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OPTIMIZATION TECHNIQUE IN RAINFALL – INTENSITY – DURATION – FREQUENCY MODELLING FOR IKEJA, SOUTH-WEST, NIGERIA

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ABSTRACT
The adequate estimation of rainfall intensity over a particular catchment is a necessary procedure in the design of water resources engineering control structures. To develop the probability and non – probability distribution function models for rainfall intensity – duration – frequency for Ikeja, 25 year daily rainfall data were collected from Nigerian Meteorological Agency (NIMET) Abuja for Ikeja. The annual maximum rainfall amounts with durations of 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300 and 420 minutes were extracted and subjected to frequency analysis using the Excel Optimization Solver wizard. To develop the rainfall intensity, duration and frequency (IDF) models, specific and general IDF models were obtained for return periods of 2, 5, 10, 25, 50 and 100 years using the Gumbel Extreme Value Type - 1, Normal and Log Pearson Type - 3 distributions. The Anderson Darling goodness of fit test was used to ascertain the best fit probability distribution. The R^2 values ranged from 0.992 – 0.993 and the Mean Squared Error, MSE from 26.43 – 115.94 for the Gumbel; 0.992 – 0.993 with MSE of 28.64 – 85.23 for Normal distribution and 0.991 – 0.993 with MSE of 28.24 – 154.85 for Log Pearson Type – 3.The prediction of rainfall intensity with the Probability Distribution Functions showed a good match with observed intensity values. The intensity – duration curve in all cases has a negative slope. The GEVT – 1 and Normal distribution models ranked first while Log-Pearson Type 3 ranked third with respect to R^2 and MSE in the non-specified return period. The probability distribution models are recommended for the prediction of rainfall intensities for Ikeja metropolis.

Keywords: *IDF models, Gumbel Extreme Value Type - 1, Normal, Log Pearson Type - 3 distributions, Excel Optimization Solver, goodness of fit test, Ikeja.*

1. INTRODUCTION

The Rainfall Intensity Duration Frequency (IDF) relationship is one of the most commonly used tools for the design of hydraulic and water resources engineering control structures. Mathematical knowledge could be employed in the development of a relationship between the rainfall intensity, duration and the frequency (return period). The establishment of such relationship was done as early as 1932 [1]. The knowledge of the frequency of extreme events such as floods, droughts, rainstorm and high winds is required in the adequate planning and design for these extreme events [2]. The planning and designing of various water resources projects require the use of IDF relationship [3]. This relationship is determined through frequency analysis of data from meteorological stations. The IDF formulae are the empirical equations representing a relationship among maximum rainfall intensity (as dependent variable) and other parameters of interest, such as rainfall duration and frequency (as independent variables). There are

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several commonly used functions found in the literature of hydrology applications [1]. Owing to its wide applications, accurate estimation of intensity-duration-frequency relationship has received attention from researchers and scientists from all over the world [4]. All functions have been widely applied in hydrology. In Nigeria, a lot of work has been done in South – East and South – South. For instance, the IDF models of [5] in Port Harcourt and that of [6] at Eket in Akwa Ibom State. All these models generated IDF curves that confirm the theory for shorter recurrence periods of 2 to 10 years.

2. MATERIALS AND METHODS

2.1 Study Area

Ikeja is the capital of Lagos State in South – West Nigeria. It covers an estimated area of about 40.60 km^2. It is located at 41m above the sea level and falls within latitude 6.59° N and longitudes 3.34°E (see Figure 1). Ikeja lies in the plane which is developed on rocks of the basement complex found in the Savannah zone. The area is characterized by relatively high temperature with mean annual temperature of 30°C and rainfall of 1,314.4 mm [7].

2.2 Data Collection

The major material used for this work is rainfall data comprising of the amount and duration. A twenty five (25) year rainfall data which included data ranging from 1986 to 2010 were obtained from Nigeria Meteorological Agency (NIMET) office Abuja, Nigeria. The data were sorted and arranged according to years, rainfall intensities and durations. The rainfall intensities selected for the analysis were the maximum values for each year for all the years analysed.

2.3 Data Analysis

The annual maximum rainfall amount was obtained by selecting the maximum amount of rainfall for each year for 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300, and 420 durations (minutes) for the 25 year period. The IDF relation is mathematically expressed as follows:

$$I = f(T,d) \quad (1)$$

Where I = rainfall intensity (mm/hr), T = return period (year) and d = duration (minutes).

The rainfall amount is converted to intensity (mm/hr) by dividing the amount by the duration (minutes) then multiplying by 60 as a conversion factor. For instance, given rainfall amount of 35.9mm for 5 minutes duration yields an intensity of (35.9/5) x 60 = 430.8 mm/hr. Table 1 shows all the intensities for various durations.

Figure 1: Location map of Ikeja in South-Western Nigeria; Source: Google map (2019)

Table 1: Ranked Observed Annual Rainfall Intensities (mm/hr) for different Durations (minutes) for Ikeja

Year	Rainfall intensity (mm/hr)												
	5	10	15	20	30	45	60	90	120	180	240	300	420
1	430.8	280.8	229.2	188.7	134.2	99.7	85.2	65.1	53.7	39.6	33.5	29.3	33.9
2	345.6	271.6	224.8	178.2	133.6	95.1	82.3	64.3	51.8	39.4	32.6	28.1	22.5

