliability functions/indices, availability and maintainability factors were obtained to described the failure behaviour and ascertain the performance efficiency of the system. Also, a preventive replacement (PR) model for the system yielded an optimal replacement time of 189.112 hours at a minimum total cost of 178.134 naira per unit time. A combination of both estimates of the reliability indices and the PR model is recommended to provide complementary information for efficient maintenance operation of the TV transmitter and similar systems.

Keywords: Cost of maintenance, failure times distribution, reliability function, replacement model, Television transmitter

1.0 INTRODUCTION

The operational efficiency of any system depends on the proper functioning of its components. Hence, knowledge of the failure distributions of systems is required before delving into the reliability of such systems. The failure distribution of a system can be any probability model, f(t) defined over time $t = \lfloor o, \infty \rfloor$ based on the failure mechanism. Lifetime distribution models are classified based on the failure models of systems where interest is in the random variable which is the interval between successive failures. Firstly, we have failure models of systems which have constant rate of failure. This accord to the postulates of a Poisson process, hence the exponential model works well for such events [1].

Secondly, is the failure model of systems whose components do not exhibit constant failure rate and their operational efficiency degrades with time and usage. In other words, they deteriorate with time and usage like those found in ball bearing and vehicle tyre. This type of failure can be modeled using the Weibull distribution as discussed by Weibull (1951) in [2].

Thirdly, is the failure model of systems whose components do not fail at constant rate and their efficiency does not decrease (deteriorate) with time. This kind of systems undergo sudden and complete, but non-constant failure rate. Examples of such failures are found in electronic systems. It is assumed that this type of failure could be modeled with lognormal distribution model which can be theoretically derived under the assumption matching many failure processes common to electronics failure mechanisms.

A system shares within its components common characteristics such as structure, behavior and interconnectivity. A system can be classified as repairable or non-repairable based on the type of components of the system. A repairable system is a system whose components can be restored to satisfactory operation by any action after failure. On the other hand, a non-repairable system is a system in which either the entire system or the individual component that fails is removed permanently from the system, while the system is restored to satisfactory operation by replacing the failed component or the entire system with another one for better usage.

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According to [3], system failure is said to occur when a functional system becomes less effective or completely useless due to sudden breakdown or gradual deterioration. System failure may be gradual or sudden. Failure mode describes the specific manner or way by which a failure occurs in the item under investigation. In reality, systems do not just fail without a cause. The various factors that initiate the mode of which failures occur are known as failure mechanism, and since the failure phenomenon of a system is stochastic in nature, the development of the concept of reliability is based on probability theory.

A reliability study is therefore concerned with random occurrences of undesirable events of failures during the lifetime of a physical system. System failure can cause a lot of damages and losses to both system users and the consumer population of the products and services of the system. Yet, system failure may be caused by very low cost components of the system due to lack of proper maintenance. This necessitates the need for probabilistic modeling of the failure rate of systems in order to determine the reliability of the system and improve the optimal life time of such systems. A reliability index with respect to fatigue and rutting within the different seasons peculiar to Nigeria was evaluated in [4] to improve empirical-mechanistic flexible pavement design approach, using First Order Reliability Method (FORM). The findings showed that season I (winter) recorded the highest component reliability index for fatigue (5.63 for normal distribution). Season II (summer) recorded the lowest component reliability index (β) for rutting (5.4 for normal distribution) and season III (spring) recorded the lowest component reliability index for fatigue (1.85 for normal distribution).

Also, [5] presents the results of safety assessment of timber columns laminated with aluminium using the First Order Reliability Methods. Three failure modes were considered in the studies: bending failure, buckling failure, and flexural buckling failure modes. The results show that the column is safer for compression failure mode and that the most critical failure mode for the column is the flexural buckling mode.

A handful of parametric models exist which have been successfully used as population models of failure times for both repairable and non-repairable systems. Example of such models are exponential, Weibull, gamma and lognormal models among others. Different systems exhibit different failure distributions according to their individual mode of failure and failure mechanisms. The choice of failure model is sometimes based on the physics of the failure mode and at other times, models are chosen solely because of their empirical success in fitting actual failure data, [6] and [7]. This work adopts the later approach in determining the lognormal distribution as the failure distribution of the TV transmitter system.

