Nigerian Journal of Technology
Vol. 40, No. 2, 2021, 2021, pp. 199–209.
www.nijotech.com

Print ISSN: 0331-8443
Electronic ISSN: 2467-8821
http://dx.doi.org/10.4314/njt.v40i2.5

Use of Higher Order Plate Theory in Dynamic Analysis of SSFS and CSFS Thick Rectangular Plates in Orthogonal Polynomials

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Abstract

This study applied polynomial expressions as displacement and shear deformation functions in the free-vibration study of thick and moderately thick isotropic rectangular plates. Rectangular plates with two different edge conditions investigated in this work are: one with simple supports at three of its edges and with no support at the other edge denoted with the acronym (SSFS) and a rectangular plate with simple supports at opposite edges while the other opposite edges has a fixed support at one edge and no support at the other edge, this is denoted with the acronym (CSFS). The total potential energy of the plate was derived using the general theory of elasticity. The general governing equation of the plate was derived by minimizing the total potential energy equation of the plate. Edge conditions of the SSFS and CSFS plates were met and substituted into the general governing equation to obtain a linear expression which was solved to generate fundamental natural frequency function for the plates with various span-depth proportion (m/t) and planar dimensions proportion (n/m). The results obtained from this research were found to agree favourably with the results of similar problems in the literature upon comparison.

Keywords: thick Plates, natural frequency, shear deformation, polynomial displacement expression, shear deformation expression, span-depth ratio

1. INTRODUCTION

There are several areas of engineering where structural plates are used. They include; bridge decks, jetties, cylindrical tanks, shear walls, retaining walls, formwork panels, sea and ocean vessels, etc., [1, 2]. Structural plates in their service life could be subjected to time-varying loads such as; load due to human beings dancing on the floor of a dancing hall, load applied on the retaining walls of a sea port by the sea waves, etc. These loads could lead to vibration of the structure. The time-varying loads could vibrate at a frequency that is the same or nearly the same in magnitude and direction with one of the natural frequencies of the structure on which the load is applied, at this point, a phenomenon called resonance takes place. Resonance results to very high amplitude vibration than could lead to total collapse of the structure. The frequency at which resonance occurs is generally called fundamental natural frequency or resonant frequency of the plate [3]. In order to forestall collapse of structural plates, there is need to carry out a detailed study on free-vibration of plates in order to estimate the frequencies that could cause resonance [4]. Kirchhoff established the Classical Plate Theory (CPT) which is extensively used in analysis of plates. The CPT offers good results when used to analyze thin plates and inaccurate results when used for thick and moderately thick plates. This is because, CPT does not take into consideration the effects of shear stresses and strains along the plate's depth to be negligible [5]. The shear stresses and strains across the thickness of the plate have substantial effect in thick and moderately thick plates, therefore the CPT is not suitable for analyzing engineering structures that involve the use of thick and moderately thick plates. According to [6], the first work on plates that considered the effect of shear strains and stresses along the plate's depth was done by Stephen Timoshenko. The theory is now widely known as Timoshenko beam theory or First Order Shear Deformation Theory (FSDT). The FSDT assumes that the displacements have a linear variation across plate's thickness and hence, a modification factor in terms of the shear is needed to fulfill the requirements of the general stress – strain relations for the plate [7]. As a result of the mentioned weaknesses in

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the use of (CPT) and (FSDT), it becomes needful to provide more refined theory that will have no need for any modification factor for the shear stresses and strains and in addition provide an expression that truly explains the behavior of the crosswise shear stresses and strains along the plate's depth. These refined theories are generally referred to as 'Higher Order Shear Deformation Theories' (HSDT). These new theories are based on the principle that; the line straight and at right angle to the plate's mid-plane prior to deformation neither remains straight nor at right angle to the plate's mid-plane after deformation [8]. [9] in their works on thick and moderately thick plates, used a trigonometric expression to describe the deformation behavior of the plate structure. [10] did a study on dynamic analysis of thick plates using HSDT deformable Plate theories. [11] studied the behavior of thick plates due to deformation by making use of trigonometric functions to describe the how shear stresses and strains varies within the plate's thickness. [12] applied Mindlin Plate Theory in their work to develop equations for vibration of rectangular thick plates. [13] used trigonometric expressions as displacement fields to explain the behavior of the crosswise shear stresses and strains along the plate's depth and derive accurate characteristic equations for plates of moderate thickness and different edge conditions. [14] derived polynomial and exact expressions as displacement equations for thick plates. [15] used polynomial displacement equations to determine the stability properties of rectangular thick plates with simple supports at all the four edges. [16] used Navier's method to compare the results obtained from plate analysis using Kirchhoff's concept and Ressiner's concept. [17] applied 'Alternative II' approach in stability and dynamic study of isotropic and orthotropic thick plates by making use of polynomial deflection expression. Galerkin-Vlasov approach was used by [18] to carry out bending analysis of rectangular plates that are carrying uniformly distributed loads. [19] used equilibrium approach to derive differential equations for isotropic thick plates. In the works of [20], Kantorvich approach was applied in the bending analysis of thin rectangular plates. [19] used polynomial expressions as displacement and shear deformation functions to carry out free-vibration study of thick rectangular plate that has simple supports at its four edges. The present work used polynomial expressions to describe the displacement and shear deformation curve to study the dynamic properties of rectangular thick plate with two different edge conditions (SSFS and CSFS) without the need for a modification factor in terms of the stresses and strains.

2. MATHEMATICAL THEORY

In Figs. 1 and 2 are shown two rectangular plates. The plate's dimensions in x and y–axes are denoted with "m" and "n" respectively. The z-direction is in the inward direction of the plane. From the diagram, x ranges from 0 to m, y ranges from 0 to n and z originates from the middle of the plate and increases downwards to a maximum of t/2 and decreases upwards to a minimum of -t/2.

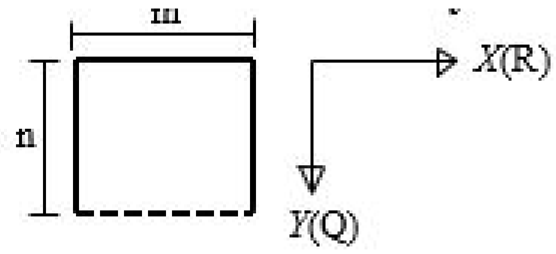

Figure 1: SSFS.

