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DETERMINATION OF THE OPTIMAL BACKORDER LEVEL FOR A LOT-SIZE REORDER POINT INVENTORY SYSTEM WITH BACKORDER PARAMETER

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ABSTRACT
This paper attempts to investigate the existence of an optimal backorder level that will minimize the expected total inventory cost of a regular (R, Q) lot-size reorder point system that is associated with backorder effect and establish the allowable backorder levels. To achieve this, the expected order cycle cost function of the lot-size reorder point system with backorder effect is optimized to determine the optimal backorder parameter. Numerical results presented showed that an optimal backorder level of 8 units exist where the expected order cycle cost is minimum with a value ₦19,036,000. Furthermore between the backorder level of 0 and 8 units, the order cycle cost decreases from ₦19,105,000 to ₦19,036,000. Thereby defining the range of the allowable backorder levels.

Key words: *Backordered Level, Backorder Effect, Cycle Stock, Stock-Out and Backordered Demand.*

1. INTRODUCTION

The lost- size-Reader point system of stock replenishment is a continuous review policy that defines a reorder point based on demand distribution during lead time, so that a lot size of a fixed order quantity Q which is often an economic order quantity *(EOQ)* is placed as soon as the inventory position reaches the reorder point, R [1]. Single delivery or regular ordering inventory models that allows for complete backordering requires that the total excess demand at the end of an order cycle (the time frame between the receipt of two consecutive regular orders) be backordered and satisfied at the commencement of the next order cycle [1 - 3]. The satisfaction of the backordered excess demand in an order cycle results in a reduction in the cycle stock (the available stock to meet demand within normal time period) of such an order cycle which is referred to as backorder effect in this study. This reduction in the cycle stock further increase the chances of higher values of excess demand at the end of the order cycle, if the prevailing demand is sustained.

The focus of previous studies is how to determine the number of units shorts at the end of the order cycle to be backordered and how to replenish the cycle stock of an order cycle where backordered excess demand was satisfied to minimize the expected cost and the stock-out level [4 - 6].

However, not all units of backordered excess demand can optimize the expected inventory cost of an operating regular ordering policy if backordering is allowed [1, 7, 8].

Secondly, there is a range of backorder values for the policy with backorder effect that defines the allowable backorder level. The highest value in that range is the maximum or optimal value of backorder, since the expected inventory cost beyond this point starts increasing.

In this paper, an optimal backorder level which is the allowable maximum backorder level is determined for a lot-size reorder point inventory system with backorder effect. The optimal backorder level will help to determine the point of minimum cost with the use of the policy and establish a range of allowable backorder for the policy with a decreasing expected cost.

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2. NOTATIONS

The following notation where used in the sequel. The following notations will be adopted in development of the proposed model.

$\mu =$ The mean regular lead time demand rate

$\gamma =$ The mean demand rate per unit time

$\tau_1 =$ The regular lead time

$l =$ Inventory rate

$l_b =$ The backorder level at the beginning of an order cycle $l_b = 1, 2, ,3...l_m$

$R_b =$ The adjusted regular reorder point for an order cycle with backorder effect

$\rho =$ The unit shortage cost per unit of item stock out at the end of a cycle

$C =$ Variable ordering cost per unit of the regular order quantity

F(R): = Probability that lead time demand is less than or equal to the regular reorder point

$F(x_L) =$ The distribution function at X_L

$f(x_L) =$ The probability density function p.d.f of X_L

$G_b(l_b) =$ The expected regular order cycle cost for the regular ordering policy only with backorder effect

h = Holding cost per unit per unit time

$l_m =$ The maximum backorder level allowable within an order cycle. $l_m = R*$

$l_b^* =$ The level of backorder that minimize the expected regular order cycle cost with back order effect

$K_1 =$ Set-up or fixed ordering cost per regular order cycle

$S_s =$ The safety stock per regular order cycle without backorder effect

$\overline{S}_s =$ The safety stock for regular order cycle with backorder effect

$X =$ The annual demand rate, a random variable

$x =$ A value of X i.e a realization on X

$X_L =$ The Lead time demand, a random variable

$x_L =$ A value of X_L

P (R): = The probability that lead time demand is greater than or equal to R

$Q =$ The regular order quantity

$Q* =$ The optimal regular order quantity.