The applicability and acceptability of lognormal distribution as a failure distribution was first shown by the life-test sampling plans, [8]. The lognormal distribution received relatively minor attention in the statistical literature until the 1970s because its applicability was limited to some rare situations in small-particles statistics, economics and biology, [2]. The lognormal distribution was considered in [9] as a failure model from the Bayesian point of view. A specific model where distributions close to the lognormal arise naturally from the program structure was proposed in [10]. They showed that the worst case bound can be estimated by a less pessimistic way, due to the mathematical complications encountered when the lognormal distribution is used as reliability growth models.

The ‘bathtub curve’ shows that the failure rate function of equipment or its components will go through three phases in a life cycle, namely; decreasing failure rate (DFR), constant failure rate (CFR) and increasing failure rate (IFR), [11]. The critical condition of the component lifetime is usually in IFR conditions (deteriorating condition) that increases the frequency of failures. The application of traditional maintenance or corrective maintenance (CM) will cause increasing maintenance cost (failure cost) and production losses (downtime). Therefore, one of the practical strategies for reducing the maintenance cost and production losses is by applying preventive maintenance (PM).

Preventive replacement (PR) maintenance model is one of the popular PM strategies which ensure a periodic replacement of non-repairable components in deteriorating condition. The main objective of PR model in this work is to reduce the frequency of failures in order to achieve a balance between the cost of failure replacement and maintenance benefits such as downtime, reliability and availability of the system. The earliest PR strategy was developed by [12] and it became a fundamental criterion for various replacement problems. In [13], a replacement model was developed that helps to establish the optimal time for the replacement of streetlight bulbs and was applied to locally sourced data. It was concluded that the model helps to reduce maintenance costs for facility managers. The PR strategy is widely applied in industrial areas and mostly applied on machine components problems. For example, [14] developed models based on PR strategy and applied it to cutting tool problem of a CNC milling process. The main objective of the model was to determine the optimal replacement intervals coupled with the forecast of tool replacement time to minimize the production cost. The PR model of [14] was modified by [15] and applied to machine tool problem in crankshaft line process. Consequently, the model in [15] was modified in this work to determine the optimum replacement time to minimize the maintenance cost.

1.1 Assumptions

The basic assumptions associated with this study are:

a. Failures in the television transmitter system occur at random.
b. Given the lifetime distribution of the television transmitter system f(t), it is assumed that failure occurs at the end of time or period, say t.
c. Failures that occur at each time, t are independent and continuous.
d. Routine preventive maintenance on the system is provided.
e. The failure of one component of the system causes the failure of the entire system.

The TV transmitter is a complex electronic device which major components are integrated circuits, diodes and fuses which are replaced after each failure. Hence, it is a non-repairable system. It fails suddenly but at a non-constant rate. The inter-failure distribution of the transmitter system was modeled using the lognormal failure distribution.

The remainder of this paper shall present the lognormal distribution as the failure distribution function for the inter-failure data of the TV transmitter system in section 2.1, obtain its failure distribution functions and its parameters estimates for the transmitter system and perform the goodness-of-fit test to ascertain its appropriateness in section 2.2. In section 2.3, the reliability function, failure rate as well as the reliability indices of the system shall be obtained while section 2.4 applies a preventive replacement model to obtain optimal replacement time and its associated cost. Section 3 presents results of the work followed by conclusion in section 4.

1.2 Data

Failure data (in hours) with associated cost of maintenance (in naira) of the transmitter system of Nigerian Television Authority, Uyo with their corresponding maintenance dates (for servicing/preventive and failure) were collected from the starting year of transmission 2004-2021. The data consist of 29 cases of replacement maintenance and 14 cases of preventive maintenance (servicing) over the period. The inter-failure hours were obtained as the difference between successive dates of maintenance.