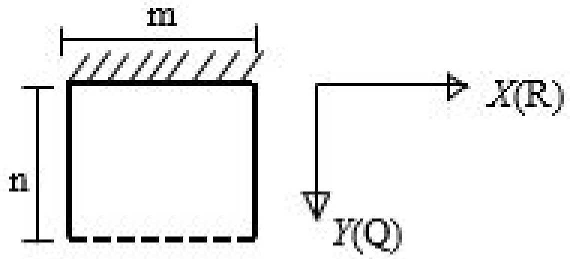

Figure 2: CSFS.

The shear deformation parameter f(z) and the displacement parameters (u, v and w) of this study are polynomial expressions derived in [21] and [14] respectively. They are given as:

$$f(z) = z - \frac{7z^3}{5t^2} \tag{1}$$

$$u(x,y,z) = -z\frac{\partial w}{\partial x} + f(z)\cdot \varnothing_x \tag{2}$$

$$u(x,y,z) = -z\frac{\partial w}{\partial x} + f(z)\cdot \varnothing_y \tag{3}$$

$$w = w_x \cdot w_y = (m_0 + m_1 R + m_2 R^2 + m_3 R^3 + m_4 R^4) \times (n_0 + n_1 Q + n_2 Q^2 + n_3 Q^3 + n_4 Q^4 + n_5 Q^5) \tag{4a}$$

Where u,v and w are the planar displacements and the out-of-plane displacement respectively. $f(z)$ which is the shear deformation expression, explains how the transverse shear stresses are distribution along the plate's thickness. $\varnothing_x$ and $\varnothing_y$ are the shear rotations in x and y-axis respectively and were derived by [16] as:

$$\varnothing_x = C_x \frac{\partial w}{\partial x},\ \varnothing_y = C_y \frac{\partial w}{\partial y} \tag{4b}$$

C_x, C_y, m_i and n_i are constants whose values are subject to the edge conditions of the plate, R and Q are non-dimensional variables in x and y-axis correspondingly, they are presented as:

$$x = mR, y = nQ \tag{4c}$$

Such that; $0 \le x \le m, 0 \le R \le 1, 0 \le y \le n, 0 \le Q \le 1$

In Elastic theory, strains are related to stresses as shown in the following equations:

$$\varepsilon_x = \frac{du}{dx} = -z\frac{\partial^2 w}{\partial x^2} + f(z)\cdot\frac{\partial \varnothing_x}{\partial x} \quad (5)$$

$$\varepsilon_y = \frac{dv}{dy} = -z\frac{\partial^2 w}{\partial y^2} + f(z)\cdot\frac{\partial \varnothing_x}{\partial x} \quad (6)$$

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx} = -2z\frac{\partial^2 w}{\partial x \partial y} + f(z)\cdot\frac{\partial \varnothing_x}{\partial y} + f(z)\cdot\frac{\partial \varnothing_y}{\partial x} \quad (7)$$

$$\gamma_{xz} = \frac{du}{dz} + \frac{dw}{dx} = f(z)\cdot\frac{\partial \varnothing_x}{\partial z} \quad (8)$$

$$\gamma_{yz} = \frac{dv}{dz} + \frac{dw}{dy} = f(z)\cdot\frac{\partial \varnothing_y}{\partial z} \quad (9)$$

In Elastic theory, the constitutive equations for a plate are presented as follows:

$$\sigma_x = \frac{E}{1-\mu^2}[\varepsilon_x + \mu\varepsilon_y] \quad (10)$$

$$\sigma_y = \frac{E}{1-\mu^2}[\mu\varepsilon_x + \varepsilon_y] \quad (11)$$

$$\tau_{xy} = \frac{E(1-\mu)}{(1-\mu^2)}\gamma_{xy} \quad (12)$$

$$\tau_{xz} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{xz} \quad (13)$$

$$\tau_{yz} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{yz} \quad (14)$$

where E and μ are the Elastic modulus and the Poisson's ratio of the plate correspondingly.

2.1. Strain Energy 'S'

The Strain energy deposited in plate's continuum is expressed as:

$$S = \frac{1}{2}\gamma_{yz}\int_x\int_y \left[\int_{-\frac{t}{2}}^{\frac{t}{2}} \left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right)dz\right]dxdy \quad (15)$$

Let S_1 represent the sum of the components of the strain energy of the plate.

$$S_1 = \sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz} \quad (16)$$

Substituting Eq. 5 to Eq. 13 into Eq. 16 gives;

$$S_1 = \frac{E}{1-\mu^2}\left[\left[z^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2zf(z)\cdot\frac{\partial \varnothing_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + f^2(z)\cdot\left(\frac{\partial \varnothing_x}{\partial x}\right)^2\right] + \left[2z^2\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - 2zf(z)\cdot\frac{\partial \varnothing_x}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - 2zf(z)\cdot\frac{\partial \varnothing_y}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right] + \left[z^2\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2zf(z)\cdot\frac{\partial \varnothing_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + f^2(z)\left(\frac{\partial \varnothing_y}{\partial y}\right)^2\right] + (1+\mu)\left[f^2\cdot\frac{\partial \varnothing_y}{\partial x}\frac{\partial \varnothing_x}{\partial y}\right] + \frac{(1-\mu)}{2}\left[f^2(z)\left(\frac{\partial \varnothing_x}{\partial y}\right)^2 + f^2(z)\left(\frac{\partial \varnothing_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2}\cdot\left(\frac{df(z)}{dz}\right)^2\cdot\left[\varnothing_x^2 + \varnothing_y^2\right]\right] \quad (17)$$

$$\text{Let; } \beta = \int_{-\frac{t}{2}}^{\frac{t}{2}} z^2 dz = \frac{t^3}{12},\ \beta A_1 = \int_{-\frac{t}{2}}^{\frac{t}{2}} z^2 dz,$$
$$\beta A_2 = \int_{-\frac{t}{2}}^{\frac{t}{2}} [zf(z)]dz,\ \beta A_3 = \int_{-\frac{t}{2}}^{\frac{t}{2}} [(f(z))^2]dz,$$
$$\beta\frac{\propto^2}{m^2}A_4 = \int_{-\frac{t}{2}}^{\frac{t}{2}} \left[\frac{df(z)}{dz}\right]^2 dz,$$
$$D = \frac{\beta E}{1-\mu^2} = \frac{Et^3}{2(1-\mu^2)} \quad (18)$$

