Systems Identification of Servomechanism Parameters using Jellyfish, Particle Swarm and Constraint Optimization

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Abstract

In this paper, DC servomechanism parameters were identified offline using Jellyfish, particle swarm and constraint optimization techniques in a MATLAB simulation environment with experimental data. Specifically, the unknown parameters of the servomechanism were identified using a two-step approach. Initially, the first-order transfer function of the servomechanism which is characterized by a DC gain and time constant was determined analytically using the experimental open-loop speed step response of the servo motor. Next, by iterative minimization of a fitness score derived from the root mean squared error between the experimental and simulated position response of the servomechanism of an equivalent state-space model structure, the servomechanism parameters were identified. The simulated angular position step response of the servomechanism with the particle swarm, Jellyfish and constraint optimization algorithm, showed excellent agreement with the experimental data in descending order and was consistent with the fitness score of 1.9035, 0.0083, and 0.00706 respectively.

Keywords: Systems Identification, Jellyfish Optimization, Particle Swarm Optimization, Constraint Optimization, Servomechanism

1.0   INTRODUCTION

Servomechanisms consist of at least a mechanical component and a control mechanism in which the input parameters are varied based on the feedback resulting from the controlled system [1]. Thus, in practice it is possible to decouple the servomechanism into two main parts; a DC servomotor and a gear mechanism that converters angular velocity to angular position [1]. Servomechanism control is therefore essential due to the certain demand requirements for fast and precise response to dynamic position and speed set-points [2].

Therefore, to accurately control a servomechanism mathematically, model parameters must closely match the characteristics of the servomechanism when simulated [3] is essential. Furthermore, the crucial model parameters which fundamentally describe the servomotor armature circuit are the winding coil series inductance and resistance, input voltage, back electromotive force. While, the rotor dynamics of the servo motor are characterized by the inertia, Torque, and angular speed/position of the DC servo motor [4].

Several approaches proposed by researchers for parameter estimation can be grouped into analytical [5] numerical or soft computing such as constraint optimization (CO) [4] and evolutionary algorithms based on nature-inspired swarm intelligence such as genetic algorithm (GA), particle swarm optimization (PSO), whale optimization algorithm (WOA) [6] etc. In [7] the total least square method showed superior performance to the ordinary least square method in identifying the parameters of an induction motor with noisy data. In [8] a comprehensive review of evolutionary algorithms was presented. Furthermore, PSO and GA were used to identify the parameters of a DC motor in [9] and [10].

The mechanical rotor parameters of a servo motor were chiefly estimated using constraint optimization via the MATLAB optimization toolbox in [5]. In [11] parameter estimation of a DC servo motor was achieved by comparing the coefficient to the transfer function

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model, parameter estimation and systems identification toolboxes in MATLAB, with the parameter estimation toolbox having the best performance error of 0.1%.

More recently, Jellyfish optimization (JFO) a novel meta-heuristic algorithm inspired by the swarm intelligence of the Jellyfish, was first introduced by Chou and Troung [12]. The JFO which is based on jellyfish foraging mechanism in the ocean has excellent explorative and exploitative features which makes it a global optimizer. JFO was evaluated against existing evolutionary algorithms and found to be the most effective in the optimization of several popularly known test functions. Until this moment, the JFO has not been evaluated with servomechanism parameter identification. Nevertheless, it was used for maximum power point tracking and parameter identification of solar photovoltaic systems in [13] and [14] respectively.

In this paper, the recent novel Jellyfish optimization, constraint optimization and particle swarm optimization methods were compared and used to illustrate the identification of servomotor parameters based on the response of the servomechanism's position control system (with velocity feedback). The parameters of the servomechanism are estimated by the optimization algorithms which iteratively adjust the state-space model parameters to consequently minimize a fitness function based on the root mean squared error difference between the model and the experimentally obtained time response of the cascaded closed-loop angular position of servomechanism. Thus, the validation of the servomechanism state model is verified in MATLAB via the closed-loop position step response plot.

Furthermore, while [4] constraint optimization was used for parameter extraction of a servomotor nonetheless, the extracted parameters excluded the coil winding inductance of the armature circuit. This paper will address the limitation by determining all the parameters of the servomotor including the coil inductance of the armature circuit via an equivalent state-space model of the servomechanism.

2.0   METHODOLOGY
This section describes the methods adopted in the paper for the actualization of the servomotor parameter estimation. The modelling of the servomechanism in state space, the working principles of the optimization algorithms and experimental procedures have been presented in this section.