2.0 METHODS

2.1 The Lognormal Distribution

The lognormal distribution is a continuous probability distribution for a random variable whose logarithm is normally distributed. A variable might be modeled as lognormal if it can be thought of as multiplicative product of many independent random variables each of which is positive by considering the central limit theorem in the log-domain, [16].

The lognormal distribution is important in the description of natural phenomena. This is because, for many natural processes of growth (failure), the growth rate (failure rate) is independent of size, [17]. In reliability theory, the lognormal distribution is often used to model time to repair of a maintainable system, [18].

2.2 The Failure Distribution Function of the Transmitter System

Given the lifetime distribution of system $f(t)$, we define a non-negative random variable t to be the time to failure (that is, the time to first failure) of a component of a system. Let F be the distribution function of t, then F (t) could be defined by;

F(t) = P(component of system fails at or before time, t) = P(T$\leq t$); $t \geq 0$

Thus, F(t) defines the failure time distribution function. The point $t = 0$ corresponds to a convenient reference point such as the time to first usage. Hence,

$$F(t) = \int_0^t f(t)dt \tag{1}$$

Eq (1) is the cumulative failure distribution at time, t which shows the probability that a randomly selected system component will fail at time, t given that it did not fail before time, t. Then, the probability density function of system failure at time, t distributed as lognormal is given by;

$$f(t;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln t-\mu}{\sigma}\right)^2\right]; t \geq 0, \sigma^2 > 0, \mu \in \Re \tag{2}$$

The failure time distribution function F(t) for the lognormal model, according to Eq (1) is given by;

$$F(t;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\int_0^t \exp\left[-\frac{1}{2}\left(\frac{\ln t-\mu}{\sigma}\right)^2\right]dt \tag{3}$$

We note that in this integral, the negative exponent of t will continue to increase showing that it is an infinite integral which can only be obtained numerically. Nevertheless, according to [19], a substitute for the lognormal distribution whose integral can be expressed in terms of a more elementary function can be obtained based on the logistic distribution to get an approximation for the CDF as given below;

$$F(t;\mu,\sigma^2)=\left[\left[\frac{e^{\mu}}{t}\right]^{\pi\sigma\sqrt{3}}+1\right]^{-1} \quad (4)$$

2.2.1 *Estimation of the lognormal parameters of the television transmitter system*

The method of maximum likelihood was used to obtain the estimate of the parameters of lognormal distribution as follows:

The likelihood function and the log likelihood of Eq (2) are respectively given as:

$$L(f(t,\mu,\sigma^2)=\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \exp[-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(Int_i-\mu)^2$$

$$InL(f(t,\mu,\sigma^2)=-nIn(\sigma\sqrt{2\pi})-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(Int_i-\mu)^2 \quad (5)$$

Differentiating Eq (5) with respect to the parameters; μ and σ and equating to zero, we obtained:

$$\hat{\mu}=\frac{\sum_{i=1}^{n}\ln t_i}{n} \quad (6)$$

and $$\hat{\sigma}^2=\sum_{i=1}^{n}\frac{(\ln t_i-\mu)^2}{n} \quad (7)$$

The parameters estimates are $\bar{x}=7.6$ and variance; $s^2=1.4$.

2.2.2 *Goodness-of-Fit Test of Lognormal Distribution*

The multinomial chi-square goodness-of-fit test was performed to ascertain if the inter-failure times of TV transmitter system follow the lognormal distribution.

The Chi-square test statistic is given by;

$$\chi^2=\sum_{i=1}^{n}\frac{(O_i-E_i)^2}{E_i}\sim\chi^2_{\alpha;n-1} \quad (8)$$

The expected frequency, E_i corresponding to the observed frequency O_i were obtained using the estimates of the parameters as:

$$E_i=N\times P[t_1<T<t_2]$$
$$=N\times P\left[\frac{lnt_1-\bar{x}}{s}<Z<\frac{lnt_2-\bar{x}}{s}\right]=N\times\left\{P\left[Z<\frac{lnt_2-\bar{x}}{s}\right]-P\left[Z<\frac{lnt_1-\bar{x}}{s}\right]\right\} \quad (9)$$

The resulting values of observed and pooled expected frequencies is given in Table 1.