Thus, from Eqs. (1) and (18), Eqs. (19a) – (19d) are derived.

$$\int_{-\frac{t}{2}}^{\frac{t}{2}} (z^2)dz = \left[\frac{z^3}{3}\right]_{-\frac{t}{2}}^{\frac{t}{2}} = \left(\frac{1}{3}\right)\left[\frac{t^3}{8} - \frac{t^3}{8}\right] = 2\left(\frac{1}{3}\right)\cdot\left(\frac{t^3}{8}\right) = \frac{t^3}{12} \quad (19a)$$

$$(f(z))^2 = z^2 - \frac{14z^4}{5t^2} + \frac{49z^6}{25t^4},$$
$$\int_{-\frac{t}{2}}^{\frac{t}{2}} (f(z))^2 dz = \left[\frac{z^3}{3} - \frac{14z^5}{3} + \frac{7z^7}{25h^4}\right]_{-\frac{t}{2}}^{\frac{t}{2}} = \frac{253t^3}{4800}\Bigg)\cdot\left(\frac{t^3}{8}\right) = \frac{t^3}{12} \quad (19b)$$

$$zf(z) = z^2 - \frac{7z^4}{5t^2},\ \int_{-\frac{t}{2}}^{\frac{t}{2}} zf(z)dz = \frac{t^3}{12} - \frac{7z^3}{400} = \frac{79t^3}{1200} \quad (19c)$$

$$\left(\frac{df(z)}{dz}\right)^2 = 1 - \frac{42z^2}{5t^2} + \frac{441z^4}{25t^4},$$

$$\int_{-\frac{t}{2}}^{\frac{t}{2}} \left(\frac{df(z)}{dz}\right)^2 dz$$

$$= \left[z - \frac{14z^3}{5t^2} + \frac{441z^5}{125t^4}\right]_{-\frac{t}{2}}^{\frac{t}{2}} = \frac{1041t}{2000} \quad (19d)$$

Substituting Eqs. (19a) – (19d) into Eq. (18) yields:

$$A_1 = \frac{\frac{t^3}{12}}{\beta} = \frac{t^3}{12} \times \frac{12}{t^3} = 1,\ A_2 = \frac{\frac{79t^3}{1200}}{\beta} = \frac{79t^3}{1200} \times \frac{12}{t^3} = 0.79 \quad (20a)$$

$$A_3 = \frac{\frac{253t^3}{4800}}{\beta} = \frac{253t^3}{4800} \times \frac{12}{t^3} = 0.6325, \quad (20b)$$

$$\frac{\propto^2}{m^2} A_4 = \frac{\frac{1041t}{2000}}{\beta} = \frac{1041t}{2000} \times \frac{12}{t^3} = \frac{6.246}{t^2},$$

$$A_4 = \frac{6.246}{t^2} \times \frac{m^2}{\propto^2} \times \frac{m^2}{1} \times \frac{t^2}{m^2} = 6.246 \quad (20c)$$

D is the plate's flexural rigidity, and $\propto = m/t$ is the span-thickness proportion.

Substituting Eq. (18) into Eq. (17) and integrating yields:

$$\int_{-\frac{t}{2}}^{\frac{t}{2}} (S_1)dz = \frac{E\beta}{1-\mu^2}\left[\left[A_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2A_2 \cdot \frac{\partial \varnothing_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + A_3 \cdot \left(\frac{\partial \varnothing_x}{\partial x}\right)^2\right] + \left[2A_1\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - 2A_2 \cdot \frac{\partial \varnothing_x}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - 2A_2 \cdot \frac{\partial \varnothing_y}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right] + \left[A_1\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2A_2 \cdot \frac{\partial \varnothing_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + A_3\left(\frac{\partial \varnothing_y}{\partial y}\right)^2\right] + (1+\mu)\left[A_3 \cdot \frac{\partial \varnothing_y}{\partial x}\frac{\partial \varnothing_x}{\partial_y}\right] + \frac{(1-\mu)}{2}\left[A_3\left(\frac{\partial \varnothing_x}{\partial y}\right)^2 + A_3\left(\frac{\partial \varnothing_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2} \cdot \frac{\propto^2}{m^2} A_4\left[\varnothing_x^2 + \varnothing_y^2\right]\right] \quad (21)$$

Substituting Eq. (21) into Eq. (15) yields:

$$S = \frac{1}{2}\int_x\int_y \left[\int_{-\frac{t}{2}}^{\frac{t}{2}} (S_1)dz\right] dxdy$$

$$= \frac{D}{2}\int_x\int_y \left[\left[A_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2A_2 \cdot \frac{\partial \varnothing_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + A_3 \cdot \left(\frac{\partial \varnothing_x}{\partial x}\right)^2\right] + \left[2A_1\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - 2A_2 \cdot \frac{\partial \varnothing_x}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - 2A_2 \cdot \frac{\partial \varnothing_y}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right] + \left[A_1\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2A_2 \cdot \frac{\partial \varnothing_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + A_3\left(\frac{\partial \varnothing_y}{\partial y}\right)^2\right] + (1+\mu)\left[A_3 \cdot \frac{\partial \varnothing_y}{\partial x}\frac{\partial \varnothing_x}{\partial_y}\right] + \frac{(1-\mu)}{2}\left[A_3\left(\frac{\partial \varnothing_x}{\partial y}\right)^2 + A_3\left(\frac{\partial \varnothing_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2} \cdot \frac{\propto^2}{m^2} A_4[\varnothing_x^2 + \varnothing_y^2]\right] dxdy \quad (22)$$

The average total work done by agitation on the plate is as shown in . (23).

$$T_w = -\frac{a}{2} \cdot \lambda^2 \int_s\int_y (w^2)\partial x \partial y \quad (23)$$

Where: a is the mass of the plate, and λ is the frequency of the cyclic motion.