Year	Rainfall intensity (mm/hr)												
	5	10	15	20	30	45	60	90	120	180	240	300	420
3	305.8	225.0	207.3	171.9	130.6	90.8	71.3	62.8	48.8	35.8	29.6	26.8	21.7
4	291.6	193.2	187.2	171.1	126.0	89.5	68.1	56.8	48.3	32.5	26.9	23.7	20.9
5	278.4	192.7	158.0	140.4	125.8	89.1	67.1	53.4	42.6	32.2	24.4	21.5	19.1
6	276.4	188.4	152.4	121.4	114.6	86.0	66.8	47.5	40.1	28.4	24.1	20.0	16.9
7	267.5	184.2	150.0	118.5	100.4	84.0	64.5	45.4	38.0	28.1	23.2	19.5	15.9
8	266.0	174.1	147.0	114.3	93.6	83.9	63.0	44.7	36.8	26.7	21.3	19.3	15.3
9	243.1	172.8	132.9	112.5	92.6	70.7	62.9	44.5	35.7	25.4	20.9	18.0	14.4
10	206.4	168.5	128.8	109.7	83.7	66.9	60.3	44.5	35.7	25.3	20.3	17.5	13.9
11	202.8	167.6	128.6	106.1	81.0	65.6	58.3	43.0	33.4	24.5	20.3	17.4	13.9
12	195.6	153.2	127.9	105.6	80.6	64.1	52.7	42.0	33.2	24.4	20.1	17.0	13.9
13	195.2	151.8	125.6	96.6	79.0	63.9	51.0	40.2	32.3	23.8	20.0	16.2	13.8
14	180.0	145.8	122.8	96.5	75.0	62.4	50.8	40.2	32.1	23.8	19.0	16.0	12.9
15	177.4	139.2	116.9	94.2	73.6	61.8	50.2	38.9	32.0	22.3	18.4	15.9	12.7
16	175.4	126.0	115.2	93.6	67.6	61.5	49.2	38.7	31.5	22.3	17.8	15.2	12.2
17	157.0	124.8	101.2	92.1	65.2	56.2	48.1	35.8	30.2	22.3	17.8	15.0	11.6
18	154.8	123.6	97.2	86.4	64.4	52.7	46.8	35.4	30.0	21.5	16.7	14.3	11.4
19	147.6	122.9	93.8	79.2	62.4	50.0	46.4	34.1	29.2	21.0	16.1	14.3	10.9
20	145.7	111.7	92.8	77.4	61.4	47.2	41.3	33.5	26.9	20.1	15.8	13.4	10.2
21	145.6	111.6	89.6	72.9	59.1	45.1	39.5	30.2	25.6	20.0	15.1	12.9	10.2
22	142.1	110.5	85.3	70.4	57.6	45.1	37.5	28.4	25.2	18.4	15.1	12.7	10.2
23	138.1	101.4	84.3	69.6	53.7	44.7	37.2	26.3	23.8	17.9	15.0	12.6	9.5
24	133.2	98.9	83.2	69.6	53.1	44.4	33.8	25.8	23.5	17.9	14.8	12.1	9.3
25	129.6	97.5	82.4	62.4	47.5	41.0	33.8	25.5	21.6	16.8	13.8	12.1	9.2
Mean	181.2	138.3	116.9	96.7	77.9	67.1	57.2	41.9	32.5	24.7	18.8	15.1	11.2
Standard Deviation	59.0	43.2	35.3	32.8	28.5	21.2	15.7	10.2	8.1	6.6	4.9	3.9	2.9
Coefficient of Skewness	1.16	0.54	0.85	0.72	0.62	0.29	0.33	0.43	0.06	2.36	2.36	2.39	2.07

The magnitudes of rainfall intensities were obtained using frequency analysis. Three probability distributions, namely Gumbel Extreme Value Type - 1 (GEVT-1), Normal and Log-Pearson Type - 3 were used to obtain the magnitude of rainfall intensities for different return periods.

2.3.1 Gumbel's Extreme Value Type - 1 Distribution

Gumbel distribution is one commonly used probability distribution for obtaining the rainfall intensity values [5]. The rainfall intensity values were obtained using Equation (2):

$$X_T = \bar{X} + K_T S \qquad (2)$$

Where X_T = rainfall intensity values (magnitude of hydrologic event); $\bar{X}$ = mean; K_T = Gumbel's frequency factor; S = standard deviation

The Gumbel's frequency factor is obtained using Equation (3):

$$K_T = \frac{\sqrt{6}}{\pi}\left\{0.5772 + In\left[In\left(\frac{T}{T-1}\right)\right]\right\} \qquad (3)$$

Where T = return period (years)

For example, Gumbel frequency factor for a 5 year return period is evaluated as:

$$K_T = \frac{\sqrt{6}}{\pi}\left\{0.5772 + In\left[In\left(\frac{5}{5-1}\right)\right]\right\} = 0.719$$

The resulting Gumbel K_T values for different return periods as calculated are shown in Table 2.

2.3.2 Normal Distribution for Ikeja

Normal distribution was applied here for frequency analysis as the probability distribution to fit the data. The rainfall intensity values are computed with Equation (2). The Normal distribution frequency factor is computed using Equation (4):

$$K_{TN} = w = \frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^2} \qquad (4)$$

where w = Intermediate Variable and is given in Equation(5) as:

$$w = \left[In\left(\frac{1}{P^2}\right)\right]^{1/2} \quad (5)$$

and P = exceedance probability given in Equation (6) as:

$$P = \frac{1}{T} \quad (6)$$

where T = return period

Example: Normal distribution frequency factor for a 5 year return period

P = $\frac{1}{5}$ = 0.2, and w = $\left[In\left(\frac{1}{0.2^2}\right)\right]^{1/2}$ = 1.794

Substituting computed w value into Equation (4) yields:

K_{TN} = w - $\frac{2.515517+0.802853(1.794)+0.010328(1.794)^2}{1+1.432788(1.794)+0.189269(1.794)^2+0.001308(1.794)^3}$

K_{TN} = 0.841457

Table 3 shows the calculated K_{TN} values for different return periods.