Table 1: Observed and pooled expected frequencies

Interval of time, t (hr)	O_i	E_i
$t<1000$	12	11.34
$1000<t<2000$	4	10.18
$2000<t<3000$	7	6.21
$3000<t>5000$	9	6.63
$t>5000$	11	8.65
Total	43	43

H_0 was accepted since $x^2=7.41<x^2{}_4(0.5)=9.45$. Hence, it was concluded that the distribution of failure times of TV transmitter system follows lognormal distribution. Also, Easyfit (5.6) software was used to validate the goodness of fit test with rank 1 as well as the parameters estimates of the distribution in section 3.1.

2.3 *Reliability Function of the Transmitter System*

Reliability is the probability of a system performing its intended function at a specified time, t. Hence, the probability that a system will fail at or before time, t plus the probability that a system will perform its intended function till time, t equals one; that is, $P(T<t)+P(T>t)=1$; such that:

$$R(t)=1-\int_0^t(t)dt=\int_t^{\infty}(t)dt \quad (10)$$

Therefore; $$R(t)=1-\left[\left[\frac{e^{\mu}}{t}\right]^{\pi\sigma\sqrt{3}}+1\right]^{-1} \quad (11)$$

2.3.1 *Failure rate of the transmitter system*

The failure rate during a given interval of time $t=[t_1,t_2]$ shows the probability that a failure per unit time occurs in the interval (t_1,t_2), conditional on the event that no failure has occurred at or before time, t_1. This means that $T>t_1$. The failure rate can be defined as follows:

$$h(t)=\frac{R(t_1)R(t_2)}{(t_2-t_1)R(t_1)}=\frac{F(t_2)-F(t_1)}{(t_2-t_1)R(t_1)}$$

Taking the limit of the failure rate as the interval, (t, $\Delta t + t$) approaches zero, where $t = t_1$ and $(t + \Delta t) = t_2$ gives the hazard function, $h(t)$. Thus,

$$h(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t+\Delta t)}{\Delta t R(t)} = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t} \cdot \frac{1}{R(t)} \quad (12)$$

$$= \lim_{\Delta t \to 0} \frac{\text{P[system will fail at (t,t+}\Delta\text{t)given that it has survived till time t]}}{\Delta t}$$

We note that $\lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t} = f(t)$.

Hence, putting this in (12) we have;

$$h(t) = \frac{f(t)}{R(t)} = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln t - \mu}{\sigma}\right)^2\right] \cdot \frac{\left[1 + \left[\frac{e^{\mu}}{t}\right]^{\pi\sigma\sqrt{3}}\right]}{\left[\frac{e^{\mu}}{t}\right]^{\pi\sigma\sqrt{3}}} \quad (13)$$

2.3.2 Reliability indices of the transmitter system using lognormal distribution

(a) ***The Mean Time to Failure:*** The expected duration of a system is the expected time during which the system will perform its intended function successfully.

It is defined by E(t) $= \int_0^\infty t f(t) dt$

The expected duration, $E(t)$ also known as the Mean Time to failure (MTTF) is given by;

$$E(t) = \int_{-\infty}^{\infty} \frac{t}{\sigma t\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{Int - \mu}{\sigma}\right)^2\right] dt$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{Int - \mu}{\sigma}\right)^2\right] dt$$

Substituting $t = e^x$

$$E(e^x) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[x - \frac{1}{2}\left(\frac{In - \mu}{\sigma}\right)\right] dt$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[2x\sigma^2 - \left(\frac{(x-\mu)^2}{\sigma}\right)\right] dt$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} exp\left\{-\frac{1}{2\sigma^2}[x - (\mu + \sigma^2)]\right\}^2 \quad (14)$$

The integral part of Eq. (14) is a cumulative function of x which is normally distributed with mean $= \mu + \sigma^2$ and variance, σ^2.