2.2. Total Potential Energy 'T_P' of the Plate

This is given as the sum of strain energy, U and the external work, T_w.

$$T_p = S + T_w = \frac{1}{2}\int_x\int_y \left[\left[A_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2A_2 \cdot \frac{\partial \varnothing_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + A_3 \cdot \left(\frac{\partial \varnothing_x}{\partial x}\right)^2\right] + \left[2A_1\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - 2A_2 \cdot \frac{\partial \varnothing_x}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - 2A_2 \cdot \frac{\partial \varnothing_y}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right] + \left[A_1\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2A_2 \cdot \frac{\partial \varnothing_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + A_3\left(\frac{\partial \varnothing_y}{\partial y}\right)^2\right] + (1+\mu)\left[A_3 \cdot \frac{\partial \varnothing_y}{\partial x}\frac{\partial \varnothing_x}{\partial_y}\right] + \frac{(1-\mu)}{2}\left[A_3\left(\frac{\partial \varnothing_x}{\partial y}\right)^2 + A_3\left(\frac{\partial \varnothing_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2} \cdot \frac{\propto^2}{m^2} A_4[\varnothing_x^2 + \varnothing_y^2]\right] dxdy - \frac{a}{2} \cdot \lambda^2 \int_x\int_y (w^2)\partial x \partial y \quad (24)$$

Let the out-of-plane displacement w be defined as:

$$w = M_1 h \quad (25a)$$

Substituting Eq. (25a) into Eq. (4b) gives;

$$\varnothing_x = C_a \cdot M_1 \frac{\partial h}{\partial x} = M_2 \frac{\partial h}{\partial x},\ \varnothing_y = C_b \cdot M_1 \frac{\partial h}{\partial y} = M_3 \frac{\partial h}{\partial y} \tag{25b}$$

Where $M_2 = C_a \cdot M_1, M_3 = C_b \cdot M_1, M_1, M_2, M_3$ are constants and 'h' represents the deflection expression for the plate.

Substituting Eq. (25a) and (25b) into Eq. (24), multiplying each term in the resulting equation by $\frac{m^4}{m^4}$ yields:

$$T_p = \frac{Dmn}{2m^4} \int_0^1 \int_0^1 [(M_1^2 A_1 - 2M_1 M_2 A_2 + M_2^2 A_3) \left(\frac{d^2h}{dR^2}\right)^2 + (M_1^2 A_1 - 2M_1 M_3 A_2 + M_3^2 A_3) \frac{1}{p^4} \left(\frac{d^2h}{dQ^2}\right)^2 + (2M_1^2 A_1 - 2M_1 M_2 A_2 - 2M_1 M_3 A_2) \cdot \frac{1}{p^2} \left(\frac{d^2h}{dR^2} \cdot \frac{d^2h}{dQ^2}\right) + (1+\mu) M_2 M_3 A_3 \cdot \frac{1}{p^2} \left(\frac{d^2h}{dR^2} \cdot \frac{d^2h}{dQ^2}\right) + \left[\left(\frac{1-\mu}{2}\right)(M_2^2 A_3 + M_3^2 A_3)\right] \cdot \frac{1}{p^2} \left(\frac{d^2h}{dR^2} \cdot \frac{d^2h}{dQ^2}\right) + \propto^2 \left(\frac{1-\mu}{2}\right) \cdot (M_2^2 A_4) \left(\frac{dh}{dR}\right)^2 + \frac{\propto^2}{p^2} \left(\frac{1-\mu}{2}\right)(M_3^2 A_4) \left(\frac{dh}{dQ}\right)^2 \Bigg] dR dQ - \frac{mnM_1^2}{2} \int_0^1 \int_0^1 [a\lambda^2 (h^2)] \partial R \partial Q \tag{26}$$

Where 'P' is the planar dimensions proportion presented as; $P = m/n$.

3. GENERAL GOVERNING EQUATIONS

Minimizing the total energy equation yields:

$$\frac{dT_p}{dM_1} = 0,\ \frac{dT_p}{dM_2} = 0,\ \frac{dT_p}{dM_3} = 0 \tag{27}$$

Evaluating Eq. (27) yields;

$$\frac{D}{m^4} \left[\left(B_1 A_1 + \frac{B_3}{P^4} A_1 + \frac{2B_2}{P^2} A_1\right) M_1 + \left(-B_1 A_2 + \frac{B_2}{P^2} A_2\right) M_2 + \left(-\frac{B_3}{P^4} A_2 - \frac{B_2}{P^2} A_2\right) M_3 \right] = (a\lambda^2 B_6) M_1 \tag{28a}$$

$$\frac{D}{m^4} \left[\left(-B_1 A_2 - \frac{B_2}{P^2} A_2\right) M_1 + \left(B_1 A_3 + \left(\frac{1-\mu}{2P^2}\right) B_2 A_3 + \left(\frac{1-\mu}{2}\right) \propto^2 B_4 A_4\right) M_2 + \left(\left(\frac{1+\mu}{2P^2}\right) B_2 A_3\right) M_3 \right] = 0 \tag{28b}$$

$$\frac{D}{m^4} \left[\left(-\frac{B_3}{P^4} A_2 - \frac{B_2}{P^2} A_2\right) M_1 + \left(\left(\frac{1+\mu}{2P^2}\right) B_2 A_3\right) M_2 + \left(\frac{B_3}{P^4} A_3 + \left(\frac{1-\mu}{2P^2}\right) B_2 A_3 + \left(\frac{1-\mu}{2P^2}\right) \propto^2 B_5 A_4\right) M_3 \right] = 0 \tag{28c}$$

Where:

$$B_1 = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial R^2}\right)^2 \partial R \partial Q,$$
$$B_2 = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial R^2} \cdot \frac{\partial^2}{\partial Q^2}\right) \partial R \partial Q,$$
$$B_3 = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial Q^2}\right)^2 \partial R \partial Q,$$
$$B_4 = \int_0^1 \int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 \partial R \partial Q,$$
$$B_5 = \int_0^1 \int_0^1 \left(\frac{\partial h}{\partial Q}\right)^2 \partial R \partial Q,$$
$$B_6 = \int_0^1 \int_0^1 (h)^2 \partial R \partial Q, \tag{29}$$

The matrix form of Eqs. (28a) – (28c) is as shown in Eq. (30a).