2.1   Servo Motor Modelling
The servomechanism system consists of the servomotor, specifically armature-controlled DC Motor, DCM150F, servo amplifier SA150D, function generator, the control unit as well as a servomotor speed to angle converter system [1] as illustrated in Figure 1.
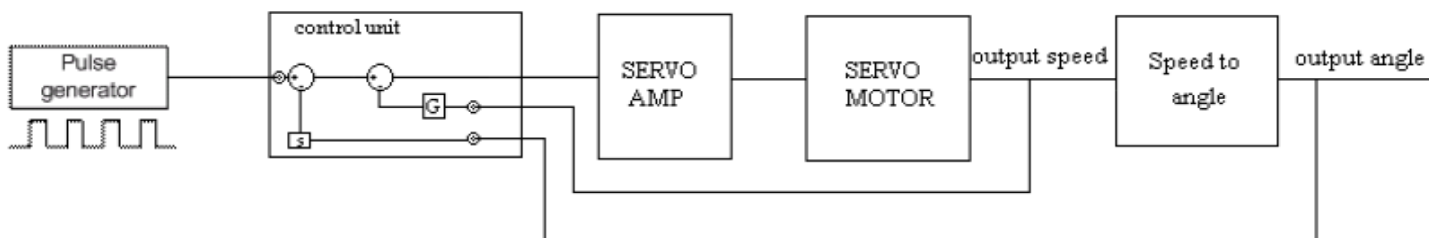

Figure 1: Generic Servomechanism architecture [1]

Therefore, the DC servo motor is modelled with mathematically equations using Kirchhoff's voltage law and Newton's second law of motion concerning the electrical armature windings and the mechanical rotor dynamics respectively.

The differential equations governing the electrical armature of the DC Servo motor are expressed in (1) as follows:

The differential equations governing the electrical armature of the DC Servo motor are expressed in (1) as follows:

$$\frac{d\iota}{dt} = \frac{1}{L}(V - R\iota - E) \qquad (1)$$

The mechanical rotor differential equations are presented in (2) as follows:

$$\frac{d\omega}{dt} = \frac{1}{J}(Ki - B\omega) \qquad (2)$$

Let, $\theta = x_1, \frac{d\theta}{dt} = \omega = \dot{x}_1 = x_2, \frac{di}{dt} = \dot{x}_3$ and $i = x_3$

To nest (1) and (2) in the state-space form:

Therefore, the state-space model of the DC servo motor is as follows:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -B/J & K/J \\ 0 & -K/L & -R/L \end{bmatrix} \begin{bmatrix} \theta \\ \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L. \end{bmatrix} V \quad (3)$$

Where L - coil inductance (in Henry), R - the coil resistance (in Ohms), E - back E.M.F (in Volts), I - the armature current (in amperes), V - input voltage (in volts), J - the moment of inertia (In Kgm^2), B - motor friction constant (In Nm/(rad/sec)), ω - the angular velocity (rad/sec), Θ - angular position (rad).

2.2   Servo Motor Time Constant and Gain

The transfer function $G(s)$ for relating angular velocity and the step input voltage of the servomechanism is based on a first-order system structure and is given by:

$$G(s) = \frac{\omega_m}{V_m} = \frac{K_m}{1 + S\tau_m} \quad (4)$$

where, ω_m is output angular velocity, V_m is the input voltage, K_m is servomotor gain and τ_m is the electro-mechanical time constant.

The servo gain can be calculated using (5)

$$K_m = \omega \,/\, V_m \quad (5)$$

Or by using (6)

$$K_m = V_t/(V_m * K_t) \quad (6)$$

where, V_t, K_t are tachometer voltage and sensitivity respectively.

Thus, transfer function H(s) for the servo gear which is known to be coupled to the G(s), to derive the position open-loop transfer function is T(s):

$$T(s) = \frac{K_m}{1 + S\tau_m} * H(s) \quad (7)$$

where,

$$H(s) = \frac{6.65}{s} \quad (8)$$

2.3   Jelly Fish Optimisation (JFO)

The novel jellyfish optimizer (JFO) proposed by Chou and Troung [12] is based on the foraging behaviour of the jellyfish in the Ocean [14]. The foraging behaviour of the JFO involves two types of movements; the ocean current and movements inside the swarm, as well as a time

control mechanism for deciding the movement type. From studies of several chaotic maps in [12], the logistic map was found to accelerate convergence and avoid local minima in the optimization search space rather than use random initialization points. Thereafter, the motion is governed by the ocean current or by the advancement inside the swarm as per the time control mechanism.

