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ECONOMIC LOAD DISPATCH OF NIGERIA INTEGRATED HIGH VOLTAGE GENERATION AND TRANSMISSION GRID USING BAT ALGORITHM

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ABSTRACT

A BAT algorithm to solve economic load dispatch problem of the Nigerian integrated power system is presented in this paper. Data from Transmission Company of Nigeria (TCN) national control centre, Osogbo was collected from January 2010 to December 2015 and was used to develop cost function for the twenty-one (21) thermal stations contributing power to the national grid. The cost functions were optimized in MATLAB® R2012a environment using Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Simulated Annealing (SA) and Bat Algorithm (BA). The result obtained shows that using the optimization tools GA, PSO, SA and BA the cost of fuel for generation are N13,239,100.0, N13,193,163.2, N13,139,672.0 and N13,105,495.2 respectively, which shows that BA gave the best result of minimizing fuel cost.

Keywords: *economic, load dispatch, bat algorithm, simulated annealing, optimization*

1. INTRODUCTION

In recent times around the world, power system has grown in complexity as regarding interconnection and increased power demand. Operators have shifted their focus towards better performance, increased customer centred operation, reduced cost, reliable and clean power [1]. The sole aim of Economic Load dispatch (ELD) is the operation of generation to produce energy at the lowest cost while satisfying system constraints by fulfilling the demand within several limits. This is not an easy task since there are a lot of factors need to be considered especially in the large interconnected power system. The primary objective of ELD is to schedule the committed generating units output so as to meet the required load demand at minimum cost satisfying all unit and system operational constraints [2].

The following conventional methods as reported by many authors have been used to solve the ELD problems: Bundle method, nonlinear programming, mixed integer linear programming, dynamic programming, quadratic programming, LaGrange relaxation method, network flow method, direct search method [3].

In practice ELD problems are nonlinear, non-convex problems that approaches local minimum severally due to the inclusion of valve point loading effect, multiple fuel options with many equality and inequality constraints. Conventional methods have failed to solve such problems as they are sensitive to initial estimates and converge into local optimal solution and computational complexity [4].

Literature is now full of many Biology-inspired metaheuristic algorithms because many researchers see it as an efficient way to deal with many hard

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combinatorial optimization problems and non-linear optimization constrained problems in general [5]. These algorithms are based on a particular successful mechanism of a biological phenomenon of Mother Nature in order to achieve optimization, such as the family of honey-bee algorithms, where the finding of an optimal solution is based on the foraging and storing the maximum amount of flowers' nectar [3]. A new algorithm that belongs in this category of the so-called nature inspired algorithms is the bat algorithm which is based on the echolocation behaviour of bats [3, 6].

The Nigerian national grid as at the year 2015 includes the twenty-one (21) thermal power station contributing power to the Nigeria Integrated power system, which includes eight privatised companies-thermal stations (Egbin ST, Sapele ST, Delta II-III, Delta IV, Geregu, Omotosho, Olorunsogo and Afam IV-V), Six NIPP-Stations (Sapele GT, Alaoji, Geregu, Olorunsogo, Omotosho and Ihovbor) and Seven IPP-Stations (Okpai, Afam VI, AES, Omoku, Ibom, Trans Amadi and Rivers). The grid is a rapidly growing power systems faced with complex operational challenges at different operating regimes. Indeed, it suffers from inadequate reactive power compensation leading to wide spread voltage fluctuations coupled with high technical losses and component over overloads during heavy system loading mode [7]. In Nigeria, the needs of electricity unity customers have been the reliability of electricity supply at an acceptable cost. The scheduling problem in electric power authority is as a result of the following constraints; fragile and inflexible state of the transmission network to wheel increased power levels, absence of state of the art dispatch facilities and high technical losses. Since power generation is dynamic, there is the need of sending the results of the ELD to all the power stations so as to meet up with the changes in the demand [2].

In this research paper therefore we present how the BAT algorithm can be used to solve the economic load dispatch of the Nigeria integrated power grid optimization problem. The effectiveness of proposed algorithm is demonstrated using the twenty-one thermal stations contributing power to the national grid as at the year 2015.

