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A NEW TEST ANALYSIS PROCEDURE FOR PRESSURE DRAWDOWN TEST OF A HORIZONTAL WELL IN AN INFINITE-ACTING RESERVOIR

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ABSTRACT

This paper develops a new well test analysis procedure of a horizontal well in an infinite-acting reservoir. Hitherto, horizontal well pressure drawdown test analysis during infinite-acting flow was accomplished based on a straight line method obtained from a plot of flowing wellbore pressure versus log of flow time. In that procedure, only limited system characterization was possible. Furthermore, the rigorous type curve matching was resorted to when a straight line did not appear. In the method developed here, a plot of flowing wellbore pressures, p_{wf}, against dimensionless wellbore pressures, p_D, is made for analysis. Results show that a straight line relationship exists between the two plotted parameters on a linear paper. The straight line slope across the middle time region can be taken to perform test analysis. From the plot, reservoir pressure, near wellbore permeability, reservoir capacity, fluid mobility and transmissibility can be calculated. Mathematical procedure leading to this method of analysis is based on selection of relevant source and Green's functions for a horizontal well during infinite-acting flow and purely as a line source. A case pressure test selected for analysis using the method developed here yielded very close reservoir character compared to the reservoir being characterized

Keywords: *Dimensionless pressure, dimensionless derivatives, infinite-acting, horizontal well, early radial flow*

1. INTRODUCTION

Horizontal well pressure drawdown and buildup analyses during infinite-acting flow were accomplished based on a straight line method. In that procedure, only limited system characterization was possible. Furthermore, the rigorous type curve matching was resorted to when a straight line did not appear. This paper develops a new procedure for horizontal well test analysis during infinite-acting flow by developing necessary mathematical expression other than that relating flowing bottomhole pressure and flow time. Pressure drawdown test is considered in our derivation.

Describing transient pressure behavior of horizontal well is considerably more complicated than vertical wells because of the occurrence of several flow periods It was established that, once the reservoir boundaries are identified, a horizontal well flow is duplicated by a 3D multiplication of the characteristic functions in accordance with Newman product rule [1]. An author produced dimensionless pressures derivative for a horizontal well subject to bottom water drive [2]. Mathematical models for horizontal flow are also discussed in many other literature.[4-12]· All the models are based solutions to the diffusivity equation for oil flow in a reservoir, and are the basis for any form of well test analysis

2.RESERVIOR PHYSICAL MODEL DESCRIPTION

Figure 1 shows the physical model of an infinite-acting reservoir with a horizontal well. The reservoir is assumed to be rectangular in geometry. A horizontal well of length L is completed at the centre of the reservoir. During flow, the well does not feel the effect of external boundaries as these boundaries are assumed to be infinitely far away during the flow period. In this case only near wellbore, reservoir and well properties can be estimated. The well length

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coincide with the x-axis, the width of the well coincide with the y-axis and considered to be negligible (line source). The thickness of the reservoir is along the z-axis. The well is z_w away from the bottom of the reservoir (well stand-off).

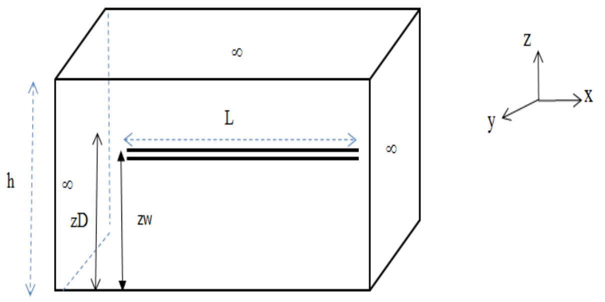

Fig 1:Horizontal Well model in an infinite acting resererrvoir

2.1 Reservoir Mathematical Model Description

Horizontal well flow in Reservoir engineering conforms to the second order linear, heterogeneous partial differential equation called the diffusivity equation. This equation is usually presented in 3-D and is given by the following for an anisotropic reservoir [1].

$$k_x \frac{\partial^2 P}{\partial x^2} + k_y \frac{\partial^2 P}{\partial y^2} + k_z \frac{\partial^2 P}{\partial z^2} = \phi \mu c_t \frac{\partial P}{\partial t} \quad (1)$$