2.3.3 Log Pearson Type - 3 distribution

"If log x follows a Pearson Type - 3 distribution, then x is said to follow a log-Pearson Type - 3 distribution" [8]. In the United States, this distribution is the standard distribution for frequency analysis of annual maximum floods [1].

2.3.4 Calibration of Sherman (1932) IDF model

According to [1], Sherman's IDF model is given as;

$$I = \frac{CT_r{}^m}{T_d{}^a} \quad (7)$$

Where c, m and a are model parameters.

Equation (7) is non-linear quotient power law that was calibrated for c, m, and a parameters using intensity, duration and return period values in Table 1 and Excel Optimization Solver. The Generalized Reduced Gradient (GRG) solver is an optimization tool embedded in Microsoft excel. It can be used to obtain the optimum values of parameters of linear or nonlinear equations. There are two solver methods namely linear programming solver (LP) for linear equations; GRG and Evolutionary solver for nonlinear Equations [4]. The premium solver [8] has details of optimization algorithms in Microsoft Excel.

2.3.5 Goodness of fit test

The data in Table 1 was subjected to Anderson-Darling test to ascertain the probability distribution that best fit the rainfall annual maximum amount. This is a nonparametric test of the equality of continuous, one dimensional probability distributions that can be used to compare a sample with a reference probability distribution. GEVT-1, Log Pearson Type-3 and Normal distributions best fit the rainfall intensities with significant values of 0.7570, 0.7538 and 0.7115 at 5% confidence level respectively in descending order.

3. RESULTS

3.1 Computation of rainfall intensities

The rainfall intensity values were computed by evaluating Equation (1). The graphical illustration of the procedure is as shown in Figure 2. Rainfall intensity using GEVT-1 distribution with the mean and standard deviation are obtained from Table 1. For a 5 minutes duration and 2 year return period, the probability equivalent of rainfall intensity via GEVT-1 is $X_T = \bar{X} + K_T S$ ➢ X_T = 200.3 + (-0.16425× 147.52)) ➢ X_T = 200.3 − 24.23 ➢ X_T = 176.07 mm/hr. Figure 3 shows rainfall intensity distributions and return periods using GEVT-1 distribution. Figure 4 shows rainfall intensity distributions and return periods using Normal distribution.

Table 2: Gumbel frequency factor for Ikeja IDF modelling

Return Period (year)	2	5	10	25	50	100
K_T values	-0.16425	0.719	1.304	2.044	2.592	3.1363

Table 3: Normal distribution frequency factor

Return Period	2	5	10	25	50	100
P	0.5	0.2	0.1	0.04	0.02	0.01
W	1.17741	1.794123	2.145966	2.537272	2.79715	3.034854
K_{TN} values	-1E-07	0.841457	1.281729	1.751077	2.054189	2.326785

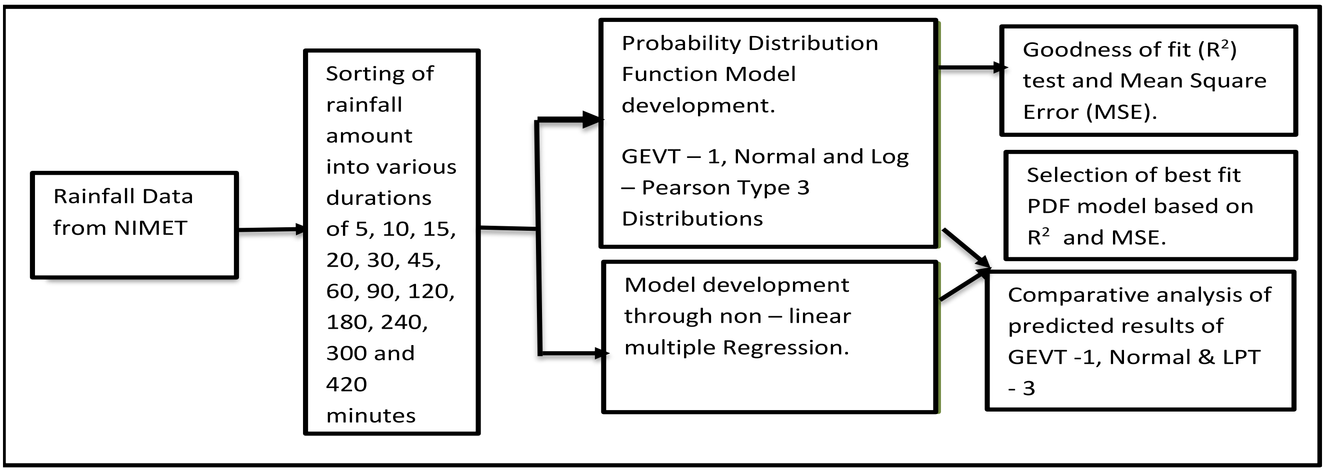

Figure 2: Graphical representation of model development, IDF

3.2 Calibration of Sherman's IDF models:

3.2.1 Specified Return periods

The calibrated Sherman (1932) IDF models for specified return periods are as presented in Table 3. Equally included in the table is coefficient of determination R^2 and mean square error (MSE) for model performance assessment. Similarly, Tables 4 and 5 showcase calibrated IDF models for Normal and Log – Pearson Type – 3 distributions alongside R^2 and MSE values.

3.3.2 Non – Specified return period (General IDF models)

A general IDF model was also developed. A total of 13 durations multiplied by 6 return periods yielded 78 input data points. The entire input data were taken from Table 1. The general IDF model was developed using Excel Optimization Solver. The least squares equations were programmed accordingly and the resulting equation is:

$$I = \frac{551.809 T_r^{\,0.188}}{T_d^{\,0.596}} \tag{8}$$

For Equation (8), the coefficient of determinant (R^2) = 0.990 and Mean Squared Error = 95.27 mm/hr. The plot of the predicted intensity values of Equation (8) is as shown in Figure 5.