2. MATHEMATICAL FORMULATION OF THE ECONOMIC LOAD DISPATCH PROBLEM

2.1 Economic Dispatch

The main objective of economic load dispatch of electric power generation is to schedule the online generating unit outputs so as to meet the load demand at minimum operating cost while satisfying all units and operational constraints of the power system. The economic dispatch problem is a constrained optimization problem and the mathematical optimization problem is as follows:

$$\min F_T = \sum_{n=1}^{n} F_n(P_n) \tag{1}$$

Where F_T: total generation cost (Naira/hr) n: number of generators P_n: real power generation of nth generator (MW) $F_n(P_n)$: generation cost for P_n Subject to a number of power systems network equality and inequality constraints [8]. These constraints include:

2.1.1 System Active Power Balance

To satisfy the power balance equation, the total power generated should be the same as total load demand plus the total line

$$P_D + P_L - \sum_{n-1}^{n} P_n = 0 \tag{2}$$

Where P_D: total system demand (MW) P_L: transmission loss of the system (MW) [9].

2.1.2 Generation limits

Generation output of each generator should be laid between maximum and minimum limits. The corresponding inequality constraints for each generator are

$$P_{n,\min} \le P_n \le P_{n,\max} \tag{3}$$

Where $P_{n,min}$: minimum power output limit of nth generator (MW) $P_{n,max}$: maximum power output limit of nth generator (MW). The generator cost function F_n (P_n) is usually expressed as a quadratic polynomial:

$$F_n(P_n) = a_n P_n^2 + b_n P_n + c_n \tag{4}$$

Where a_n, b_n and c_n are fuel cost coefficients [10, 11].

2.1.3 Network Losses

Usually all the power generating stations are not located in the same place they are spread out geographically, the transmission network losses must be taken into account to achieve true economic dispatch. Network loss is a function of unit generation. To calculate network losses, two methods are in general used.

One is the penalty factors method and the other is the B coefficients method, network losses are expressed as a quadratic function [12]:

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \tag{5}$$

Where B_{mn} constants are called B coefficients or loss coefficients [12-14].

2.2 Bat Algorithm

Bats are extremely interesting and attractive animals. They are the only animals that give birth to live babies with wings and they also have advanced capability of echolocation (the use of reflected sound waves for finding things) [3, 15]. Most of bats use echolocation to a certain degree; among all the species, microbats are famous example as microbats use echolocation extensively, while megabats do not. Microbats use a type of sonar, called echolocation, to detect pry, avoid obstacles, and locate their roosting crevices in the dark [3, 16]. If we idealize some of the echolocation characteristics of microbats, virous bat-inpired algorithms or bat algorithms can be developed. For simplicity, in our approach, the following approximate or idealized rules were used:

1. All bats echolocation to sense distance, and they also know the difference between food/prey and background barriers.
2. Bats fly randomly with velocity vi at position xi with a fixed frequency f_{min} (or wavelength λ), varying wavelength λ (of frequency) of their emitted pulses and adjust the rate of pulse emission r $\subset [0,1]$, depending on the proximity of their targets;
3. Although the loudness can vary in many ways, we assume that the loudness varies from a large (positive) A_0 to a minimum value Amin.

Another obvious simplification is that no ray tracing is used in estimating the time delay and three dimensional topographies. In addition to these simplified assumptions, the following approximations have been used, for simplicity. In general the frequency f in a range of [20kHz, 500kHz] corresponds to a range of wavelengths $[\lambda_{\min}, \lambda_{\max}]$. For example, a frequency range of [20kHz, 500kHz] corresponds to a range of wavelengths from 0.7 mm to 1.7 mm [15].

In simulations, we use virtual bats naturally. We have to define the rules how their positions xi and velocities vi in a d-dimensional search space are updated. The new solutions x_i^0 and velocities v_i^0 at time step t are given by:

$$f_i = f_{\min} + (f_{\max} - f_{\min})\beta \qquad (6)$$

$$v_t^t = v_i^{t-1} + (x_i^t - x_0) f_i \qquad (7)$$

Where $\beta \subset [0,1]$ is a random vector drawn from a uniform distribution. Here x_0 is the current global best location (solution) which is located after comparing all the solutions among all the n bats. As the product $\lambda_i f_i$ is the other velocity increment, we can use either fi (or λ_i) to adjust the velocity change while fixing the other factor $\lambda_i (or, f_i)$, depending on the type of the problem of interest [17]. For the local search part, once a solution is selected among the current best solutions, a new solution for each bat is generated locally using random walk:

$$X_{new} = X_{old} + E.A^t \qquad (8)$$

Where $E \subset [0,1]$ is a random number, while = $A^t = \pi A_j^t \phi$ is the average loudness of all the bats at this time step [18].