Solution to Equation (1) has been obtained depending on the objective to be achieved. However, Newman product of instantaneous source functions based on the nature of the reservoir boundaries, is a major solution method popular in developing well test analysis procedure [1]. We shall consider only infinite-acting flow solution (early radial flow solution) to develop a new test analysis procedure in this paper

3. EARLY RADIAL FLOW SOLUTION

This occurs when the well is initially opened for production and the flow has not experienced any external boundary effect. The flow is completely radial and is equivalent to that of a fully penetrating vertical well in an infinite-acting reservoir. Thus, it could be referred to as vertical radial flow regime [3]. From the physical description of the reservoir system above, the well experience an infinite slab source along the x-axis, and infinite plane sources along the y- axis and z-axes, all at early time flow

This solution has been used by several authors as follows for ab anisotropic reservoir case:

$$p_D = 2\pi h_D \int_0^{t_D} \frac{1}{2}\left[erf\left(\frac{\sqrt{k/k_x} + X_D}{2\sqrt{\tau}}\right) + erf\left(\frac{\sqrt{k/k_x} - X_D}{2\sqrt{\tau}}\right)\right] \times \frac{1}{2\sqrt{\pi\tau}} e^{-\frac{(y_D - y_{wD})^2}{4\tau}} \times \frac{1}{2\sqrt{\pi\tau}} e^{-\frac{(Z_D - Z_{wD})^2}{4\tau}} d\tau \quad (2)$$

At early flow time

$$P_D = \frac{h_D}{2} \int_0^{t_D} ß * \frac{e^{-\frac{[(y_D - y_{wD})^2 + (Z_D - Z_{wD})^2]}{4\tau}}}{\tau} d\tau \quad (3)$$

Where,

$$ß = erf\left(\frac{\sqrt{k/k_x} + X_D}{2\sqrt{\tau}}\right) + erf\left(\frac{\sqrt{k/k_x} - X_D}{2\sqrt{\tau}}\right) \quad (3a)$$

By direct integration Equation (3) can be expressed as follows:

$$P_D = \frac{-ßh_D}{4} . Ei\left(-\frac{(z_D - z_{wD})^2 + (y_D - y_{wD})^2}{4t_D}\right) \quad (4)$$

For strictly wellbore flow, i.e. at $y_D = y_{wD}$ and infinite conductivity condition $ß = 2$, equation (4) becomes

$$P_D = \frac{-h_D}{2} . Ei - \left(\frac{(z_D - z_{wD})^2}{4t_D}\right) \quad (5)$$

From the physical model above, $z_D - z_{wD} = r_{wD}$.

This is renders equation (5) as

$$P_D = \frac{-h_D}{2} . Ei - \left(\frac{(rwD)^2}{4t_D}\right) \quad (6)$$

3.2 Derivation of Pressure Drawdown Well Test Analysis Procedure

The dimensionless pressure drop is expressed as:

$$P_D = \frac{kh\Delta P}{141.2quB} \quad (7)$$

Where,

$$\Delta P = pi - Pwf \quad (8)$$

Solving from equation (7),

$$pw = pi - mp_D \quad (9)$$

$$m = -\frac{141.2quB}{kh} \quad (10)$$

Equation (9) is the new procedure for analyzing pressure drawdown test data. A plot of p_{wf} against p_D on a Cartesian axes gives a slope of m. from the slope, k, kh, k/h kh/μcan be estimated. Furthermore, from the plot, extrapolate the straight line to p_{wf}= 0; that is, point where the dimensionless pressure at time = 0 (pDi). The expression for pDi is given as:

$$PDi = \frac{kh\Delta P}{141.2quB} \quad (11)$$

From equation (11), we can solve for the initial reservoir pressure, p_i. In Equation 9, pD and p_{wf} are obtained as recordings from well test.

4. DEMOSTRATION OF PROBLEM

A drawdown test is conducted on a horizontal well. Result obtained is tabulated in Table 1. Other pertinent information is as follows: reservoir pay thickness = 20ft, well length = 2000ft, oil viscosity = 0.2cp, compressibility = 6x10^{-6}/psi, oil formation volume factor = 1.35bbl//stb, porosity = 15%, production rate = 500bb/day. Infinite conductivity is assumed, that is x_D = 0.732. The well is centrally located along the vertical axis of the reservoir, that is z_{wD} = 0.5. Wellbore radius = 0.25ft. Directional permeabilities are k_x = 0.05md, k_y = 0.08md and k_z = 0.07md. It is desired to analyze the test data to obtain initial reservoir pressure, average reservoir permeability, and other reservoir, and fluid properties