Similarly, Equations (9 and 10) show the general IDF models for Normal and Log – Pearson distributions while Figures 6 and 7 show the result in plotted form.

Normal distribution:

$$I = \frac{579.532 T_r^{\,0.130}}{T_d^{\,0.577}} \tag{9}$$

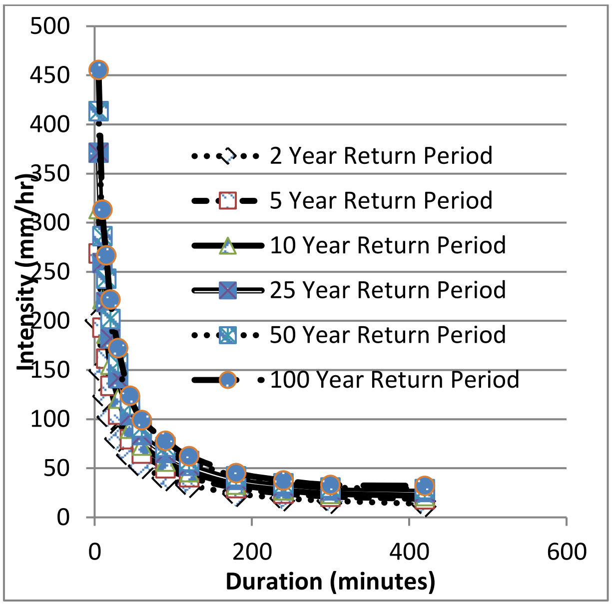

Figure 3: Intensity Duration Frequency (IDF) curves for GEVT - 1 distribution for Ikeja.

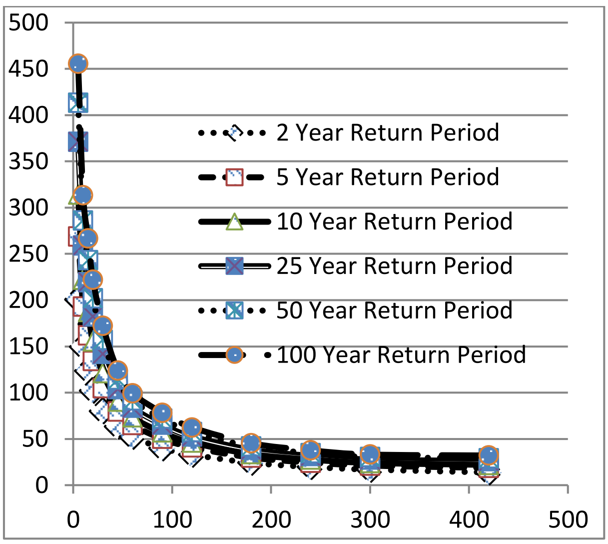

Figure 4: Intensity Duration Frequency (IDF) curves for Normal distribution for Ikeja.

Table 3: GEVT-1 calibrated IDF Models for different return periods for Ikeja.

Return Period	IDF Model	Coefficient of Determination (R^2)	Mean Squared Error (MSE)
2	$I = \frac{4.873T_r^{6.700}}{T_d^{0.550}}$	0.992	26.43
5	$I = \frac{2.240T_r^{3.564}}{T_d^{0.550}}$	0.993	41.12
10	$I = \frac{1.688T_r^{2.686}}{T_d^{0.575}}$	0.993	54.59
25	$I = \frac{1.313T_r^{2.055}}{T_d^{0.583}}$	0.993	75.85
50	$I = \frac{1.193T_r^{1.744}}{T_d^{0.587}}$	0.993	94.68
100	$I = \frac{1.110T_r^{1.519}}{T_d^{0.590}}$	0.993	115.94

±: return period specific IDF models; T_r = return period (year) and T_d = duration (minutes)

Table 4: Normal distribution calibrated IDF Models for different return periods for Ikeja.

Return Period	IDF Model ±	Coefficient of Determination (R^2)	Mean Squared Error (MSE)
2	$I = \frac{4.906T_r^{6.786}}{T_d^{0.554}}$	0.992	28.64
5	$I = \frac{2.245T_r^{3.586}}{T_d^{0.569}}$	0.993	43.68
10	$I = \frac{1.685T_r^{2.684}}{T_d^{0.575}}$	0.993	54.01
25	$I = \frac{1.307T_r^{2.035}}{T_d^{0.580}}$	0.993	66.87
50	$I = \frac{1.189T_r^{1.716}}{T_d^{0.583}}$	0.993	76.19
100	$I = \frac{1.104T_r^{1.487}}{T_d^{0.585}}$	0.993	85.25

± return period specific IDF models

Table 5: IDF Models for different return periods using Log-Pearson distribution for Ikeja

Return Period	IDF Model ±	Coefficient of Determination (R^2)	Mean Squared Error (MSE)
2	$I = \frac{4.858T_r^{6.656}}{T_d^{0.542}}$	0.991	28.24
5	$I = \frac{2.233T_r^{3.550}}{T_d^{0.560}}$	0.992	43.36
10	$I = \frac{1.689T_r^{2.685}}{T_d^{0.573}}$	0.993	56.84
25	$I = \frac{1.316T_r^{2.067}}{T_d^{0.590}}$	0.993	82.17
50	$I = \frac{1.196T_r^{1.764}}{T_d^{0.602}}$	0.992	111.76
100	$I = \frac{1.116T_r^{1.545}}{T_d^{0.614}}$	0.992	154.85

± return period specific IDF models

For Equation (9), the coefficient of determinant (R^2) = 0.990 and Mean Squared Error = 95.29 mm/hr.