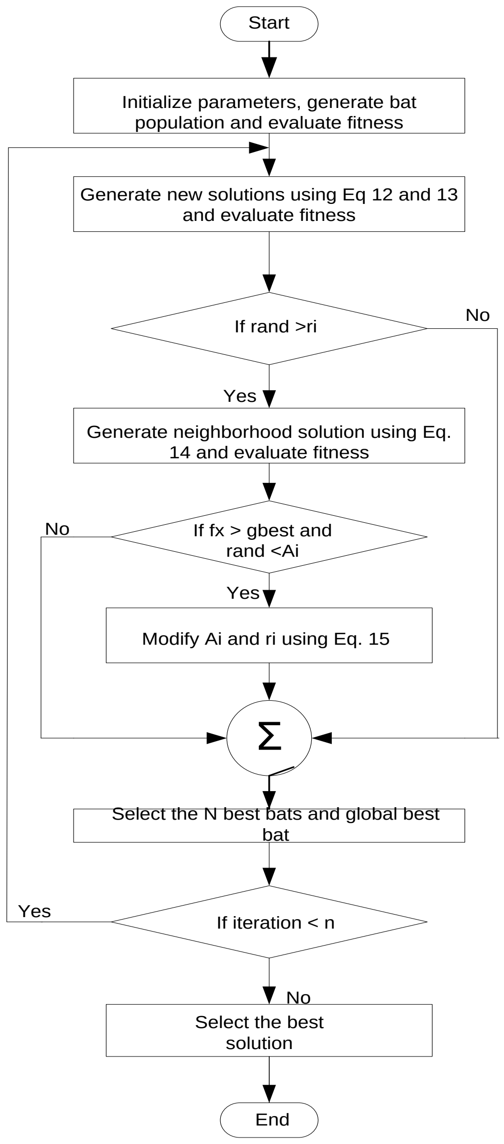

Figure 1: Flow Chart of Bat Algorithm

Based on the above approximations and idealization, the pseudo-code of the Bat Algorithm (BA) can be summarized below. The loudness usually decreases once a bat found its prey and rate of pulse emission increases. In our experiments, the loudness and pulse emission rate are varied once a solution is improved. The bat is moving towards optimal solution according to:

$$A^{t+1} = \alpha A^{t}, r^{t+1} = r^{0}\left[1 - e^{\gamma}\right] \qquad (9)$$

where α and γ are constants [19]. Figure 1 represents flowchart of the Bat Algorithm.

3. RESULTS AND DISCUSSION

This paper presents the development of Bat Algorithm for solving an economic dispatch problem with the standardized 2015 model of the Nigerian network which comprises of 24 integrated power stations, out of which 3 are hydro whilst the remaining stations are thermal. The programs are written in MATLAB software package.

The generator cost coefficients; generation limits which is the quadratic cost functions for the various thermal units have been developed using least square curve fit to their actual operating cost data over a period of seven years. Table 1 presents the cost coefficients so obtained for the twenty-one thermal units and their minimum and maximum loading limits. The loss coefficients is obtained via loss parameter estimation based on several power flow scenarios [20] and B-coefficient matrix of the power stations are given in (10). Economic Load Dispatch solution for the twenty-one thermal stations is solved using BAT algorithm.

The cost coefficients and power limits of 21 thermal power stations are given Table 1. The present work has been implemented in MATLAB 7.12.0.499 (R2012a) environment on a laptop computer (Window 8 pro, 64-bits operating system with processor speed of 2.8GHz and 4.0GB RAM) for the solution of economic load dispatch problem of the Nigerian power system. The programs have been written (in m file) to calculate the solution of economic load dispatch problems and its results are presented. The techniques have been successfully applied to dispatch problems by considering all equality and inequality constraints.

Table 2 shows the power generated with the method of bat algorithm by varying demand.

Table 1: Cost coefficients and power limits of Nigerian 21 thermal power stations.