4.1 Testing Analysis

The average reservoir permeability is calculated as:

$$k = \sqrt[3]{k_x k_y k_z} \qquad (12)$$

Hence, $k = 0.065md$

For pay thickness wellbore, $h = 20f$, h_D is calculated from our definition of dimensionless parameters as

$$h_D = \frac{2 \times 20}{2000}\sqrt{\frac{0.07}{0.065}} = 0.021$$

For dimensionless wellbore radius, r_{Wd}

$$r_{wD} = \frac{2.(0.25)}{2000}\sqrt{\frac{0.08}{0.065}} = 0.000277$$

Table 2 also shows computed values of dimensionless time, t_D and dimensionless pressure from the flowing wellbore pressures recorded from the test. Note that the dimensionless pressure is computed from equation (6) according to [6].

Table 1: Well Test Result and Computed Dimensionless time and Dimensionless pressure

Flow time (hr)	Flowing pressure, psi	Dimensionless time, t_D	Dimensionless pressure, p_D
0.001	1330	9.5E-08	0.006421
0.002	1225.2	1.9E-07	0.009571
0.003	1118.6	2.85E-07	0.012049
0.004	1008.236	3.8E-07	0.012957
0.005	980.45	4.75E-07	0.014075
0.006	978.36	5.7E-07	0.015535
0.007	947.4536	6.65E-07	0.015887
0.008	958.56	7.6E-07	0.016469
0.009	886.71	8.55E-07	0.01712
0.01	915.36	9.5E-07	0.017614
0.02	908.43	1.9E-06	0.021200
0.03	866.521	2.85E-06	0.023861
0.04	920.323	3.8E-06	0.024812
0.05	820.312	4.75E-06	0.025977
0.06	825.35	5.7E-06	0.028009
0.07	816.47	6.65E-06	0.029605
0.08	813.5	7.6E-06	0.029064
0.09	817.216	8.55E-06	0.028613
0.1	822.73	9.5E-06	0.029605
0.2	803.53	0.000019	0.033243
0.3	803.545	2.85E-05	0.035364
0.4	791.66	0.000038	0.036875
0.5	789.56	4.75E-05	0.038046
0.6	770.46	0.000057	0.039003
0.7	785.54	6.65E-05	0.039813
0.8	771.506	0.000076	0.040514
0.9	775.211	8.55E-05	0.041132
1	766.73	0.000095	0.041685
2	660.65	0.00019	0.045324
3	765.81	0.000285	0.047453
4	667.44	0.00038	0.048963
5	661.76	0.000475	0.050135
6	657.56	0.00057	0.051092

Table 1: Continues: Well Test Result and Computed Dimensionless time and Dimensionless pressure.

Flow time (hr)	Flowing pressure, psi	Dimensionless time, tD	Dimensionless pressure, pD
7	679.811	0.000665	0.051901
8	651.123	0.00076	0.052602
9	645.45	0.000855	0.*053221*
10	640.32	0.000950	0.053774
20	644.433	0.001900	0.057413
30	634.233	0.00285	0.059541
40	632.567	0.0038	0.061052
50	623.45	0.00475	0.062223
60	622.655	0.0057	0.063180
70	612.95	0.00665	0.063990
80	603.876	0.0076	0.064691
90	611.1197	0.00855	0.065309
100	600.0197	0.0095	0.065862
200	590.002	0.019	0.069501
300	570.5007	0.0285	0.07163
400	563.002	0.038	0.07314
500	521.62	0.0475	0.074312
600	565.152	0.057	0.075269
700	523.052	0.0665	0.076078
800	521.4407	0.076	0.076779
900	514.5907	0.0855	0.077398
1000	502.4907	0.095	0.077951
2000	536.482	0.19	0.08159
3000	497.422	0.285	0.083719
4000	497.572	0.38	0.085229
5000	509.422	0.475	0.0864

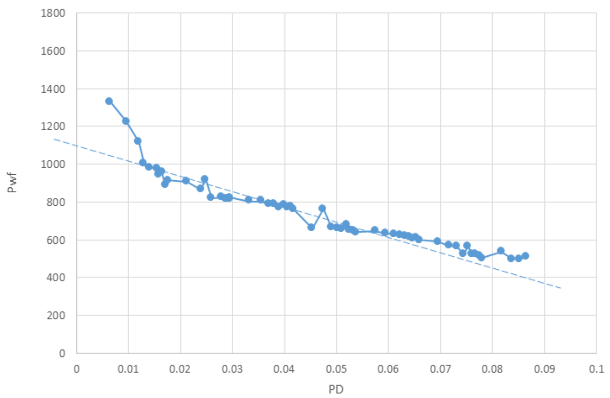

Figure 2: Flowing Wellbore pressure versus dimensionless pressure.