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \frac{m^4}{D} \begin{bmatrix} a\lambda^2 B_6 \\ 0 \\ 0 \end{bmatrix} \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \end{Bmatrix} \tag{30a}$$

Eq. (30a) is the general governing Equation.

Where:

$$K_{11} = A_1 \left(B_1 + \frac{2B_2}{P^2} + \frac{B_3}{P^4}\right),\ K_{12} = -A_2 \left(B_1 + \frac{B_2}{P^2}\right),$$
$$K_{13} = -A_2 \left(\frac{B_2}{P^2} + \frac{B_3}{P^4}\right),\ K_{21} = K_{12},$$
$$K_{22} = B_1 A_3 + \left(\frac{1-\mu}{2P^2}\right) B_2 A_3 + \left(\frac{1-\mu}{2}\right) \propto^2 B_4 A_4,$$
$$K_{23} = \left(\frac{1+\mu}{2P^2}\right) B_2 A_3,\ K_{31} = K_{13},\ K_{32} = K_{23},$$
$$K_{33} = \left(\frac{1-\mu}{2P^2}\right) B_2 A_3 + \frac{B_3}{P^4} A_3 + \left(\frac{1-\mu}{2P^2}\right) \propto^2 B_5 A_4 \tag{30b}$$

Eq. (30a) can be rewritten as:

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{bmatrix} \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \frac{am^4\lambda^2}{D} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \end{Bmatrix} \tag{31a}$$

$$\text{Where: } U_{ij} = K_{ij} \times \frac{1}{B_6} \tag{31b}$$

Solving Eq. (31a) using substitution method yields:

$$M_2 = \left[\frac{-U_{23}\cdot U_{31}+U_{33}\cdot U_{21}}{U_{32}^2-U_{33}\cdot U_{22}}\right]M_1,$$
$$M_3 = \left[\frac{-U_{23}\cdot U_{21}+U_{22}\cdot U_{31}}{U_{32}^2-U_{33}\cdot U_{22}}\right]M_1 \qquad (31c)$$

Putting Eq. (31c) into the first equation in the matrix of Eq.. (31a) yields:

$$U_{11}+U_{12}\cdot\left[\frac{-U_{23}\cdot U_{31}+U_{33}\cdot U_{21}}{U_{32}^2-U_{33}\cdot U_{22}}\right] + U_{13}\cdot\left[\frac{-U_{23}\cdot U_{21}+U_{22}\cdot U_{31}}{U_{32}^2-U_{33}\cdot U_{22}}\right] = \frac{am^4\lambda^2}{D} = \Lambda^2 \qquad (31d)$$

Where λ^2 is a non-dimensional natural frequency parameter given as:

$$\Lambda^2 = \frac{am^4\lambda^2}{D} \qquad (31e)$$

$$\Lambda = \sqrt{\Lambda^2}\sqrt{\frac{am^4\lambda^2}{D}} \qquad (31f)$$

3.1. Boundary Conditions

Three major edge conditions were examined. They are; simple support, free support and Clamped support denoted by (S), (F) and (C) respectively. A beam is made up of any two of these support conditions, resulting to three different beams in total. They are shown in Figs. 3, 4 and 5. In the Figure, the range of R is; $0 \le R \le 1$.

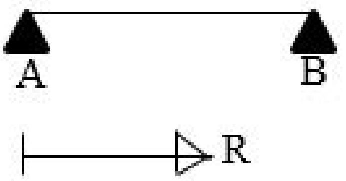

Figure 3: S - S Beam.

Figs. 3, 4 and 5 represent a beam that has simple supports at both ends (S - S beam), a beam whose one edge has simple support and the other edge has no support (S - F beam) and a beam whose one edge is fixed and the other edge has no support (C - F beam) respectively. A rectangular plate is made up of a series beams arranged at right angle to one another. Looking at Eq. (4a), one can observe that it is a product of two beams that are at right angles to each other. The out-of-plane displacement, w for the two independent beams are presented in Eqs. (32a) and (32b).

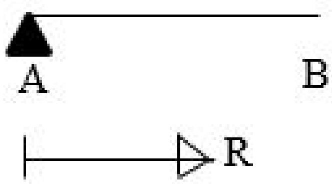

Figure 4: S - F Beam.

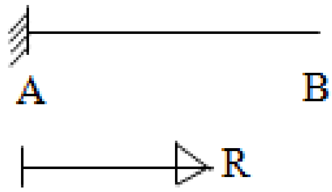

Figure 5: C - F Beam.

$$w_x = (m_0 + m_1R + m_2R^2 + m_3R^3 + m_4R^4) \qquad (32a)$$

$$w_y = (n_0 + n_1Q + n_2Q^2 + n_3Q^3 + n_4Q^4 + n_5Q^5) \qquad (32b)$$

Differentiating Eq. (32a) and (32b) yields:

$$\frac{\partial w_x}{\partial R} = (m_1 + m_2 2R + m_3 3R^2 + m_4 4R^3) \qquad (33a)$$

$$\frac{\partial^2 w_x}{\partial R} = (2m_2 + m_3 6R + m_4 12R^2) \qquad (33b)$$

$$\frac{\partial w_y}{\partial Q} = (n_1 + 2n_2Q + 3n_3Q^2 + 4n_4Q^3 + 5n_5Q^4) \qquad (34a)$$

$$\frac{\partial^2 w_y}{\partial Q^2} = (2n_2 + 6n_3Q + 12n_4Q^2 + 20n_5Q^3) \qquad (34b)$$

$$\frac{\partial^3 w_y}{\partial Q^3} = (6n_3 + 24n_4Q + 60n_5Q^2) \qquad (34c)$$

From the works of [22], when one edge has no support and the other edge has a simple support (S-F beam), the magnitude of the slope at the free edge is equivalent to negative two-third (-2/3) of the deflection coefficient. Also [22] stated that when one edge has no support and the other edge is clamped, (C-F beam), the magnitude of the slope at the free edge is equivalent to negative one-fifth (-1/5) of the deflection coefficient.