$R =$ The regular reorder point

$R* =$ The optimal regular reorder point

$t =$ Time

T = The regular order cycle length; a random variable.

$T_b =$ The expected total inventory cost per unit time for the regular order policy only with backorder effect

LTCP = Low Tension Concrete pole

EOQ = Economic Order Quantity

3. ASSUMPTIONS

1. The maximum allowable backorder level l_m is equal to the optimal reorder level R^*.
2. The expected order cycle cost of the lot-size reorder point system with backorder effect is to have been formulated already following the approximation method of the cost formulation.
3. All optimal policy parameters besides that of the optimal backorder level are have been determined already.
4. The demand rate per annum (X) is discrete and follows a Poisson process and is approximated by a continuous process within lead time for convenience of computation.

4. MODEL DEVELOPMENT

Figure 1 shows the inventory realization process of an order cycle replenished with a lot-size reorder point system with backorder effect. In the policy operation, the starting inventory of the order cycle is reduced to $\left(Q^* - l_b\right)$ from *Q** due to the satisfaction of the backordered excess demand *l_b*. It is expected for the optimal or the minimum allowable backorder value l_b^* to be determined, such that $l_b \le l_b*$ are the allowable backorder values.

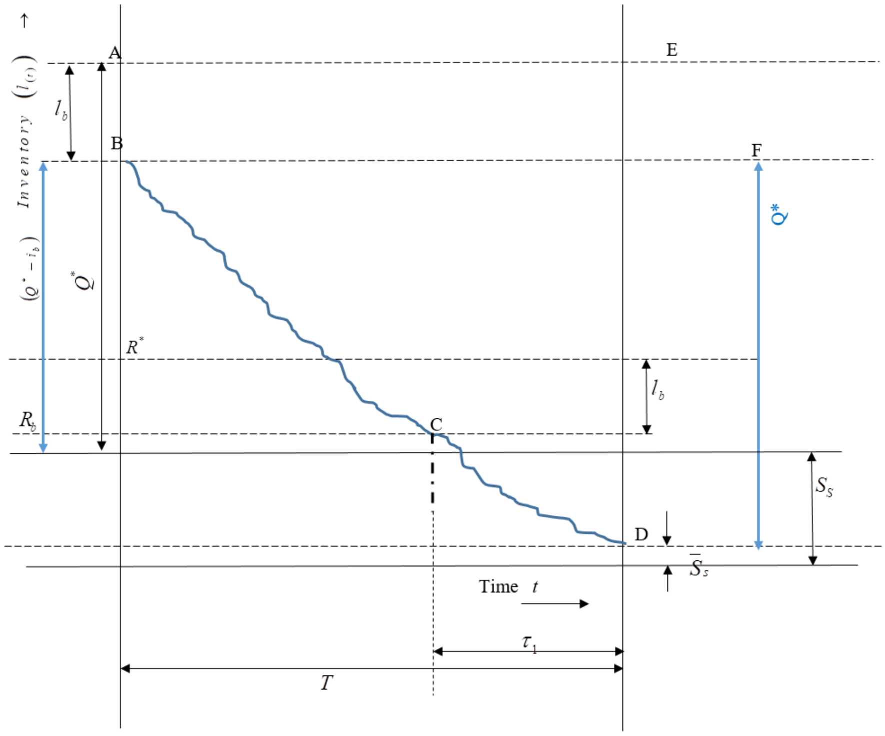

Figure 1: Inventory process for the restructured regular ordering policy for a Lot – size reorder point system with backorder effect.

4. DETERMINATION OF THE OPTIMAL BACKORDER LEVEL (l_b^*)

The order cycle cost of a lot-size reorder point system formulated alongside the approximation method stated in [9] is restructured by the introduction of a backorder parameter in this paper and stated in equation (1) as follows;

$$G_b(l_b) = (CQ^* + K_1) + h\left[\frac{Q^* - 3l_b}{2} + R^* - \gamma\tau_1\right] + \rho\int_{(R^*-l_b)}^{\infty}[x_L - (R^* - l_b)]\, f(x_L)dx_L \qquad (1)$$

Where: $(CQ^* + K_1)$, $h\left[\frac{Q^*-3l_b}{2} + R^* - \gamma\tau_1\right]$ and $+\rho\int_{(R^*-l_b)}^{\infty}[x_L - (R^* - l_b)]\, f(x_L)dx_L$ are the expected ordering, holding and shortage cost respectively.