S/N	Power Stations	a_i	b_i	c_i	$P_i$min (MW)	$P_i$max (MW)
1	EGBIN ST(GAS)	0.0000109	0.0284	3.92	118	1100
2	SAPELE ST	0.0000591	0.0226	8.10	33	223
3	DELTA II-III	0.0000757	0.0326	6.47	10	110
4	DELTA IV	0.0000743	0.0334	9.85	22	434
5	GEREGU	0.0000201	0.0313	1.25	14	450
6	OMOTOSHO	0.0000514	0.0312	4.70	29	480
7	OLORUNSOGO	0.0000294	0.0313	2.80	10	293
8	AFAM IV-V	0.0000834	0.0289	2.03	24	453
9	SAPELE GT NIPP	0.0000105	0.0227	5.60	30	373
10	ALAOJI NIPP	0.0000200	0.0332	3.00	34	87
11	GEREGU NIPP	0.0000223	0.0314	1.00	94	272
12	OLORUNSOGO NIPP	0.0000287	0.0313	1.70	31	422
13	OMOTOSHO NIPP	0.0000179	0.0313	2.64	20	225
14	IHOVBE NIPP	0.0000200	0.0294	1.00	91	120
15	OKPAI	0.0000326	0.0286	4.53	100	475
16	AFAM VI	0.0000115	0.0286	8.00	45	656
17	AES	0.0000133	0.0286	4.30	51	242
18	OMOKU	0.0000442	0.0314	1.30	3	65
19	IBOM	0.0000189	0.0312	4.60	10	101
20	TRANS AMADI	0.0000315	0.0311	1.00	4	31
21	RIVERS IPP	0.0000215	0.0318	6.00	20	160

Table 2: Results of Bat algorithm for the Nigerian 21 thermal power stations: Power dispatch (MW)

S/N	Power Stations	Load on Power Stations			
		2500 MW	3000 MW	3500 MW	4000 MW
1.	EGBIN ST (GAS)	496.5275	122.8602	605.3239	288.83
2.	SAPELE ST	34.9763	133.5573	185.8841	196.448
3.	DELTA II-III	10.001	19.9982	98.9096	88.8314
4.	DELTA IV	22.0039	61.0326	84.1693	107.2724
5.	GEREGU	191.2735	189.9218	360.2352	442.2986
6.	OMOTOSHO	320.8359	68.6196	181.1366	179.6815
7.	OLORUNSOGO	10.0625	29.9109	232.2028	278.7401
8.	AFAM IV-V	24	324.9404	156.3046	296.502
9.	SAPELE GT NIPP	101.8654	190.6673	140.6664	116.6221
10.	ALAOJI NIPP	34.0039	47.5819	44.4524	48.1721
11	GEREGU NIPP	94.0002	221.9574	136.9487	198.4023
12	OLORUNSOGO NIPP	31.0156	233.4077	418.4326	365.4274
13	OMOTOSHO NIPP	183.6467	119.7283	194.2104	141.7964
14	IHOVBE NIPP	92.1441	103.9853	93.2798	112.531
15	OKPAI	128.1011	272.8071	113.4729	233.0529
16	AFAM VI	480.9069	497.5713	74.8461	585.8328
17	AES	193.03	192.9541	171.6475	119.293
18	OMOKU	3	49.1825	8.7339	23.1311
19	IBOM	24.605	58.368	53.3774	63.081
20	TRANS AMADI	4	9.1717	8.6454	28.773
21	RIVERS IPP	20.0003	51.7764	137.1203	85.2807
TOTAL COST(Naira/hr)		170.8845	192.1326	210.8092	234.0267
P_{Loss}(MW)		11.74	12.33	13.67	15.73
CPU_{Time}(Sec.)		18.92	20.56	19.53	21.34
ITERATION		90	100	90	100

Table 3: Comparison results between BA, GA, PSO and SA

	COST IN NAIRA			
METHODS / LOADS	GA	PSO	SA	BA
2500MW	9,664,048,8	9,638,876.8	9,628,892.0	9,569,532.0
3000MW	10,877,059.2	10,858,428.0	10,779,596.8	10,759,425.6
3000MW	11,909,447.2	11,880,534.4	11,869,642.4	11,805,315.2
4000MW	13,239,100.0	13,193,163.2	13,139,672.0	13,105,495.2

In Table 2 it is seen that the results obtained by the method of Bat algorithm are satisfactory.