We note that

$$\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} exp\left\{-\frac{1}{2\sigma^2}[x - (\mu + \sigma^2)]^2\right\} dx = 1$$

Hence, $E(t) = e^{\mu + \frac{\sigma^2}{2}}$ (15)

(b) ***The Mean Time Between Failures***

The mean time between failures of the system (MTBF) is given as; $\frac{1}{\lambda}$

Where $\lambda = \frac{N}{T-D}$, T= total time, D = total downtime and N= number of failures in the system;

$T - D =$ the uptime $= \sum_{i=1}^{N} ti \Rightarrow \lambda = \frac{N}{\sum_{i=1}^{N} ti}$

Hence, $MTBF = \frac{\sum_{i=1}^{N} t_i}{N}$ (16)

(c) ***The Mean Time to Repair***

The mean time to repair MTTR is given by $\frac{D}{N}$

where $D = \sum_{i=1}^{N} d_i$

and N= the total number of failures in the system.

Hence, $MTTR = \frac{\sum_{i=1}^{N} d_i}{N}$ (17)

(d) ***Availability Factor***

Availability measures the percentage of effectiveness of the system within a given period of time. The availability of the system is given by:

$$A = \frac{MTBF}{MTTR + MTBF} \times 100\% \quad (18)$$

(e) ***The Maintainability Factor***

Maintainability is the probability that a system will be restored to a specified condition within a given period of time when maintenance is performed in accordance with prescribed procedures and resources, [11]. The maintainability factor is given by:

$$M = \frac{MTTR}{MTTR + MTBF} \times 100\% \quad (19)$$

2.4 *Replacement Model for the Television Transmitter System*

In developing a replacement model, the decision criterion is defined by $E[c(t)]$, which is the expected cost/cycle time of replacing a part of the system in cycle period (0, t). It was shown in [15] that the expected number of failure $E[N(t)]$ occurring in the cycle period (0, t) is equal to the probability of occurrence of a failure before time, t given by F(t).

The number of failures occurring during the period (0, t) is defined as N(t), which is a discrete random variable. Its probability distribution function is defined as;

$$P[N(t)=n]=G(n); n=1,2,3\ldots$$

Its average value (mathematical expectation) is then equal to;

$$E[N(t)]=\sum_{N(t)=0}^{N(t)=n} N(t).G[N(t)]$$

Where G[N(t)] is the probability distribution function of N(t) failures occurring in the period (0,t). It is assumed that each interval is made as short as possible so that the probability of having more than one failure is negligible. In this situation the probability of having two failures is small compared to having a single failure; That is;

$$\Pr[N(t)=2]<\Pr[N(t)=1]$$

And the probability of having three failures is small compared to having two failures;

$\Pr[N(t)=3]<\Pr[N(t)=2]$ and so on.

Therefore,

$$\Pr[N(t)=1]>\Pr[N(t)=2]>\Pr[N(t)=3]>\Pr[N(t)=4]>\ldots$$

Consider preventive replacement at τ, the expected number of failures in the period (0, τ) can be estimated from the following:

$$G(1)=\Pr[N(\tau)=1]\approx F(\tau)$$
$$and\ \ G(0)=\Pr[N(\tau)=0]\approx 1-F(\tau)$$

$$Then,\ E[N(\tau)]=\sum_{n=0}^{\infty} n\times G(n)=0\times[1-F(\tau)]+1\times F(\tau)$$

$$E[N(\tau)]=F(\tau) \quad (20)$$

This implies that, $E[N(\tau)]$, the mean number of failures occurring during the cycle period (0,τ] is equal to the probability of occurrence of a failure before time, τ.
According to [15], the total expected cost per unit time for preventive replacement at replacement time, τ is defined as;

$$E[C(\tau)]=\frac{total\ expected\ cost}{replacement\ time}=\frac{C_P+C_f\,E[N(\tau)]}{\tau} \quad (21)$$