3.1.1. *S – S beam*

For the beam whose two edges have simple support, the edge conditions are given as:

$$\text{(i) At R} = 0;\ w_x = 0,\ \text{(ii) At R} = 0;\ \frac{\partial^2 w_x}{\partial R^2} = 0$$
$$\text{(iii) At R} = 1; w_x = 0, \text{(iv) At R} = 1,\ \frac{\partial^2 w_x}{\partial R^2} = 0 \qquad (35)$$

Putting conditions (i), (ii), (iii), and (iv) into Eqs. (32a), (33b) and solving yields:

$$m_0 = 0;\ m_2 = 0,\ m_1 = m_4,\ m_3 = -2m_4 \quad (36)$$

Putting Eqs. (36) into Eq. (32a), yields the deflection for the beam having simple supports at both ends. This is presented in Eq. (37).

$$w_x = m_4(R - 2R^3 + R^4 \quad (37)$$

3.1.2. S − F beam

For the beam whose one edge has simple support and the other edge has no support, presented in Fig. 4, the edge conditions are given as:

$$\text{(i) At Q} = 0;\ w_y = 0,\ \text{(ii) At Q} = 0;\ \frac{\partial^2 w_y}{\partial Q^2} = 0 \quad (38a)$$

$$\text{(iii) At Q} = 1;\ \frac{\partial^2 w_y}{\partial Q^2} = 0,\ \text{(iv) At Q} = 1;\ \frac{\partial^3 w_y}{\partial Q^3} = 0,$$
$$\text{(v) At Q} = 1,\ \frac{\partial w_y}{\partial Q} = -\frac{2n_s}{3}, \quad (38b)$$

Putting conditions (i), (ii), (iii), (iv) and (v) into Eqs. (32b), (34b) and (34c) and solving yields:

$$n_0 = 0;\ n_1 = \frac{-7n_s}{3},\ n_0 = 0,\ n_3 = \frac{10}{3}n_3,\ n_4 = \frac{-10}{3}n_s \quad (39)$$

Substituting Eq. 39 into Eq. 32b yields:

$$w_y = \left(0 - \frac{7n_s}{3}Q + 0 + \frac{10}{3}n_3 Q^3 - \frac{10}{3}n_s Q^4 + n_s Q^5\right) \quad (40)$$

Factorizing Eq. (40) the deflection for the (S-F) beam becomes as presented in Eq. (41).

$$w_y = -n_5\left(\frac{7}{3}Q - \frac{7}{3}Q^3 + \frac{7}{3}Q^4 - Q^5\right) \quad (41)$$

3.1.3. C − F beam

For the beam whose one edge is fixed and the other edge has no support, presented in Fig. 5, the edge conditions are given as:

$$\text{(i) At Q} = 0;\ w_y = 0,\ \text{(ii) At Q} = 0;\ \frac{\partial^2 w_y}{\partial Q} = 0 \quad (42a)$$

$$\text{(iii) At Q} = 1;\ \frac{\partial^2 w_y}{\partial Q^2} = 0,\ \text{(iv) At Q} = 1;\ \frac{\partial^3 w_y}{\partial Q^3} = 0,$$
$$\text{(v) At Q} = 1,\ \frac{\partial w_y}{\partial Q} = -\frac{n_5}{5}, \quad (42b)$$

Substituting conditions (i), (ii), (iii), (iv) and (v) into Eqs. (32b), (34a), (34c) and (33a) and solving yields:

$$n_0 = 0;\ n_1 = 0,\ n_3 = 5.2n_s,\ n_4 = -3.8n_s,\ n_2 = -2.8n_s \quad (43)$$

Substituting Eq. (43) into Eq. (32b) yields:

$$w_y = -n_5(2.8Q^2 - 5.2Q^3 + 3.8Q^4 - Q^5) \quad (44a)$$

Evaluating Eq. (44a) the deflection for the (C-F) beam becomes as shown in

$$w_y = -n_5\left(\frac{14}{5}Q^2 - \frac{26}{5}Q^3 + \frac{19}{5}Q^4 - Q^5\right) \quad (44b)$$

3.2. Free-Vibration Analysis of SSFS Plate

Figure 1 shows SSFS plate which is formed by multiplying one S-S beam in the horizontal direction (x(R) - axis) with one S-F beam in the vertical direction (y(Q) – axis). Hence, the deflection expression becomes the product of Eq. (37) and Eq. (41). This is shown as Eq. (45).

$$w = m_4(R - 2R^3 + R^4)\cdot -n_5\left(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right) \quad (45)$$

Factorizing Eq. (45), Eq. (46) was obtained.

$$w = Bh_1 = B(R - 2R^3 + R^4)\cdot\left(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right) \quad (46)$$

where:

$$h_1 = (R - 2R^3 + R^4)\cdot\left(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right) \quad (47a)$$

$B = -m_4 n_5$ is the amplitude and 'h' represents the deflection function for the SSFS thick plate.

Putting Eq. (47a) into Eq. (29) and solving, yields the stiffness coefficients (Bi) values for the SSFS plate

$$B_1 = 4.025782;\ B_2 = 0.601361;\ B_3 = 0.187453;$$
$$B_4 = 0.407371;\ B_5 = 0.104661;\ B_6 = 0.041270 \quad (47b)$$

Putting the 'Bi' and 'Ai' values of Eqs. (20a), (20b), (20c) and (47b) into Eq. (30b), the K_{ij} values are obtained and shown in Eq. (47c).

$$K_{11} = 5.4160, K_{12} = K_{21} = -3.6554,$$
$$K_{13} = K_{31} = -0.6232, K_{22} = 91.7348,$$
$$K_{23} = K_{32} = 0.2472, K_{33} = 23.1316 \quad (47c)$$

Putting the K_{ij} values into Eq. (31b), the U_{ij} values are generated as given in Eq. (48).

$$U_{11} = 131.2323, U_{12} = U_{21} - 88.5739,$$
$$U_{13} = U_{31} = -15.0997, U_{22} = 2222.7960,$$
$$U_{23} = U_{32} = 5.9907, U_{33} = 560.4951 \quad (48)$$

Putting the U_{ij} values into Eq. (31d), the values of Λ^2 and Λ at $n/m = 1.0$ and $m/t = 10$, are obtained and presented in Eq. (49).

$$\Lambda^2 = 127.3088;\ \Lambda = \sqrt{127.3088} = 11.2831 \quad (49)$$

Table 1 gives values of 'Λ' of the SSFS plate at various values of $P = n/m, m/t$ and at $\mu = 0.3$ derived from the present research. A comparison between the results from this work and the works of [13] at various values of $P, m/t$ and at $\mu = 0.3$ are presented on Table 2.