To verify the effectiveness of our method (BA), we vary power demand from 2500 MW to 4000 MW and the results obtained by bat algorithm are compared with other methods such as Genetic Algorithm (GA), Particle Swarm Optimization PSO and Simulated Annealing (SA), and the results of this comparison are shown in Table 3.

From the technical report of the Transmission Company of Nigeria a flat rate of Two hundred and eighty Naira (N280) was the cost of one millimeter standard cubic feet (mmscf) of gas as supplied by the Gas Company of Nigeria (GCN) for the years 2014 and 2015. One hundred and Ninety five Naira (N195) was the cost of one millimeter Standared Cubic feet (mmscf) of Gas as supplied by the Gas Company of Nigeria in 2013 and 2012. This goes for One hundred and thirty five Naira (N135) and Thirty Naira (N30) for the years 2011 and 2010 respectively. A flat rate of N200 for one US Dollar has been used for 2014 and 2015 [21]. N198 for One US Dollar has been use to calculate the actual cost of Gas for runing the Power Stations and the results are shown in Table 3 and Figure 2 have been used to illustrate this and it all show that BAT algorithm is the best optimization tool for the Nigerian Intergrated High Voltage Power System in terms of fuel cost as compared to GA, PSO and SA.

The results of comparison of table 3 are represented by the graph of the figure 2.

The result of comparison shows that BA gives a cost of N13,105,495.2, GA gives a cost of N 13,239,100.0, PSO gives N13,193,163.2 and SA gives N13,139,672.0 for 4000 MW power demand, which shows that the method of BAT algorithm gives better results as it minimizes the cost of fuel more than the other methods.

Additionally, the advantages of BAT algorithm are that BAT algorithm is easier to implement and there are fewer parameters to adjust.

4. CONCLUSIONS

The results obtained from applying the proposed BAT algorithm were compared to those obtained from GA, PSO and SA. The BAT algorithm has superior features, including quality of solution, stable iteration characteristics, and good computational efficiency. The comparison shows that BAT algorithm performs better than the other techniques.

Therefore, we are recommending BAT algorithm for the optimization of the Nigerian Power System as it is a promising technique for solving complicated problems in power system.

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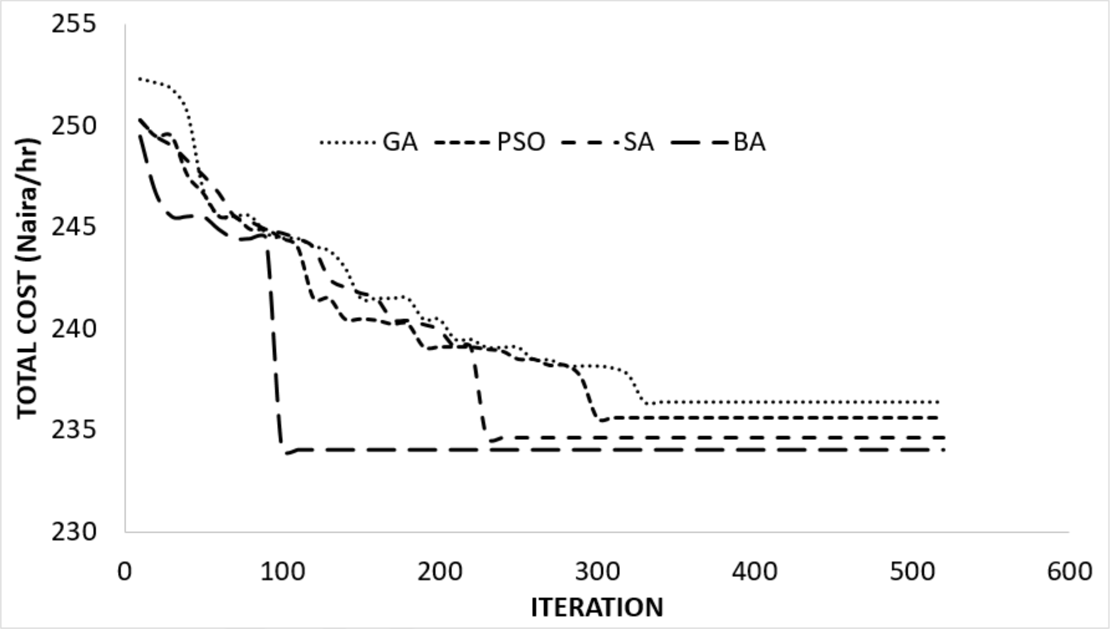

Figure 2: Graph showing the result of comparison between BA, GA, PSO and SA.