Therefore;

$$E[C(\tau)]=\frac{C_p+C_f F(\tau)}{\tau} \quad (22)$$

2.4.1 *Minimization of the expected cost function*

In order to obtain optimal replacement time, we differentiate Eq. (21) as follows:

$$\frac{d}{d\tau}E[C(\tau)]=\frac{\tau C_f\frac{d}{d\tau}E[N(\tau)]-[C_p+C_f E[N(\tau)]]}{\tau^2}=0$$

$$\Rightarrow \tau C_f\frac{d}{d\tau}E[N(\tau)]-[C_p+C_f E[N(\tau)]]=0$$

$$\Rightarrow \tau=\frac{[C_p+C_f E[N(\tau)]]}{C_f\frac{d}{d\tau}E[N(\tau)]}$$

$$\therefore \quad \tau^*=\frac{\frac{C_p}{C_f}+E[N(\tau)]}{\frac{d}{d\tau}E[N(\tau)]} \quad (23)$$

But, $E[N(t)]=F(t);\ \frac{d}{d\tau}E[N(t)]=\frac{d}{d\tau}F(t)=f(t)$

Hence,

$$\tau^*=\frac{\frac{C_p}{C_f}+F(t)}{f(t)} \quad (24)$$

3.0 RESULTS

3.1 Estimation of the Lognormal Parameters of the TV Transmitter System

The method of maximum likelihood was used to obtain the parameters estimates of the lognormal distribution as $\bar{x} = 7.6$ and variance; $s^2 = 1.4$. Easyfit version 5.6 was also used to validate the parameters estimates.

3.2 Evaluation of Failure Functions of the Transmitter System

For each value of t, the failure density function, $f(t)$, the failure distribution function $F(t)$, the reliability function $R(t)$, and the failure rate $h(t)$, were evaluated according to Eqs. (2), (4), (11) and (13) and the graphs are shown in Figures 1, 2, 3 and 4.

Figure 1 shows that the likelihood of occurrence of failures in the transmitter system increased suddenly to 1 within the first 1000hours of operation and reduces gradually as time increases. This is a positively skewed density function of a lognormal distribution with $\bar{x} = 7.6$ and variance; $s^2 = 1.4$. Also, Figure 2 shows a stable distribution of failures for the first 1000hrs before it suddenly increased to1 between the intervals of 1000hrs to 2000hrs and again attained stability as time increases from 2000hrs. This shows a sudden increase in the distribution of failures in the system. Figure 3 shows that the reliability of the TV transmitter system was stable at the first 1000hrs; it decreases sharply within the time interval of 1000hrs to 2500hrs and then reduces gradually as time increases from 2500hrs. Figure 4 shows that the hazard function or failure rate of the system was very low (about 0.07) and stable for the first 3000hrs; this shows high reliability of the system. It increased gradually from 3000hrs to 4500hrs at the rate of 0.4 followed by a sudden increase to a very high rate of 1 as time increases from 4500hrs to 5000hrs suggesting the need for replacement maintenance of the system at this interval.

3.3 Estimates of Reliability Indices

In addition to the probability functions, reliability factor and its indices were obtained respectively from Eqs. (11), (15), (16), (17), (18) and (19) as shown in Table 2.

Table 2: Values of Reliability indices of TV transmission system

R_f	(MTTF)	MTBF	MTTR	A_f	M_f
99%	3949.57	3109.35	114.30	96%	4%

The mean time to failure of the system, MTTF=3949.57 hrs implies that the expected time for the first failure to occur in the system is 3949.57 hours. The mean time between subsequent failures in the system; MTBF=3109.35 hrs implies that the transmitter system will fail after every 3109.35hrs. The mean time to repair given by MTTR =114hrs means that it will take about 5 days of maintenance to restored the transmitter system to normal working condition after failure. Also, the system performance level (reliability, R_f = 99%) is very high with availability factor, A_f =96%. This implies that the television transmitter system is in a working state for about 96% of the time while only 4% of the time is used for maintenance activities.

3.4 Replacement model and optimal probability functions of the TV transmitter system

The following associated costs of maintenance : failure replacement cost, $C_f = \sum_{i=1}^{29} C_i = 566950$ and preventive replacement cost, $C_p = \sum_{i=1}^{14} C_j = 24850$ were used to obtain the expected cost of maintenance, $E[C(\tau)]$ and optimal replacement time, τ^* in Eqs. (22) and (24) respectively at constant interval time as shown in Table 4.