And Log – Pearson distribution:

$$I = \frac{519.214T_r^{0.204}}{T_d^{0.592}} \tag{10}$$

For Equation (10), R^2 = 0.990 and Mean Squared Error = 106.86 mm/hr.

3.2.3 Evaluation of iterative Equation Solver in Excel

Excel Solver model parameters trial solution for return period (2 year) specific IDF model has fourteen (14) iterations before convergence (see Table 6). Similarly, there are thirty-five (35) iterations in the development of the general IDF model given in Equation (8).

The coefficient of determination is computed from Equation (11) and Table 7.

$$R^2 = \frac{\sum_{i=1}^{n}(y - y_{avg})^2 - \sum_{i=1}^{n}(y - y_{pred})^2}{\sum_{i=1}^{n}(y - y_{avg})^2} \quad (11)$$

The tabular evaluation of the various terms involved in the computation of coefficient of determination (R^2) such as observed rainfall intensity (I); predicted intensity (I_p); average intensity (I_{avg}); $(I - I_p)^2$ and $(I - Iavg)^2$ respectively are as presented in Table 7. Given Table 7 and Equation (11), the evaluation of R^2 and Mean Square Error (MSE) are as follows:

$$R^2 = \frac{(41807.74 - 1098.365)}{41807.74} = 0.973$$

and

$$MSE = \frac{\sum_{i=1}^{n}(y - y_{pred})^2}{n} \quad (12)$$

$$\text{MSE} = \frac{1098.365}{13} = 84.49$$

3.3 Comparison of Observed and Predicted Rainfall Intensities

The general IDF model enables one to predict the intensity of rainfall of any duration and any return period. The verification of the developed model is carried out by plotting the observed and predicted intensities on the same graph as shown in Figures 8 to 10. Similarly, a comparartive plots for GEVT – 1, Normal and Log – Pearson Type 3 distributions for 5 and 100 year return periods are as shown in Figures 11 and 12.

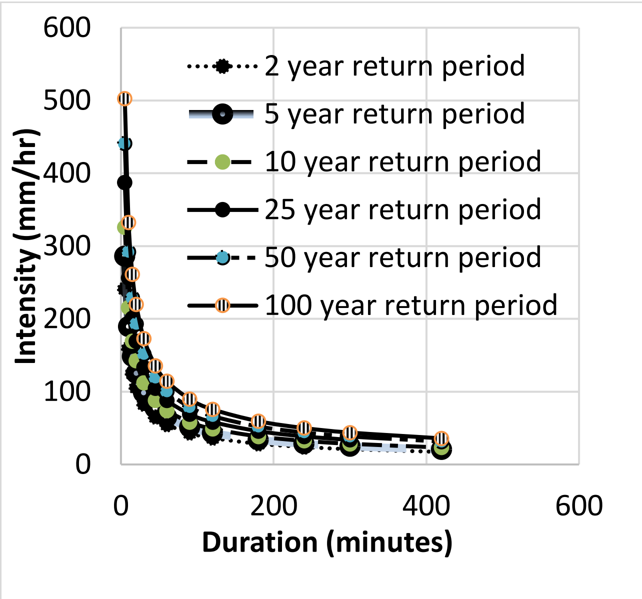

Figure 5: Intensity Duration Curve for Gumbel Extreme Value Type 1 IDF general model for Ikeja.

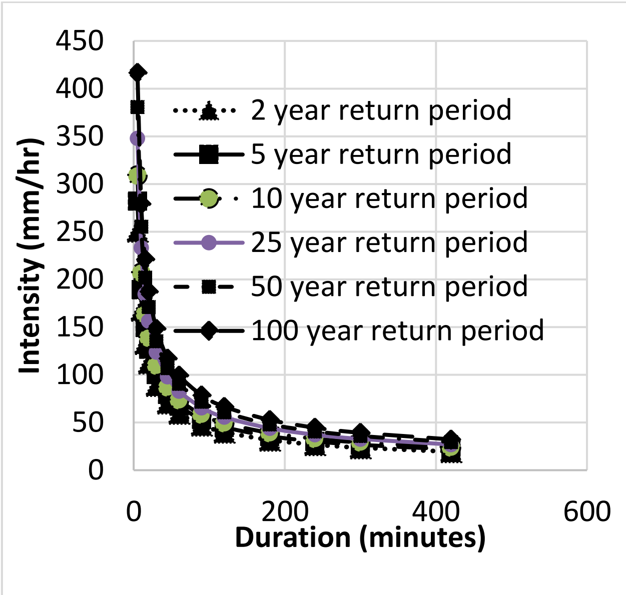

Figure 6: Intensity Duration Curve for Normal Distribution IDF general model for Ikeja.

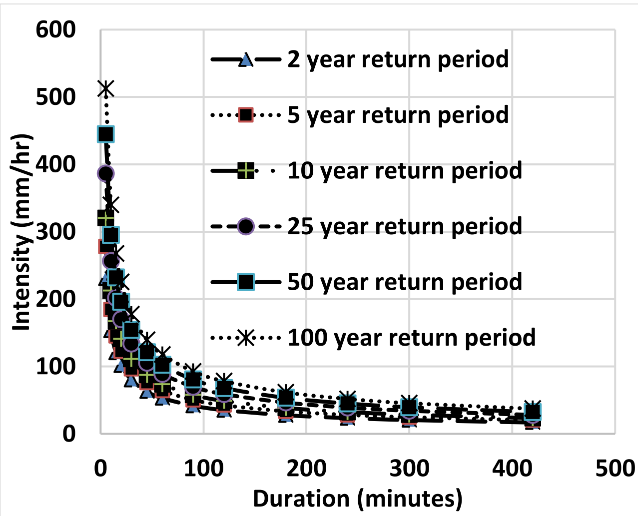

Figure 7: Intensity Duration Curve for Log – Pearson Type 3 Distribution IDF general model for Ikeja.