3.3. Free-Vibration Analysis of CSFS Plate

Figure 2 shows CSFS plate which is formed by multiplying one S-S beam in the horizontal direction (x(R) - axis) with one C-F beam in the vertical direction (y(Q) – axis). Hence, the deflection expression becomes the product of Eq. (37) and Eq. (44b) and is presented as Eq. (50).

$$w = m_4(R - 2R^3 + R^4) \cdot -n_s\left(\frac{14}{5}Q^2 - \frac{26}{5}Q^3 + \frac{19}{5}Q^4 - Q^5\right) \quad (50)$$

Factorizing Eq. (50), Eq. (51) is obtained.

$$w = Bh_2 = B(R - 2R^3 + R^4) \cdot \left(\frac{14}{5}Q^2 - \frac{26}{5}Q^3 + \frac{19}{5}Q^4 - Q^5\right) \quad (51)$$

where

$$h_2 = (R - 2R^3 + R^4) \cdot \left(\frac{14}{5}Q^2 - \frac{26}{5}Q^3 + \frac{19}{5}Q^4 - Q^5\right) \quad (52a)$$

$B = -m_4 n_5$ is the amplitude and 'h' represents the deflection expression for the CSFS thick plate.

Substituting Eq. (52a) into Eq. (29) and evaluating, the B_i values for the CSFS plate are derived and presented as Eq. (52b).

$$B_1 = 0.328478;\ B_2 = 0.053043;\ B_3 = 0.128668;$$
$$B_4 = 0.033239;\ B_5 = 0.009310;\ B_6 = 0.003367 \quad (52b)$$

Substituting the A_i and B_i values of Eqs. (20a), (20b), (20c) and (52b) into Eq. (30b), the values of A_{ij} are obtained and presented as Eq.(52c).;

$$K_{11} = 0.5632,\ K_{12} = K_{21} - 0.3014,$$
$$K_{13} = K_{31} = -0.1436,\ K_{22} = 7.4859,$$
$$K_{23} = K_{32} = 0.0218,\ K_{33} = 2.1284 \quad (52c)$$

Substituting the K_{ij} values into Eq. (31b), the values of U_{ij} are obtained and presented in Eq. (52d).

$$U_{11} = 167.2801,\ U_{12} = U_{21} - 89.5164,$$
$$U_{13} = U_31 = -42.6349,\ U_{22} = 2223.3090,$$
$$U_{23} = U_{32} = 6.4768,\ U_{33} = 632.1307 \quad (52d)$$

Substituting the values of U_{ij} into Eq. (31d), the value of the non-dimensional fundamental natural frequency parameters Λ and Λ at $n/m = 1.0$ and $m/t = 10$ are obtained and presented as Eq. (53).

$$\Lambda^2 = 160.8353;\ \Lambda = \sqrt{160.8353} = 12.6821 \quad (53)$$

The value of 'Λ' for the CSFS plate at various values of $P, a/t$ and at $\mu = 0.3$ obtained from this work were compared with the results of [13] in Table 4. Presented on Table 3 are the values of value of 'Λ' for the CSFS plate at various values of $P, a/t$ and at $\mu = 0.3$ obtained from this work.

4. RESULTS AND DISCUSSION

From Table 1, it is inferred that, at a constant value of planar dimension ratio ($P = n/m$), there is an increase in the values of Λ when (m/t) increases. The implication of this is that the impact created by vibratory load on the plate rises as $\propto (= m/t)$ rises. Also, at a given $\propto (= m/t)$, there is a decrease in the values of Λ as $P (= n/m)$ increases having its highest value at $P = 1$ (Square plates). This implies that the ability of the plate to withstand vibration decreases as $P (= n/m)$ increases, with the square plate having the highest capacity to resist vibration.

A close look at Table 2 reveals that upon comparison of the results from this work with the works of [13] for SSFS plates, the percentage difference ranges from 0.05 to 1.93. These differences are quite negligible and thus, the results obtained are of high accuracy. This indicates that the present study is an efficient and a reliable technique for free-vibration study of SSFS thick plates.

From Table 3, it is inferred that, at a constant value of planar dimension ratio $P (= n/m)$, there is an increase in the values of Λ when $\propto (= m/t)$, increases. The implication of this is that the impact created by vibratory load on the plate rises as the span – depth ratio rises. Also, at a given value of $\propto (= m/t)$, there is a decrease in the values of Λ as $P (= n/m)$ increases, its highest value occurring at $P = 1.0$ (Square plates). This implies that the ability of the plate to withstand vibration decrease as $P (= n/m)$ increases, with the square plate having the highest capacity to resist vibration.

A close look at Table 4 reveals that upon comparison of the results from this work with the works of [13] for CSFS plates, the percentage difference ranges from 0.09 to 5.16. These differences are quite negligible and thus, the results obtained are of high accuracy. This indicates that the present study is an efficient and a reliable method for free-vibration study of the SSFS and CSFS thick plates.

Table 1: Non–dimensional natural frequencies of SSFS thick plate.