Optimal probability functions were also obtained at the respective optimum values for the replacement model under consideration in Table 3.

Table 3: Optimal probability functions and expected cost functions for replacement model

τ^*	$f(\tau^*)$	$F(\tau^*)$	$E[C(\tau^*)]$
189.112	0.018582	1.76×10^{-8}	178.134

Detail computation of the optimal replacement time, $\tau^* = 189.112\, hrs$ for a unit/component of the transmitter system at minimum cost per cycle of 178.134 Naira and optimal probability values were obtained for the television transmitter system in Table 4.

3.5 Propose Maintenance Policy for the Television Transmitter System

Maintenance policy is a set of administrative, technical and managerial action to apply during the life cycle of a machine used to guide maintenance management and decision making towards retaining certain operational conditions of a machine or dedicated to restoring the machine to said condition. The proposed maintenance policy states that "*the television transmitter system which has 99% reliability value and 96% availability index would operate optimally for* $T \leq 189.112$ *hours before any* *preventive replacement is carried out at a minimum cost of 178.134 naira while failure maintenance should be performed at any time t<T if it fails".*

4.0 CONCLUSIONS

This work models the failure times of a non-repairable television transmitter system which exhibit a sudden but non-constant rate of failure. The multinomial χ^2 goodness-of-fit tests showed that the failure distribution of the system follows a lognormal distribution with mean, $\bar{x} = 7.6$ and variance $s^2 = 1.4$. Probability functions of the system were obtained as primary indicators of its operating condition which describe the failure behavior as well. Also, reliability indices were obtained for the transmitter system. The mean operational time between failures was about 3109.35 hrs (130 days) as against 189 days by the PR model. While the mean time to repair or restored the system to normal working condition was an average of about 5 days after failure as against 4 days by the PR model. The transmitter system was found to have a very high performance level of 99% and a working state index of 96%. Furthermore, the minimum total cost per unit time of replacement based on the PR model is 178.134 naira at optimal replacement time of 189.112 hrs. The results from these two perspectives provide a useful guide to maintenance action instead of depending on one set of estimates. Therefore, a combination of both estimates of the reliability indices and the PR model is recommended to provide complementary information for efficient maintenance operation of the TV transmitter and similar systems.