Table 6: Trial solution result for Sherman's specific IDF model calibration for Ikeja

Iteration	c	m	a
1	1	1	1
2	1.461474	1.31987	0
3	3.546129	3.431661	0
4	3.825354	4.117993	0
5	3.830287	4.130401	0.05
6	4.528795	5.887498	0.312129
7	4.713106	6.348498	0.400196
8	4.838772	6.614912	0.52986
9	4.859924	6.669481	0.538164
10	4.857193	6.663613	0.535575
11	4.856903	6.662889	0.535429
12	4.856903	6.662889	0.535429
13	4.856903	6.662889	0.535429
14	4.856903	6.662889	0.535429

Table 7: Evaluated terms for determining coefficient of determination for 2 year return period

Intensity, I	Intensity$_{pred}$, I_p	$(I - Ip)^2$	$(I-Iavg)^2$
192.1498641	207.892929	247.8440829	14668.11
155.0966423	143.436046	135.9695073	7065.876
128.463877	115.444493	169.5043489	3297.745
112.3163251	98.9639205	178.2867085	1703.91
81.16415026	79.6511058	2.28930367	102.5414
65.78223051	64.1071879	2.805767634	27.62183
52.68677814	54.9554029	5.146658379	336.7629
39.42640188	44.2308529	23.08274969	999.2854
30.27733462	37.9165648	58.35783719	1661.422
21.74873497	30.517145	76.88501435	2429.42
18.13831768	26.1605922	64.35688805	2798.363
15.11094943	23.2144685	65.66702178	3127.821
11.13080687	19.3872836	68.16940809	3588.857
Average = 71.038		Sum = 1098.365	Sum = 41807.74

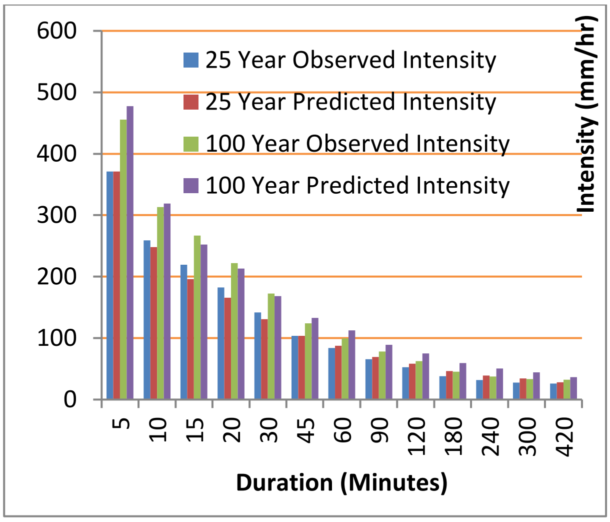

Figure 8: Observed rainfall intensity against predicted rainfall intensity for 25 and 100 year return periods for Gumbel distribution for Ikeja

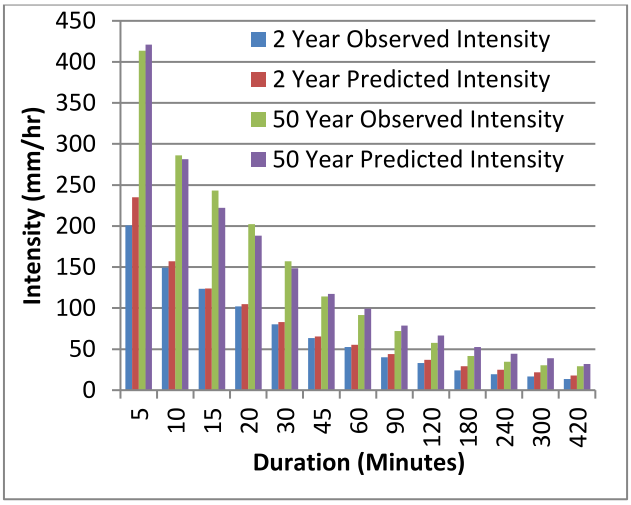

Figure 10: Observed rainfall intensity and predicted rainfall intensity for 2 and 50 year return periods for Gumbel distribution for Ikeja

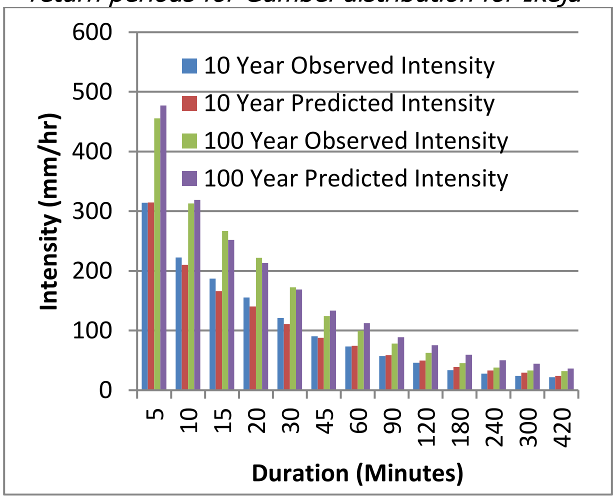

Figure 9: Observed rainfall intensity against predicted rainfall intensity for 10 and 100 year return periods for Gumbel distribution for Ikeja