$\propto$ = **m/t**	**n/m = 1.0**	**n/m = 1.2**	**n/m = 1.5**	**n/m = 1.6**	**n/m = 1.8**	**n/m = 2.0**	**n/m = 2.2**	**n/m = 2.5**
				$\lambda = \frac{\Lambda}{m^2}\sqrt{\frac{D}{a}}$ Λ				
5	10.8099	10.3513	9.9860	9.9087	9.7918	9.7090	9.6481	9.5831
10	11.2831	10.7928	10.4031	10.3207	10.1962	10.1080	10.0432	9.9741
15	11.3780	10.8811	10.4864	10.4029	10.2769	10.1876	10.1220	10.0520
20	11.4118	10.9125	10.5160	10.4322	10.3056	10.2159	10.1500	10.0797
25	11.4275	10.9272	10.5298	10.4458	10.3190	10.2291	10.1631	10.0926
30	11.4361	10.9351	10.5373	10.4532	10.3262	10.2363	10.1702	10.0996
40	11.4446	10.9431	10.5448	10.4606	10.3335	10.2434	10.1773	10.1066
50	11.4486	10.9468	10.5483	10.4641	10.3369	10.2468	10.1805	10.1099
60	11.4508	10.9488	10.5502	10.4659	10.3387	10.2486	10.1823	10.1116
70	11.4521	10.9500	10.5513	10.4670	10.3398	10.2496	10.1834	10.1127
80	11.4529	10.9508	10.5521	10.4678	10.3405	10.2503	10.1841	10.1134
90	11.4535	10.9513	10.5526	10.4683	10.3410	10.2508	10.1846	10.1139
100	11.4539	10.9517	10.5529	10.4686	10.3414	10.2512	10.1849	10.1142

Table 2: Comparison of the Present Study's results with the results of [13] for SSFS Thick Plates.

$\frac{n}{m}$	$\frac{m}{t}$	$\lambda = \frac{\Lambda}{m^2}\sqrt{\frac{D}{a}}$ Λ **Present Study (P.S)**	[13]	% **DIfference** $\frac{(P.S - H.A.)\times 100}{P.S}$
	100	11.4539	11.6746	-1.93
	20	11.4118	11.5877	-1.54
1	10	11.2831	11.3810	-0.87
	6.67	11.0786	11.0843	-0.05
	5	10.8099	10.7218	0.81
	100	10.5529	10.6655	-1.07
	20	10.5160	10.6028	-0.83
1.5	10	10.4031	10.4404	, -0.36
	6.67	10.2232	10.1988	0.24
	5	9.9860	9.8972	0.89
	100	10.2512	10.2948	-0.43
	20	10.2159	10.2402	-0.24
2	10	10.1080	10.0929	0.15
	6.67	9.9360	9.8705	0.66
	5	9.7090	9.5902	1.22
	100	10.1142	10.1222	-0.08
	20	10.0797	10.0713	0.08
2.5	10	9.9741	9.9310	0.43
	6.67	9.8056	9.7173	0.90
	5	9.5831	9.4470	1.42

Table 3: Non–dimensional natural frequencies of CSFS thick plate.

$\propto$ = **m/t**	**n/m = 1.0**	**n/m = 1.2**	**n/m = 1.5**	**n/m = 1.6**	**n/m = 1.8**	**n/m = 2.0**	**n/m = 2.2**	**n/m = 2.5**
				$\lambda = \frac{\Lambda}{m^2}\sqrt{\frac{D}{a}}$ Λ				
5	12.0129	11.0190	10.3011	10.1616	9.9610	9.8273	9.7340	9.6394
10	12.6821	11.5473	10.7502	10.5976	10.3799	10.2357	10.1355	10.0343
15	12.8199	11.6540	10.8401	10.6849	10.4636	10.3172	10.2155	10.1130
20	12.8693	11.6921	10.8722	10.7159	10.4933	10.3462	10.2440	10.1410
25	12.8923	11.7099	10.8871	10.7304	10.5072	10.3597	10.2573	10.1540
30	12.9049	11.7196	10.8952	10.7383	10.5148	10.3670	10.2645	10.1611
40	12.9175	11.7292	10.9033	10.7461	10.5223	10.3744	10.2717	10.1682
50	12.9233	11.7337	10.9071	10.7498	10.5258	10.3778	10.2751	10.1715
60	12.9265	11.7362	10.9091	10.7517	10.5277	10.3796	10.2769	10.1732
70	12.9284	11.7376	10.9103	10.7529	10.5288	10.3807	10.2780	10.1743
80	12.9296	11.7386	10.9111	10.7537	10.5296	10.3814	10.2787	10.1750
90	12.9305	11.7392	10.9117	10.7542	10.5301	10.3819	10.2792	10.1755
100	12.9311	11.7397	10.9121	10.7546	10.5304	10.3823	10.2795	10.1758

Table 4: Comparison of the Present Study's results with the results of [13] for CSFS Thick Plates.

$\frac{n}{m}$	$\frac{m}{t}$	$\lambda = \frac{\Lambda}{m^2}\sqrt{\frac{D}{a}}$ Λ Present Study (P.S)	[13]	% DIfference $\frac{(P.S-[13])*100}{P.S}$
1	100	12.9311	12.6728	2.00
	20	12.8693	12.5482	2.49
	10	12.6821	12.2606	3.32
	6.6667	12.3889	11.8620	4.25
	5	12.0129	11.3931	5.16
1.5	100	10.9121	10.9682	-0.51
	20	10.8722	10.8951	-0.21
	10	10.7502	10.7099	0.37
	6.6667	10.5559	10.4390	1.11
	5	10.3011	10.1060	1.89
2	100	10.3823	10.4206	-0.37
	20	10.3462	10.3618	-0.15
	10	10.2357	10.2054	0.30
	6.6667	10.0593	9.9712	0.88
	5	9.8273	9.6782	1.52
2.5	100	10.1758	10.1848	-0.09
	20	10.1410	10.1319	0.09
	10	10.0343	9.9871	0.47
	6.6667	9.8638	9.7676	0.98
	5	9.6394	9.4910	1.54

5. CONCLUSION

From the present study, one could conclude that:

(i) The use polynomial expressions as the displacement and shear deformation expressions makes the mathematical formulations easy to manipulate. It also provides for easy satisfaction of the boundary conditions.

(ii) The simple linear equation formulated and applied herein, yields fast and reliable results for free-vibration study of thick SSFS and CSFS plates at various span-thickness ratios and planar dimensions ratios.

(iii) The results gotten from the present work are in close agreement with the results of previous researchers and therefore are very consistent.

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