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Appendix

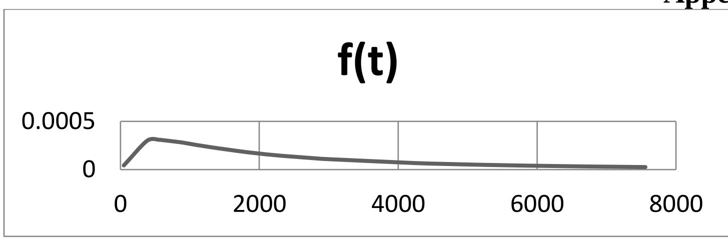

Figure 1: Graph of f(t) of TV transmitter system

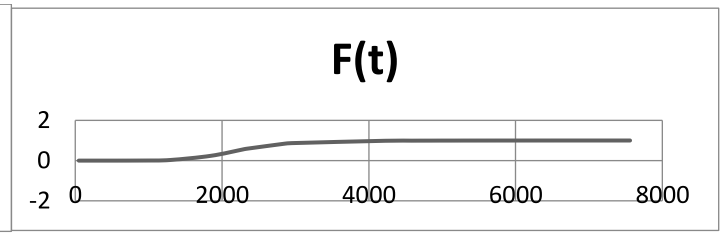

Figure 2: Graph of F(T) of TV transmitter system

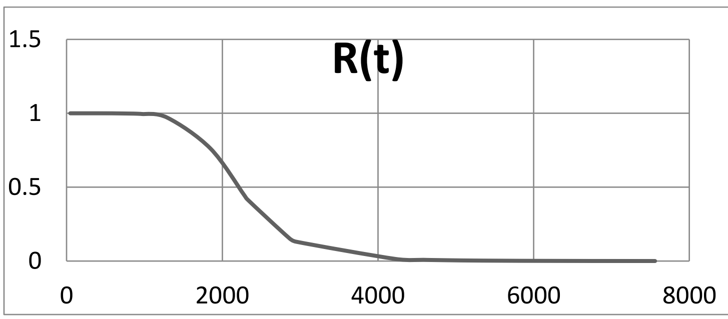

Figure 3: Graph of R(t) of TV transmitter system

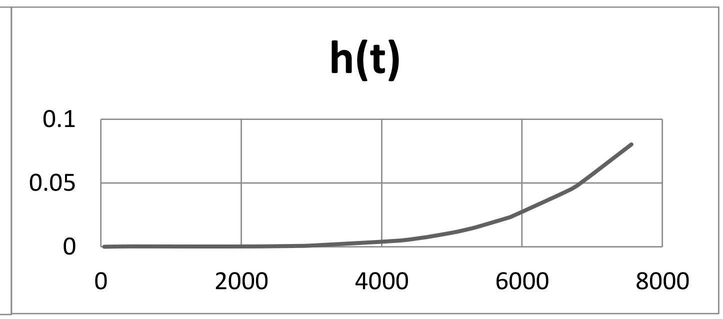

Figure 4: Graph of h(t) of TV transmitter system

Table 4: Preventive replacement times τ of the television transmitter system and cost of replacement at constant interval time

t_r (hr)	$f(t_r)$	$F(t_r)$	τ	E(c(τ))
5	6.28E-07	3.09E-17	63640.86	45.666
10	1.12E-05	2.51E-15	3576.558	75.546
15	5.11E-05	3.29E-14	1899.335	100.345
20	0.00014	2.04E-13	990.635	120.3306
25	0.000292	8.40E-13	782.1007	139.878
30	0.00052	2.67E-12	639.5315	145.903
35	0.000829	7.10E-12	467.932	147.109
40	0.001226	1.66E-11	400.453	151.211
45	0.001712	3.50E-11	367.232	154.003
50	0.002288	6.83E-11	356.111	157.776
55	0.002954	1.25E-10	300.233	159.799
60	0.003708	2.17E-10	267.345	163.005
65	0.004548	3.61E-10	231.005	164.604
70	0.005471	5.77E-10	245.125	166.455
75	0.006475	8.94E-10	211.566	169.001
80	0.007556	1.35E-09	205.92	171.934

t_r (hr)	$f(t_r)$	$F(t_r)$	τ	$E(c(\tau))$
85	0.00871	1.98E-09	198.02	173.591
90	0.009936	2.84E-09	194.78	174.455
95	0.011229	4.00E-09	192.55	176.955
100	0.012585	5.55E-09	190.443	177.566
105	0.014002	7.56E-09	190.301	178.093
110	0.015476	1.02E-08	189.546	178.085
115	0.017003	1.35E-08	189.403	178.113
120	0.018582	1.76E-08	189.112	178.134
125	0.020208	2.28E-08	189.411	178.101
130	0.021879	2.93E-08	191.004	178.092
135	0.023593	3.72E-08	195.006	178.075
140	0.025345	4.69E-08	197.673	178.054
145	0.027135	5.86E-08	199.101	178.043
150	0.028958	7.26E-08	200.556	178.0334
155	0.030814	8.94E-08	205.567	178.021
160	0.032699	1.09E-07	211.678	178.009
165	0.034612	1.33E-07	230.455	178.006
170	0.03655	1.61E-07	236.777	178.002
175	0.038511	1.93E-07	240.546	177.981
180	0.040494	2.31E-07	256.704	177.971
185	0.042497	2.75E-07	259.001	177.969
190	0.044518	3.25E-07	278.441	177.95
195	0.046555	3.84E-07	280.551	177.901
200	0.048607	4.50E-07	282.995	177.805
205	0.050673	5.27E-07	286.495	176.678
210	0.052751	6.14E-07	289.007	176.563
215	0.054839	7.13E-07	311.345	176.546
220	0.056936	8.25E-07	345.111	176.331