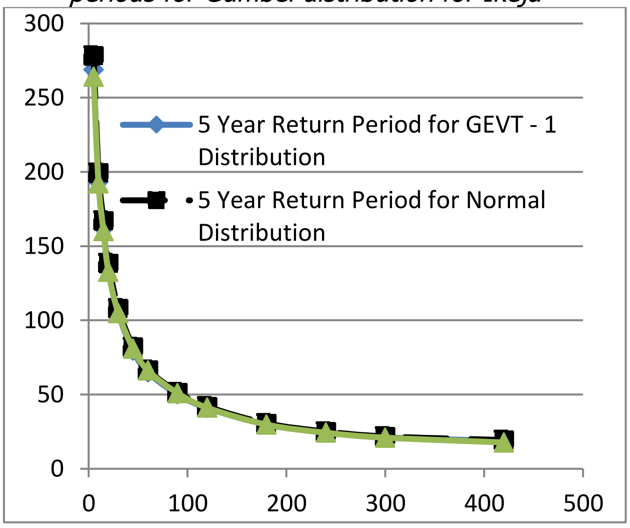

Figure 11: Plot of 5 year return period for GEVT – 1, Normal and Log – Pearson type 3 distributions

4. DISCUSSION OF RESULTS

Basically Table 1 is all about descriptive statistics giving information on mean and standard deviation of rainfall intensities for different durations. The result of these tables served as input data for rainfall intensity transformation using Equation 2 to obtain probability distribution function equivalent (GEVT – 1, Normal and Log – Pearson Type 3).The resulting IDF based intensity values are as plotted in Figures 3, 4, 5 and 6.

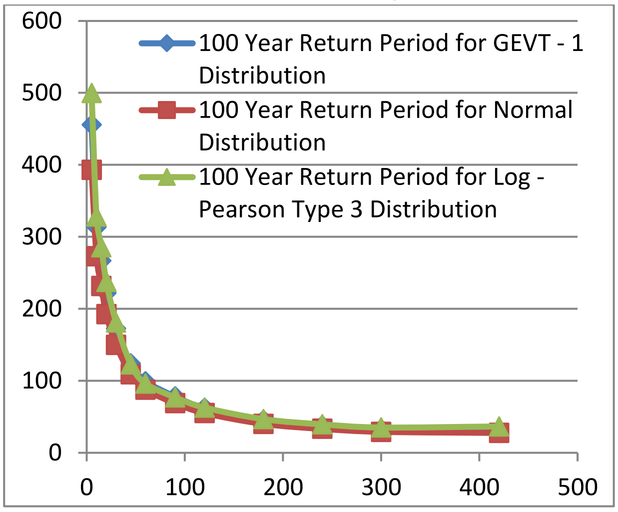

Figure 12: Plot of 100 year return period for GEVT – 1, Normal and Log – Pearson type 3 distributions

Table 8: Results from regression approach and excel solver optimization approach (GEVT-1, 2 year return period)

Method	c	m	a	R^2	MSE
Regression	63.30	3.550	0.685	0.820	320.10
Solver	4.873	6.700	0.550	0.992	26.43

3.3.1. Performance of Regression approach against Excel Optimization Solver via R^2 and MSE

Table 8 (an extension of Table 6) shows the result of the iterative method observed with Excel Optimization Solver as against regression approach. Unlike the specified return period model which is constrained by the given return period, the non – specified return period (general models) is unrestricted by the return periods. In other words, any selected return period value or duration can be used to evaluate the rainfall intensity. For GEVT – 1 and Normal distribution models selected return period of 2, 5, 10, 25, 50 and 100 years were used while various durations (minimum 5 to maximum 420 minutes), were employed to evaluate Equations 8 and 9.

The plotted graph (Figures 3 and 4) each containing a total of six plots for different return periods. Among the common features of the IDF curves observed in the plots are:

(i) Intensity decreases with increase in duration;

(ii) Intensity increases with increase in return period for a given duration

Maximum intensities occur at short duration with large variations with return period, while the flattened shape observed in Figures 2, 3, 4, 5 & 6 toward 420 minutes is because with long duration there is no much difference in intensities with return period [9].

MSE arising from evaluation of Equation (12) is reflected in Table 3 for GEVT – 1, Table 4 for Normal and Table 5 for Log – Pearson Type 3 distributions. In all the two year return periods, specific models gave the least MSE of 26.43 while the 100 year equivalent gave 115.94. However, the goodness of fit values for 2, 5, 10, 25, 50 and 100 year return periods range from 0.992 to 0.993. For Normal distribution in Table 4, similar observations were noted with MSE of 28.64 and 85.25 for 2 and 100 year return periods and R^2 value ranging from 0.992 to 0.993. Similar observations were made for Log – Pearson Type 3 (see Table 5), the MSE values for 2 and 100 year return periods respectively are 28.24 and 154.85 while the R^2 values range from 0.991 to 0.993. Equations (8, 9 and 10) shows that the non- specified models ranked GEVT -1 and Normal Distributions first with R^2 value of 0.990 each and MSE values of 95.27 and 95.29 while the Log-Pearson Type 3 model ranked third with R^2 value of 0.990 and MSE of 106.86.

The distribution of the observed and predicted rainfall intensities are both influenced by duration and return period. In all the plots (Figures 8 to 10), it was observed that rainfall intensity decreases with increasing duration. It was also shown in the figures that, for a given duration, the higher return period yielded corresponding higher intensity values. This observation is supported by the works of [10-17]. In Figures 11 and 12, Log – Pearson Type 3 gave the highest predicted Intensity values followed by GEVT – 1 and Normal distribution. For instance at 5 minutes duration and 100 year return period, Log – Pearson Type 3 predicted 499.65 mm/hr intensity followed by GEVT – 1 with 455.67 mm/hr intensity while Normal distribution predicted 393.11 mm/hr intensity. Similarly, at 60 minutes duration and 100 year return period, GEVT – 1 predicted 99.38 mm/hr intensity followed by Log – Pearson Type 3 with 96.1 mm/hr intensity while Normal distribution predicted 87.85 mm/hr intensity. And finally at 300 minutes duration and 100 year return period, Log – Pearson Type 3 predicted 35.15 mm/hr intensity followed by GEVT – 1 with 33.15 mm/hr intensity while Normal distribution predicted 29.14 mm/hr intensity. Apparently, the superiority of Log – Pearson Type 3 over the other two models in predicting higher intensity values at short, medium and higher durations is very consistent.