The condition for achieving l_b^* (optimal backorder level) is to differentiate $G_b(l_b)$ with respect to l_b and set the differential equal to zero. The steps are as follows:

$$\frac{dG_b(l_b)}{dl_b} = -3\frac{h}{2} + \rho\int_{(R^*-l_b)}^{\infty} f(x_L)dx_L \qquad (2)$$

(from first principle and the product rule of differentiation).

But $f(x_L) = \frac{dF(x_L)}{dx_L}$ since it was assumed that the lead time demand is approximated by a continuous process.

Hence $\int f(x_L)dx_L = F(x_L)$

$$\Rightarrow \rho\int_{(R^*-l_b)}^{\infty} f(x_L)dx_L = \rho[F(x_L)]_{R^*-l_b}^{\infty}$$
$$= \rho[F(\infty) - F(R^* - l_b)]$$
$$= \rho[1 - F(R^* - l_b)] \qquad (3)$$

Since $F(\infty) = 1$ [9]

$$\Rightarrow \frac{dG_b(l_b)}{dl_b} = -3\frac{h}{2} + \rho[1 - F(R^* - l_b)] \qquad (4)$$

For optimality:

$$\frac{dG_b(l_b)}{dl_b} = 0 \qquad (5)$$

Hence, equation (4)

$$\rho[1 - F(R^* - l_b^*)] = \frac{3h}{2} \qquad (6)$$

But from probability theory, $1 - F(R^* - l_b^*) = P(R^* - l_b^*)$

$F\left(R^{*}-l_{b}^{*}\right)=P\left\{X_{L}\leq\left(R^{*}-l_{b}^{*}\right)\right\}$

$P\left(R^{*}-l_{b}^{*}\right)=P\left\{X_{L}\geq\left(R^{*}-l_{b}^{*}\right)\right\}$ and l_b^* is the optimal backorder level.

Therefore,

$$\rho\left[P\left(R^{*}-l_{b}^{*}\right)\right]=\frac{3h}{2} \tag{7}$$

$$P\left(R^{*}-l_{b}^{*}\right)=\frac{3h}{2\rho} \tag{8}$$

Equation (8) can be expressed in terms of probability statement as;

$$P\left(R^{*}-l_{b}^{*}\right)=P\left\{X_{L}\geq\left(R^{*}-l_{b}^{*}\right)=\frac{3h}{2\rho}\right\} \tag{9}$$

Since R^* is already known, the probability value corresponding to $\frac{3h}{2\rho}$ on the Poisson table under the mean lead time demand of μ is subtracted from R^* to obtain l_b^* since demand is assumed to follow a Poisson process. For $l_b = l_b^*$, $G_b(l_b)$ will be minimum.

5. FEATURES OF THE COST FUNCTION OF THE REGULAR ORDER ONLY WITH BACKORDER EFFECT

Figure 2 shows the shape of the expected inventory cost per order cycle $G_b(l_b)$ for the regular ordering policy only with backorder effect against various backorder value l_b.

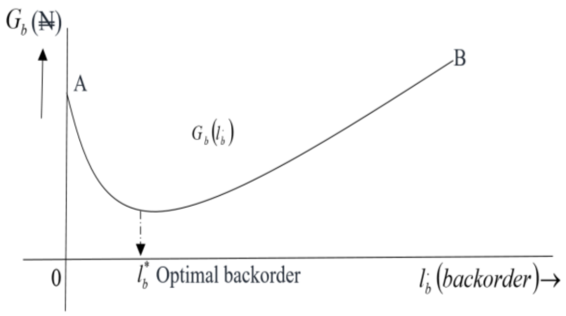

Figure 2: The curve of G_b with respect to l_b for the regular ordering policy only with backorder effect.

The optimal backorder level is the point of minimize cost on the cost curve beyond this point the cost start increasing.

6. COMPUTATIONAL RESULT

The monthly demand of low tension concrete poles (LTCP) for seven years (2010 - 2016) obtained from a pole manufacturing firm is shown in Table 1.