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APPENDIX

The loss co-efficient matrix of Nigerian 21 thermal power stations:

$$B_{ij}=\left[\begin{matrix}
0.0037 & 0.0002 & -0.0074 & 0.0005 & 0.012 & -0.0076 & -0.0036 & 0.0037 & 0.0074 & 0.0005 & 0.0120\\
0.0002 & 0.0103 & 0.0033 & -0.0031 & 0.0022 & 0.0005 & -0.0011 & 0.0074 & 0.0076 & 0.0023 & 0.0130\\
-0.0074 & 0.0033 & -0.0076 & 0.0023 & 0.013 & 0.0042 & 0.0152 & 0.0005 & 0.0023 & 0.0056 & 0.0149\\
0.0005 & -0.0031 & 0.0023 & 0.0056 & 0.0149 & 0.0004 & -0.0096 & 0.0120 & 0.0130 & 0.0149 & 0.0935\\
0.012 & 0.0022 & 0.013 & 0.0149 & 0.0935 & -0.0248 & -0.1354 & 0.0076 & 0.0042 & 0.0005 & 0.0248\\
-0.0076 & 0.0006 & 0.0042 & 0.0004 & -0.0248 & 0.0127 & 0.0649 & 0.0036 & 0.0152 & 0.0096 & 0.1354\\
-0.0036 & -0.0011 & 0.0152 & -0.0096 & -0.1354 & 0.0649 & 0.0769 & 0.0014 & 0.0017 & 0.0015 & 0.0019\\
0.0037 & 0.0074 & 0.0005 & 0.0120 & 0.0076 & 0.0036 & 0.0175 & 0.0017 & 0.0060 & 0.0013 & 0.0016\\
0.0074 & 0.0076 & 0.0023 & 0.0130 & 0.0042 & 0.0152 & 0.0184 & 0.0015 & 0.0013 & 0.0065 & 0.0017\\
0.0005 & 0.0023 & 0.0056 & 0.0149 & 0.0004 & 0.0096 & 0.0175 & 0.0019 & 0.0016 & 0.0017 & 0.0071\\
0.0120 & 0.0130 & 0.0149 & 0.0935 & 0.0248 & 0.1354 & 0.0154 & 0.0026 & 0.0015 & 0.0024 & 0.0030\\
0.0076 & 0.0042 & 0.0005 & 0.0248 & 0.0127 & 0.0649 & 0.0283 & 0.0022 & 0.0020 & 0.0019 & 0.0025\\
0.0036 & 0.0152 & 0.0096 & 0.1354 & 0.0649 & 0.0769 & 0.0037 & 0.0002 & -0.0074 & 0.0005 & 0.012\\
0.0014 & 0.0017 & 0.0015 & 0.0019 & 0.0026 & 0.0022 & 0.0002 & 0.0103 & 0.0033 & -0.0031 & 0.0022\\
0.0017 & 0.0060 & 0.0013 & 0.0016 & 0.0015 & 0.0020 & -0.0074 & 0.0033 & -0.0076 & 0.0023 & 0.0013\\
0.0015 & 0.0013 & 0.0065 & 0.0017 & 0.0024 & 0.0019 & 0.0005 & -0.0031 & 0.0023 & 0.0056 & 0.0149\\
0.0019 & 0.0016 & 0.0017 & 0.0071 & 0.00.30 & 0.0025 & 0.012 & 0.0022 & 0.013 & 0.0149 & 0.0935\\
0.0026 & 0.0015 & 0.0024 & 0.0030 & 0.0069 & 0.0032 & -0.0076 & 0.0005 & 0.0042 & 0.0004 & -0.0248\\
0.0022 & 0.0020 & 0.0019 & 0.0025 & 0.0032 & 0.0085 & -0.0036 & -0.0011 & 0.0152 & -0.0096 & -0.1354\\
0.0218 & 0.0107 & -0.0036 & -0.0011 & 0.0055 & 0.0033 & -0.0075 & 0.0005 & 0.0015 & 0.0005 & 0.0075\\
0.0107 & 0.0107 & -0.0001 & -0.0079 & 0.0026 & 0.0028 & 0.0019 & 0.0001 & 0.0045 & 0.0001 & 0.0136
\end{matrix}\right.$$