Figure 2 is a Plot of p_{wf} against p_D

From Figure 2, the slope, $\mathrm{m} = -7738.45\ \mathrm{psi}$, From the slope (k)

$$k = \frac{141.2 q\mu B}{mh} = \frac{141.2 \times 500 \times 0.2 \times 1.35}{7738.45 \times 20} = 0.123\mathrm{md}$$

Fluid mobility, M is also obtained as

$$\mathrm{M} = \frac{k}{u} = \frac{0.123}{0.2} = 0.614\mathrm{mD/cp}$$

Fluid capacity (kh),

$$\mathrm{kh} = 0.123 \times 20 = 2.46\mathrm{mD/ft}$$

Fluid transmissibility,

$$\frac{kh}{u} = \frac{0.123 \times 20}{0.2} = 12.3\ \mathrm{mDft/cp}$$

From Equation (11),

$$\mathrm{Pi} = \mathrm{PD}\frac{141.2 quB}{kh} + \mathrm{Pwf} \qquad (13)$$

Substituting into equation (11) at which $\mathrm{pDi} = 0$, from the plot $\mathrm{pwf} = 1100\mathrm{psi}$, $\mathrm{k} = 0.065\mathrm{mD}$, then $\mathrm{pi} = 1100\mathrm{psi}$. Finally, the average reservoir permeability obtained from the test (0.123mD) is in close agreement with the cube root of the product of all the directional permeabilities (0.065mD).

5. CONCLUSION

1. A new horizontal well drawdown test analysis during infinite-acting flow has been developed.
2. A case well drawdown test analysis was demonstrated successfully using the new procedure.
3. A plot of wellbore flowing pressure versus dimensionless pressure on linear axes is required. This is in contrast from the conventional semilog plot in the literature.
4. The new method offers a possibility of estimating initial reservoir pressure, pi.

6. APPENDIX

$$x_D = \frac{2x}{L}\sqrt{\frac{k}{k_x}} \quad \text{A1}$$

$$y_D = \frac{2y}{L}\sqrt{\frac{k}{k_y}} \quad \text{A2}$$

$$z_D = \frac{z}{h} \quad \text{A3}$$

$$z_{wD} = \frac{z_w}{h} \quad \text{A4}$$

$$L_D = \frac{L}{2h}\sqrt{\frac{k}{k_y}} \quad \text{A5}$$

$$t_D = \frac{0.001056kt}{\phi\mu ctL^2} \quad \text{A6}$$

$$h_D = \frac{2h}{L}\sqrt{\frac{k_z}{k}} \quad \text{A7}$$

$$h_D = \frac{1}{LD} \quad \text{A8}$$

$$P_D = \frac{kh\Delta p}{141.2q\mu B} \quad \text{A9}$$

$$r_{wD} = \frac{2r_w}{L}\sqrt{\frac{k_y}{k}} \quad \text{A10}$$

7. NOMENCLATURE

E_i = exponential integral function
h = reservoir pay thickness, ft
h_D = dimensionless pay thickness
k_x = permeability in the x direction in the areal plane, md
k_y = permeability in the y direction in the areal plane, md
k_H = effective horizontal permeability in the areal plane parallel to the bedding (md).
k_v = k_z = permeability in the z direction., md
L = horizontal well length, ft
L_D = dimensionless well length
P_D = dimensionless pressure
P'_D = dimensionless pressure derivative
P_i = initial reservoir pressure, psi
r_w = wellbore radius, ft
r_{wD} = dimensionless wellbore radius
t = flow time, hrs
t_D = dimensionless time
x_D = perforation length, ft
z_w = vertical distance from the wellbore to the bottom boundary (well stand-off),
zD = dimensionless vertical distance from the wellbore to any part of the boundary
∞ = infinity

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