2.3.1   Ocean current

The jellyfish drifts in the direction of the ocean current which has a lot of nutrients. The drift is determined by the error between the jellyfish with the best position and the average of all the vector positions of each jellyfish in the ocean. This is expressed mathematically as follows:

$$\overrightarrow{drift} = 1/N \sum \overrightarrow{drift}_k = 1/N \sum (x_* - a_c x_k = x_* - a_c \sum \frac{x_k}{N})) = x_* - a_c \mu)) \quad (9)$$

where, x_* denotes the jellyfish with the current-best position in the swarm; N is the total number of the jellyfish; a_c is an attraction coefficient; μ is the mean position of the jellyfish swarm. Let:
Set

$$Df = a_c \mu \quad (10)$$

Thus, assuming that the jellyfish are normally distributed in all dimensions, a distance of $\pm \beta \sigma$ around μ position contains a certain likelihood of the entire jellyfish and Df is the mean position of the jellyfish from the jellyfish with the best position.
Therefore,

$$.Df = \beta \sigma \, r1(0,1) \quad (11)$$

Set $\sigma = r2.(0,1) \quad (12)$

Where, $r1, r2$ are randomly generated values between 0 and 1.
Therefore, the new position of the jellyfish is as follows:

$$X_i(t+1) = X_i(t) + r1(0,1).\overrightarrow{drift} \quad (13)$$

Also,

$$X_i(t+1) = X_i(t) + r1(0,1).(x_* - \beta \sigma \, r1(0,1)) \quad (14)$$

2.3.2   Jellyfish swarm

The movement of the jellyfish inside the swarm is either active (type A) or passive (type B). The active motion is exhibited during the initial formation of the

swarm around its location. Nevertheless, the jellyfish over time exhibits passive motion which is based on updating its location to that of a random jellyfish in the swarm with a better direction to find food (better fitness score).

The type A motion is expressed as follows:

$$X_i(t+1) = X_i(t) + \gamma . r1(0,1) . (UB - LB) \qquad (15)$$

Where γ is the motion coefficient concerning the distance around the jellyfish's location, UB and LB are upper and lower bounds for the search space.

Also, type B motion is represented mathematically as follows:

$$\overrightarrow{Step} = X_i(t+1) - X_i(t) \qquad (16)$$

Where,

$$\overrightarrow{Step} = r1(0,1) . \overrightarrow{direction} \qquad (17)$$

$$\overrightarrow{direction} = \begin{cases} X_j(t) - X_i(t) & if\ f(X_i(t)) \geq f(X_j(t)) \\ X_i(t) - X_j(t) & if\ f(X_i(t)) < f(X_j(t)) \end{cases} \qquad (18)$$

Where j is the index of the jth jellyfish in the swarm and f is an objective function of position X. Furthermore, the next position of the jellyfish is updated as follows:

$$X_i(t+1) = X_i(t) + \overrightarrow{Step} \qquad (19)$$

2.3.3 Time control mechanism
The time control mechanism is not only used to switch the motion between type A and B but also towards the ocean current which contains an enormous quantity of nutritious food. The time control mechanism involves the use of a constant value Co and a time control function $c(t)$ that fluctuates randomly between 0 and 1, and is expressed mathematically as follows:

$$c(t) = \left(1 - \frac{t}{t_{max}}\right) . (2 . r2 - 1) \qquad (20)$$

Where, t, t_{max} are the iteration and maximum iteration time specified respectively, $r2$ is a random value (0,1).

Similar to c(t), the function $(1 - c(t))$ simulates both type A and B inside a swarm such that if r2(0, 1) > $(1 - c(t))$,

then the jellyfish will exhibit type A motion and if r2(0, 1) < $(1 - c(t))$, jellyfish will opt for type B motion. Since the value of $(1 - c(t))$ increases with time, the probability that the (1-c(t)) value will exceed the randomly generated value is also greater thus favouring the selection of type B motion over type A which was initially favoured. Figure 2 shows the schematics of the JFO algorithm.

2.4 Particle Swarm Optimisation (PSO)
The particle swarm optimization (PSO) technique is a population-based meta-heuristic optimiser in which particles (individuals) signify solutions in a search space (feeding haven) [15]. The PSO ab initio is initialised with random population particles which in turn possess random velocity and flight through the problem search space. Each particle adjusts its flight pattern p$_{best}$ based on self-experience and that of other particles while keeping track of the optimal flight trajectory g$_{best}$ that results in the best fitness score [15].