$$\left.\begin{matrix}
0.0076 & 0.0036 & 0.0014 & 0.0017 & 0.0015 & 0.0019 & 0.0026 & 0.0022 & 0.0218 & 0.0107\\
0.0042 & 0.0152 & 0.0017 & 0.0060 & 0.0013 & 0.0016 & 0.0015 & 0.0020 & 0.0107 & 0.0107\\
0.0004 & 0.0096 & 0.0015 & 0.0013 & 0.0065 & 0.0017 & 0.0024 & 0.0019 & -0.0040 & -0.0020\\
0.0248 & 0.1354 & 0.0019 & 0.0016 & 0.0017 & 0.0071 & 0.0030 & 0.0025 & -0.0011 & 0.0018\\
0.0127 & 0.0649 & 0.0026 & 0.0015 & 0.0024 & 0.0030 & 0.0069 & 0.0032 & 0.0055 & 0.0026\\
0.0649 & 0.0769 & 0.0022 & 0.0020 & 0.0019 & 0.0025 & 0.0032 & 0.0085 & 0.0033 & 0.0028\\
0.0037 & 0.0002 & -0.0074 & 0.0005 & 0.012 & 0.0076 & -0.0036 & 0.0136 & 0.0175 & 0.0184\\
0.0002 & 0.0103 & 0.0033 & -0.0031 & 0.0022 & 0.0005 & -0.0011 & 0.0175 & 0.0154 & 0.0283\\
-0.0074 & 0.0033 & -0.0076 & 0.0023 & 0.013 & 0.0042 & 0.0152 & 0.0184 & 0.0283 & 0.0161\\
0.0005 & -0.0031 & 0.0023 & 0.0056 & 0.0149 & 0.0004 & -0.0096 & 0.0075 & 0.0005 & 0.0015\\
0.012 & 0.0022 & 0.013 & 0.0149 & 0.0935 & -0.0248 & 0.1354 & 0.0019 & 0.0001 & 0.0045\\
-0.0076 & 0.0005 & 0.0042 & 0.0004 & 0.0248 & 0.0127 & 0.0649 & 0.0005 & 0.0075 & 0.0001\\
-0.0036 & -0.0011 & 0.0152 & -0.0096 & -0.1354 & 0.0649 & 0.0769 & 0.0075 & 0.0005 & 0.0015\\
0.0014 & 0.0017 & 0.0015 & 0.0019 & 0.0026 & 0.0022 & 0.0218 & 0.0107 & -0.036 & -0.0011\\
0.0017 & 0.0060 & 0.0013 & 0.0016 & 0.0015 & 0.0020 & 0.0107 & 0.0107 & -0.0001 & -0.0079\\
0.0015 & 0.0013 & 0.0065 & 0.0017 & 0.0024 & 0.0019 & -0.0004 & -0.0002 & 0.2459 & -0.0133\\
0.0019 & 0.0016 & 0.0017 & 0.0071 & 0.0030 & 0.0025 & -0.0011 & -0.0018 & -0.1328 & 0.0265\\
0.0026 & 0.0015 & 0.0024 & 0.0030 & 0.0069 & 0.0032 & 0.0055 & 0.0026 & 0.0118 & 0.0098\\
0.0022 & 0.0020 & 0.0019 & 0.0025 & 0.0032 & 0.0085 & 0.0033 & 0.0028 & -0.0792 & 0.0045\\
0.0055 & 0.0033 & 0.0026 & 0.0028 & -0.0118 & -0.0079 & 0.0098 & 0.0045 & 0.0216 & -0.0001\\
-0.0012 & 0.0297 & 0.0218 & 0.0093 & 0.0028 & 0.0093 & 0.0228 & 0.0017 & 0.0028 & 0.0017
\end{matrix}\right]$$

$$B_{i0}=[-0.0154 \quad -0.0119 \quad -0.0812 \quad -0.0881 \quad 0.0049 \quad -0.2679 \quad 0.2456 \quad 0.0218 \quad 0.0093 \quad 0.0028 \quad 0.0093$$
$$0.0228 \quad 0.0017 \quad 0.0028 \quad 0.0017 \quad 0.0179 \quad -0.0005 \quad -0.0006 \quad 0.0129 \quad 0.0067 \quad -0.0032]$$

$$B_{00}=0.2278$$