The following information is given about the firm's inventory process.

K_1 = ₦200,000, C = ₦90,000, ρ = ₦170,000, h = ₦9,000, τ_1 = 1 month

The value of the mean demand per year was estimated from Table 1 as γ =865units/yr. *Q**= 198 units, *R** = 93 units and l_b^* = 8 units.

Table 2 shows the expected order cycle cost for the regular ordering policy with backorder effect $G_b(l_b)$ for the considered backorder levels.

The lowest expected order cycle cost from the entries in Table 2 is N19,036,000 it occurs at a backorder level of 8 units, which is the optimal backorder level as determined. From backorder level of 0 to 8 units the expected order cycle cost decreases progressively. This range defines the allowable backorder level with the use of the policy with 8 units been the maximum allowable backorder level. Beyond this point, the expected order cycle cost starts increasing, hence requires a stock replenishment strategy to minimize the backorder effect.

The reason for the increasing cost is due to the higher number of units stock-out at the end of the order cycle with the increasing backorder level without stock replenishment.

The cost curve generated from plotting the various values of $G_b(l_b)$ against the corresponding values of l_b in Table 2

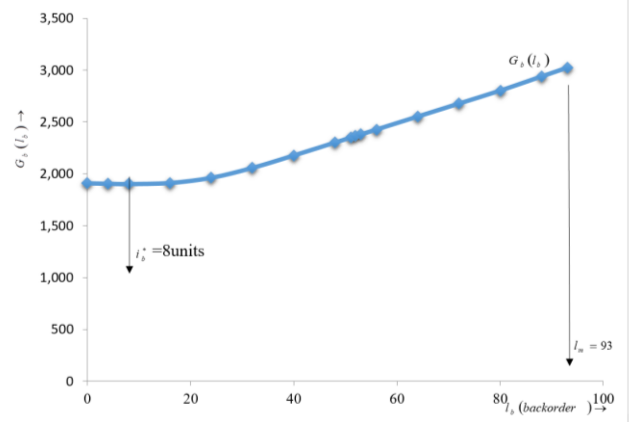

Figure 3: The curve of $G_b(l_b)$ against l_b

Table 1: Monthly Demand of Low Tension Concrete Electric Poles (LTCP) 2010 – 2016.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
2010	80	96	98	114	120	128	128	136	136	136	144	148
2011	100	104	90	62	55	55	44	40	39	42	50	54
2012	52	52	40	32	28	37	43	50	56	56	60	63
2013	40	35	42	38	34	47	59	60	67	78	78	80
2014	71	73	60	60	60	55	48	53	58	69	78	88
2015	95	112	121	125	130	130	132	136	140	125	108	108
2016	60	60	54	51	50	48	45	30	48	55	61	70

Source: Demand history from store department of O and O Technical Company, Kilometer 6 Benin Sapele Road, Benin City, Edo State, Nigeria.

Table 2: Different Expected Order Cycle Cost for the Regular Ordering Policy Only with Backorder Effect with Increasing Backorder Level

Backorder values l_b(unit)	Expected Order Cycle Costs for the Regular Ordering Policy $G_b(l_b)N \times 10^4$
0	1,910.5
4	1,906.1
8	1,903.6
16	1,913.7
24	1,963.7
32	2,059.7
40	2,179.4
48	2,304.2
51	2,351.3
52	2,366.8
53	2,382.5
56	2,429.4
64	2,554.6
72	2,679.8
80	2,805
88	2,941
93	3,026

7. CONCLUSION

An optimal backorder level has been determined for a lot-size reorder point system in this study. It was shown that the expected order cycle cost for the operating policy at this level of backorder is minimum. Furthermore, results showed that a range of backorder levels exist where even without stock replenishment that could attract extra cost, the operating policy cost decreases progressively. This characteristic property of the developed model makes it more attractive to use at the allowable range of backorders in terms of cost effectiveness. In the application of the Lot-size reorder point system to inventory control where complete backordering is allowed, it may not be necessary to replenish all backorder excess demand if such backorder demand are within the allowable range. Also the optimal backorder level should not be exceeded in the use of the policy for cost effectiveness.

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