The particles position and velocity modification are thus achieved as thus:

$$v_{i,g}^{(k+1)} = K * \left[v_{i,g}^{(k)} + c_1 r_1 * \left(P_{best\ i} - p_{i,g}^{(k)} \right) + c_2 r_2 * \left(g_{best\ i} - p_{i,g}^{(k)} \right) \right] \qquad (21)$$

$$p_{i,g}^{(k+1)} = p_{i,g}^{(k)} + v_{i,g}^{(k+1)} \qquad (22)$$

Where, $v_{i,g}^{(k+1)}$ is the velocity of particle i in dimension g in iteration $k+1$ and $p_{i,g}^{(k+1)}$ signifies the position of particle i in dimension g in iteration $k+1$, while c_1 and c_2 represents cognitive and social acceleration constants respectively.

The constriction factor Kf is expressed mathematically as:

$$Kf = \frac{2}{\left| 2 - \varphi \sqrt{\varphi^2 - 4\varphi} \right|} \quad where\ \varphi = c_1 + c_2, \varphi > 4 \qquad (23)$$

where, the range of velocity is $[-Vmax, Vmax], i = 1, 2, \ldots, n; \ g = 1, 2, \ldots, d,$ and n is the amount of particle swarm while the dimension of the problem is "d" [15]
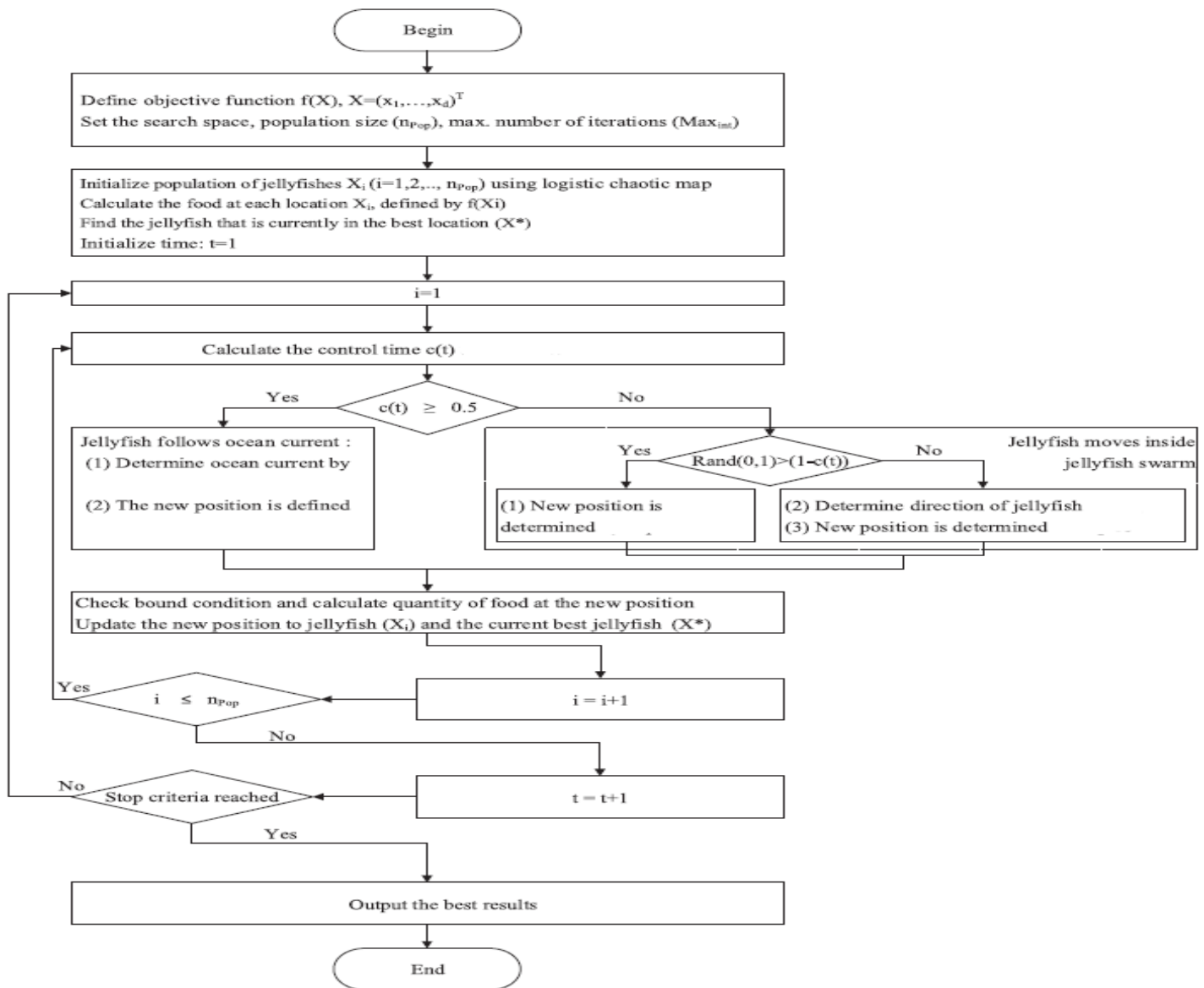
Figure 2: Jellyfish optimisation algorithms [12]