Similar observation was noted for Port Harcourt IDF models as reported by [5].

It was observed from Table 7 that the percentage difference between observed and predicted intensity values are 7.57% for 5 minutes duration, 13.34% for 20 minutes duration and 10.85% for 90 minutes duration. As per the Excel Optimization solver, a total of 14 iterations were observed which yielded the calibrated values of a, c and m; which are slightly different from the multiple regression approach. The bench mark for selecting the superior set of results is anchored on the goodness of fit (R^2) and Mean Square Error (MSE) of which the values are $R^2 = 0.992$, MSE = 26.43 for solver as against $R^2 = 0.820$, MSE = 320.10 for regression approach. Thus the Excel Solver option is superior alternative to multiple regression method. These observations is in consonance with those of [4] and [5].

5. CONCLUSION

It has been observed for Ikeja rain gauge station that for a given return period, the intensity decreases as the duration increases which is in line with what is obtainable in literatures. Models have been developed for GEVT-1, Normal and Log Pearson Type-3 distributions which are in agreement with PDF theory which shows higher intensity occurring at shorter duration and lower intensity at longer duration. The prediction of rainfall intensity with the Probability Distribution Functions showed a good match with observed intensity values. The intensity – duration curve in all cases has a negative slope. The GEVT – 1 and Normal distribution models ranked first with respect to MSE 95.27 & 95.29 and R^2 of 0.993 while the Log-Pearson Type 3 ranked third with MSE of 106.86 and R^2 of 0.990 in the non-specified return period (model).

6. REFERENCES

[1] David, A. O., Nwaogazie, I. L., & Agunwamba, J. C. (2019a). Development of Models for Rainfall Intensity-duration-frequency for Akure, South-West, Nigeria. *International Journal of Environment and Climate Change,* 9(8): 457 – 466.

[2] Hosking, J. R. M., and Wallis, J. R. (1997). *Regional frequency analysis: an approach based on L-moments.* Cambridge University Press, Cambridge, U.K.

[3] El-Syed, E. A. (1961), "Generation of Rainfall Intensity Duration Frequency Curves For Ungauged Sites", *Nile Basin Water Science & Engineering Journal,* 4(1) pp. 112-124.

[4] Mohammad Zakwan (2016): "Application of Optimization Technique to Estimate IDF Parameters", *Water and Energy International.* Vol. 59, Issue 6, pp. 69-71.

[5] Nwaogazie, Ify L. and Masi, S.G. (2019): Probability and non-probability rainfall intensity – duration – frequency modelling for Port Harcourt metropolis, Nigeria. *International Journal of Hydrology.* 3(1): 66 – 75.

[6] Nwaogazie, Ify L. and Duru, E. (2002): "Developing Rainfall Intensity – Frequency Model for Port Harcourt" *NSE Technical Transaction*, Vol. 37, No. 2, pp. 19 – 32.

[7] Www.timeanddate.com/weather/nigeria /lagos/climate.

[8] Premium Solver Platform, User Guide. 2010. Frontline Systems, Inc.

[9] Jaleel, L. A. and Farawn, M. A. (2013): "Developing Rainfall Intensity-Duration-Frequency Relationship for Basrah city. *Kufa Journal of Engineering* (K. J. E), Vol. 5, No. 1, pp. 105-112.

[10] Ven Te Chow, Larry, R., Maidment and Larry, W. (1988): Applied Hydrology, McGraw – Hill publishers, New York.

[11] David, A. O.; Nwaogazie, Ify L. and Agunwamba, J. C. (2019). Modeling Rainfall Intensity by Optimization Technique in Abeokuta, South – West, Nigeria. *Journal of Engineering Research and Reports* 6(4): 1 – 10.

[12] Nwaogazie, I. L. and Nwadike, C. C. (2010): Developing Annual and Partial Series Rainfall Models for Enugu City, *Global Journal of Engineering Research*, 9(1 & 2); 1 – 18.

[13] Okonkwo, G. I. and Mbajiorgu, C. C. (2010): Rainfall intensity – duration – frequency analysis for Southern Nigeria. *Agric Eng. Int: The CIGR Journal*; 12(1): 22 – 30.

[14] Udo Inyang, C. U. and Edem, D. I. (2012): Analysis of rainfall trends in Akwa Ibom State, Nigeria. *Journal of Environment and Earth Science;* 2(8): 1 – 12

[15] Akpan, S. U. and Okoro, B. C. (2013): Developing rainfall intensity – duration – frequency models for Calabar city, South - South, Nigeria. *American Journal of Engineering Research;* 2(6): 19 – 24.

[16] Ologhadien, I and Nwaogazie, Ify L (2014): Rainfall Intensity – Duration – Frequency Models for selected Cities in Southern Nigeria. *Standard Scientific Research and Essays* Vol 2 (10): 509 – 515.

[17] Nwaogazie, I. L. and Okonkwo, S. C. (2017): Rainfall intensity-duration-frequency modelling and comparative analysis of developed models for Abakaliki, Ebonyi State, Nigeria. *International Journal of Trend in Research and Development*, 4(2): 776 – 783.