2.5   *Constraint Optimisation Toolbox in MATLAB*

The constraint optimisation was performed using the optimisation graphics user interface in the MATLAB simulation environment. The constraint optimisation input fields were the fitness function and the upper and lower bound constraints. Typically, the problem of parameter estimation is formulated and expressed as follows:
Minimise RMSE subject to (25)

$$A_l \leq \sum_{l=1}^{T} X_l \leq Z_l \qquad \forall\, l = 1,2\dots T \qquad (25)$$

where, X_l is the lth estimated parameter bounded by corresponding values of A_l and Z_l constraints, RMSE is

the root mean squared error between the velocity and position closed-loop response of the experiment and model as follows:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{k} p(\theta^* - \theta)_i^2 + q(\omega^* - \omega)_i^2} \qquad (26)$$

where, θ^* and ω^* are the reference values for angular position and speed response of the servomechanism obtained experimentally and correspond to 1 to N data entry points and p, q are weighting factors of the coupled speed and position error from the estimation.

3.0 RESULTS AND DISCUSSIONS

3.1 Experimental determination of Servo Motor Transfer function Parameters

The analysis for Servomechanism gain K_m and Time constant T_m determination was carried out in this section. The experimental setup was realized in an open-loop based on Figure 3, with a 15V power supply connected to the attenuator feedback module GT150X. Also, 0 to 2.5V was applied to the Servo amp SA150D. Thereafter, the Servomechanism parameters were determined by observing the scope traces, voltage readings and by performing arithmetic calculations using the equations in section 2.

The threshold voltage at which the Servomotor starts to rotate is the dead-band, which was found to be 1.2V.

To determine the time constant T_m, a function generator was used to excite the servomechanism with a +/-1.2 – 2V square wave input voltage, V_m with a frequency of 0.25Hz. The values of servo gain K_m for a constant input voltage and the square wave voltage of similar magnitude and polarity were then calculated using equations (5) and (6) and found to be the same as shown in Table 1. This holds since the time constant for the servomechanism has been determined as 162ms from Figure 4, while the period of the square wave voltage is 4 seconds consisting of 2 seconds for 'on' and 2 seconds off, which was long enough for the servo to achieve a steady-state. Therefore, the square wave and the constant input affects the servo in the same way.
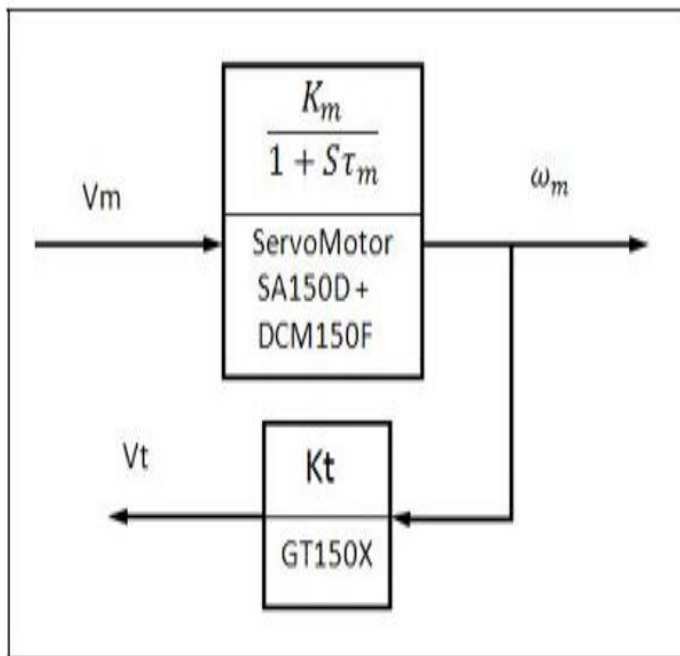

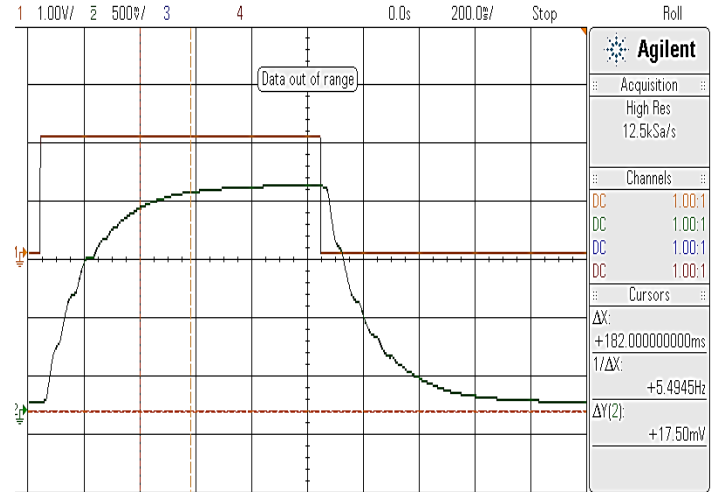**Figure 3:** SA150D+DCM150F+GT150X speed openloop block diagram [1]

Figure 4: The speed output response of the servomechanism

The servomotor angular velocity ω (RPM) versus input voltage V_m (V) plot for forward and reverse rotation is shown in Figures 5 and 6 respectively.
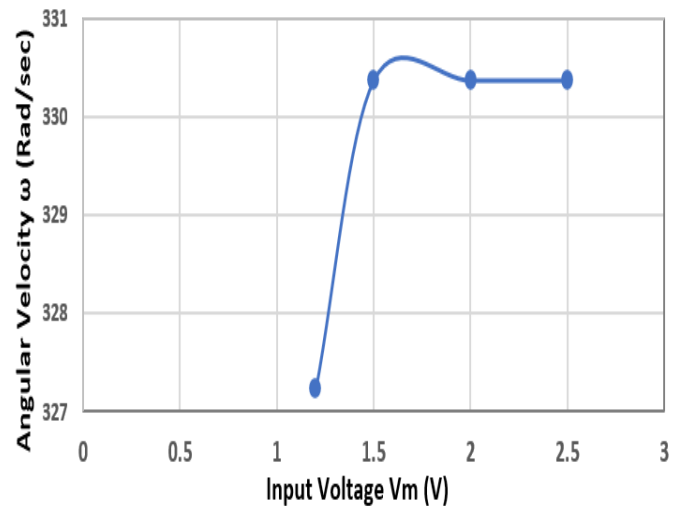

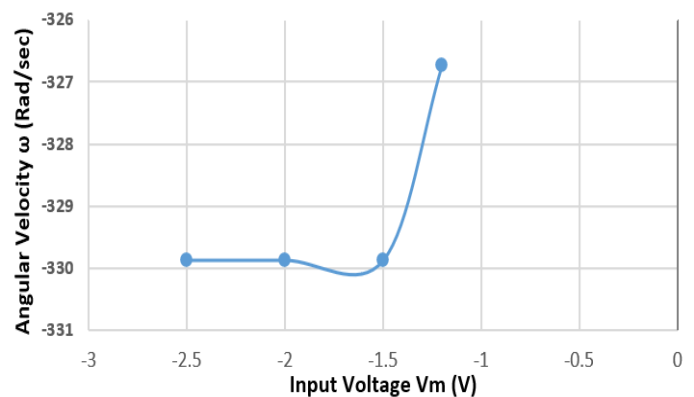**Figure 5:** Angular velocity versus input voltage for forward rotation

Figure 6: Angular velocity versus input voltage for reverse rotation

Table 1: Summarises the Experimental Results for the Servomotor

SA150D	Servomotor Input Voltage $V_m (V)$	Tachometer speed (n) (RPM)	Angular velocity $\omega = \dfrac{2\pi n}{60}$ (rad/sec)	Tacho-meter Voltage $V_t (V)$	Tacho-meter Sensitivity $K_t = (Vt/\omega)$	Servo Gain $K_m = \omega / Vm$	Servo Gain $K_m = V_t/(V_m * K_t)$
T#1	1.200	3120	326.725	8.618	0.026	272.270	272.270
T#1	1.500	3150	329.867	8.643	0.026	219.911	219.911
T#1	2.000	3150	329.867	8.668	0.026	164.933	164.933
T#1	2.500	3150	329.867	8.687	0.026	131.946	131.946
T#2	-1.200	-3130	-327.772	-8.828	0.026	273.144	273.144
T#2	-1.500	-3150	-329.867	-8.830	0.026	219.911	219.911
T#2	-2.000	-3150	-329.867	-8.831	0.026	164.933	164.933
T#2	-2.500	-3150	-329.867	-8.832	0.026	131.946	131.946

It is important to note that the sensitivity of the tachometer kt did not affect Km since the system was implemented in an open loop. Mathematically, Vt and kt cancel out thus it does not affect the open-loop gain as the open-loop servomechanism system is not sensitive to the feedback loop. This confirms that Km is constant for a specific voltage magnitude.

Thus, the speed open loop T.F of the Servo motor is determined using equation (4) as follows:

$$G(s) = \frac{272.27}{0.162s + 1} \tag{27}$$

Also, the open-loop position transfer function is:

$$T(s) = G(s) * H(s) \tag{28}$$

Therefore, the open-loop position transfer function is determined from equation (28):

$$T(s) = 272.27 * \frac{6.61}{(0.162s + 1)s} \tag{28}$$

And the closed-loop position T.F:

$$T(s) = \frac{1800}{0.162s^2 + 273.27s + 1800} \tag{29}$$

3.2 Simulation and Validation

The Jellyfish, Particle swarm and Constraint Optimisation algorithms have been compared via simulation in MATLAB concerning accurate estimation of servomechanism parameters (R, L, J, B, and K) from experimental data. The optimisation algorithms were all used to minimise the modified RMSE fitness function in (26). The JFO had the second-best performance score of 0.0083 which was only second to the CO which had 0.00706 as shown in Figure 7, and the PSO had the least performance score of 1.9035. The parameters obtained with the optimisation algorithms and transient response performances are shown summarised in Table 2. Furthermore, as shown in 8, the time response of the JFO and CO closely matched the experimental data accordingly with the PSO exhibiting overshoot.
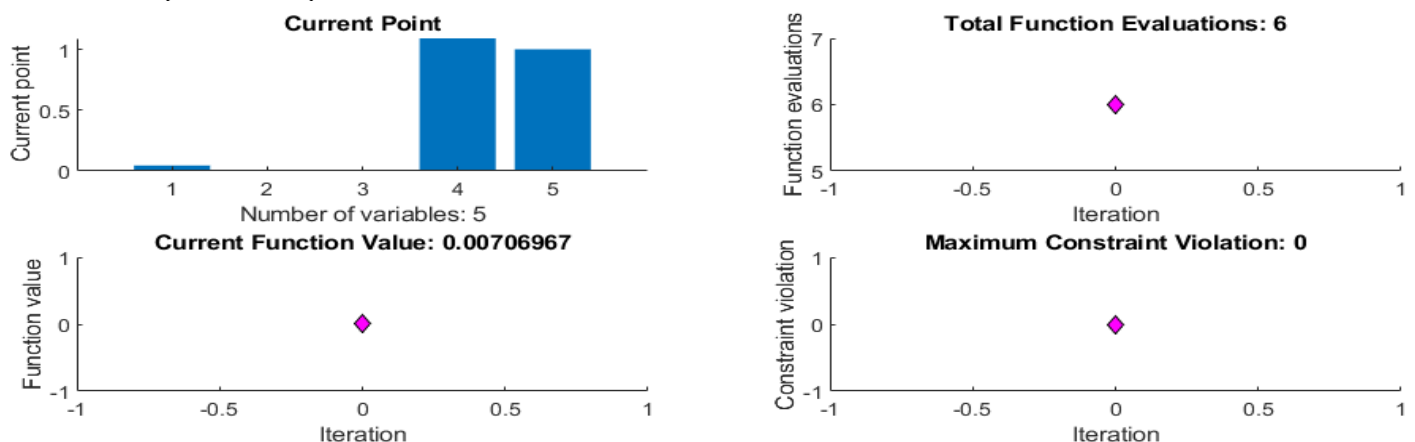

Figure 7: Closed-loop position step response with a velocity feedback loop with the CO method

Table 2: Estimated Servo Motor parameters and Performance index

Motor Parameters Value	JFO	PSO	CO
Mechanical damping (Friction) factor 'B' in *Nms*	0.087103	0.01	45e-3
Moment of inertia for motor rotor 'J' in *Kg.m2*	7.61e-5	0.01	4e-3
Inductance of the armature 'L' in *H*	0.0085	0.01	0.005
Resistance of the armature 'R' in Ω	8.4893	0.252	1.09
Back E.M.F constant 'K' in *Nm/A*	3.5909	0.01	1
RMSE Fitness Score	0.0083	1.9035	0.00706
Settling time (seconds)	0.59	14.9	8.56
Rise time (Seconds)	0.33	8.34	0.62
Percentage Overshoot (%)	-	-	48.9
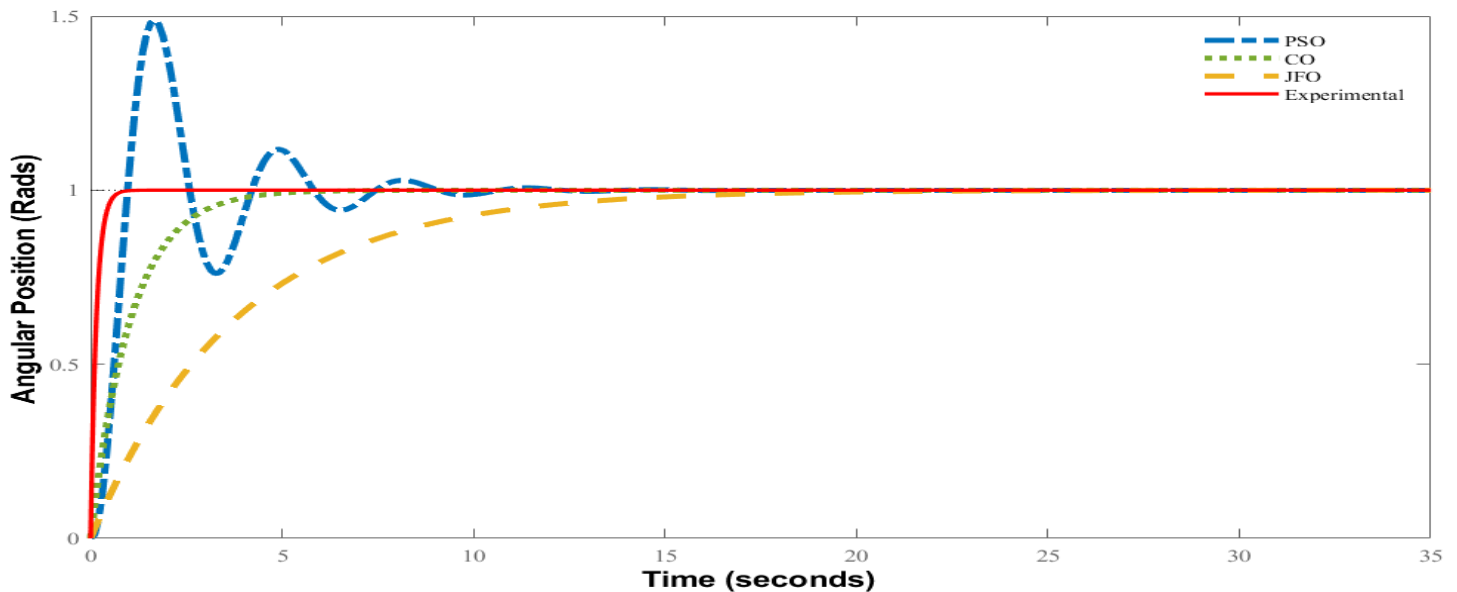
Figure 8: Validation of Angular Position Step response of Servomechanism identified with PSO, JFO, and CO methods

4.0   CONCLUSION

The paper compared the effectiveness of the PSO, CO and JFO algorithms in identifying the model parameters of a servomechanism using the root mean squared error fitness function. Firstly, the position closed-loop transfer function of the servomechanism is determined experimentally from the closed-loop speed step response.

Thereafter, the equivalent state-space model parameters of the servomechanism were identified based on a recursive minimization of a performance fitness function based on the root mean squared error between the model and the experimental position closed-loop step response. The CO algorithm resulted in the best performance with a fitness score of 0.00706 which was closely followed by the JFO which had a score of 0.0083, while the PSO had 1.9035 which was the worst score. Thus, the consequences, the angular position control step response of the identified state-space model parameters using the CO and JFO algorithms showed excellent agreement with the experimental position closed-loop step response accordingly compared to the response obtained using PSO.

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