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A FINITE ELEMENT ANALYSIS OF AXISYMMETRIC FLOW OF AIR THROUGH THE TOP RISER OF CASTING USING THE STREAM FUNCTION MODEL

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ABSTRACT
Casting is a manufacturing process for making complex shapes of metal materials. Casting has two stages – filling process requiring a gating system and solidification process requiring a riser. The air in the mould cavity during casting is displaced through the riser by the molten metal in a very rapid manner necessitating the need to understand the air flow behaviour as it exit through the riser. The finite element method and the stream function model were used to analyze the axisymmetric flow of air through the top risers of casting. Results show that the velocity profile at any cross section is parabolic in shape with the maximum velocity at the centre. Comparing results with exact solution shows that the finite element result converged towards the exact solution.

Keywords: *stream function, axisymmetric flow, top riser, air, casting*

NOMENCLATURE

D	-Riser diameter
f^e	-Nodal forces
H	-Riser height
K^e	-Stiffness matrix
N	-Interpolation function
r & z	-Axis-symmetry coordinates variables
U_0	-Initial Velocity of Air
U_1	-Final Velocity of Air
u, v, w	-r, θ, z components of the velocity respectively
u_r,& u_z	-Axis-symmetry velocity components
ρ	-Density
ψ	-Stream function

1. INTRODUCTION

A key element in producing quality castings is the proper design and sizing of the gating and riser systems. A foundry can produce the best quality moulds, cores and molten metal and still end up with a poor quality casting by using poorly designed gating and riser systems. The main objective of a gating system is to lead clean molten metal poured from ladle to the casting cavity, ensuring smooth, uniform and complete filling. [1]. A riser is a reservoir built into a metal casting mould to prevent cavities due to shrinkage, because metals are less dense as liquids than as solids (with some exceptions). This can leave a void, generally at the last point to solidify. Risers prevent this by providing molten metal at the point of likely shrinkage, so that the cavity forms in the riser, not the casting [2]. The riser apart from serving as a reservoir to compensate for shrinkage during solidification, also serve as a channel through which the air displaced as a result of filling the mould cavity with the molten metal goes out of the mould cavity [3]. Numerous efforts have been made by castings engineers and researchers on gating system design over the past few decades [4 – 9]. Although there are general casting design rules and empirical equations for the gating ratio, pouring time, gating and riser system dimensions and optimization [5, 7], consideration has not been given prior to now to the air flow behaviour in the casting mould. Hence this work focuses on the analysis of the air flow through the top risers of casting using the stream function model.

2. FINITE ELEMENT ANALYSIS

2.1 The Model Equation

The model equation is the stream function model of axisymmetric flow of air in casting mould [13]

$$\frac{\partial^2\psi}{\partial z^2} = -\frac{\partial^2\psi}{\partial r^2} \tag{1}$$

2.2Weak Formulation of the Model Equation

The weighted residual of equation (1) is

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$$N_i(r,z)\left(\frac{\partial^2\psi}{\partial z^2}-\frac{\partial^2\psi}{\partial r^2}\right) \quad = 0 \qquad (2)$$

We integrated equation (2) over the element domain Ω_e

$$\int_{\Omega_e} N_i(r,z)\left(\frac{\partial^2\psi}{\partial z^2}+\frac{\partial^2\psi}{\partial r^2}\right)drdz = \int_{\Omega_e} N\left(\frac{\partial^2\psi}{\partial z^2}\right)drdz + \int_{\Omega_e} N\left(\frac{\partial^2\psi}{\partial r^2}\right)drdz = \int_{\Omega_e} N\left(\frac{\partial}{\partial z}\right)\left(\frac{\partial\psi}{\partial z}\right)drdz + \int_{\Omega_e} N\left(\frac{\partial}{\partial r}\right)\left(\frac{\partial\psi}{\partial r}\right)drdz = 0 \qquad (3)$$

$$let\,\frac{\partial\psi}{\partial r}=\frac{\partial\psi}{\partial z}=H \qquad (4)$$

$$\therefore \int_{\Omega_e} N\left(\frac{\partial H}{\partial z}\right)drdz + \int_{\Omega_e} N\left(\frac{\partial H}{\partial r}\right)drdz = 0 \qquad (5)$$

We integrated equation (5) by parts with respect to z and r using the basic relation

$$\int_{\Omega_e} udv = -\int_{\Omega_e} udv + uv|_{\Omega_e} \qquad (6)$$

$$\int_{\Omega_e} N\left(\frac{\partial^2\psi}{\partial z^2}\right)drdz = -\int_{\Omega_e} N\left(\frac{\partial\psi}{\partial z}\right)\left(\frac{\partial N}{\partial z}\right)drdz + \int_{T_2}^{T_2} N\frac{\partial\psi}{\partial r}n_r d\Omega_e \qquad (7)$$

$$\int_{\Omega_e} N\left(\frac{\partial^2\psi}{\partial z^2}\right)drdz = -\int_{\Omega_e}\left(\frac{\partial\psi}{\partial r}\right)\left(\frac{\partial N}{\partial r}\right)drdz + \int_{T_2}^{T_2} N\frac{\partial\psi}{\partial r}n_r d\Omega_e \qquad (8)$$

We Combined equations (7) and (8) using equation (3) to give the integration by parts.

$$\int_{\Omega_e}\left(\left(\frac{\partial[N]}{dz}\right)\left(\frac{\partial N}{\partial z}\right)+\left(\frac{\partial[N]}{dr}\right)\left(\frac{\partial N}{\partial r}\right)\right)drdz\{\psi\} = \int_{T_1}^{T_2} N(un_z - wn_r)d\Omega_e \qquad (9)$$

2.3 The Finite Element Model

The finite element model of equation (9) is given in matrix form as equation (10):

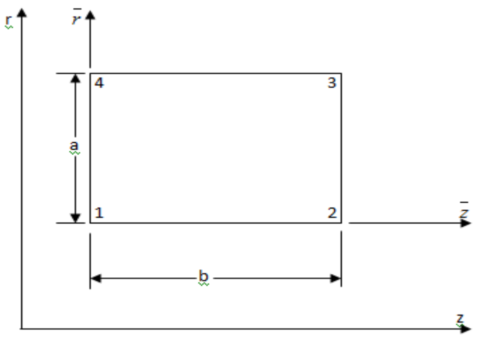

Figure 1a: Geometry of the element

$$[k^e]\{\psi\} = |f^e| \qquad (10)$$

$$[k^e] = \int_{\Omega_e}\left(\left(\frac{\partial[N]}{dz}\right)\left(\frac{\partial N}{\partial z}\right) + \left(\frac{\partial[N]}{dr}\right)\left(\frac{\partial N}{\partial r}\right)\right)drdz \qquad (11)$$

$$\left|\int^e\right| = \int_{T_1}^{T_2} N(un_z - wn_r)d\Omega\,|f^e| \qquad (12)$$

Let $(un_z - wn_r) = U_1$. Therefore,

$$|f^e| = \int_{T_1}^{T_2} N(U_1)d\Omega \qquad (13)$$

2.4 Derivation of the Finite Element Interpolation Functions

The stream function model over the domain of interest is discretized into finite elements having M nodes, using suitable interpolation model for $\psi^{(e)}$ in element e as [11]:

$$\psi \approx \psi(r,z) = \sum_{i=1}^{M} N_i(r,z)\psi_i = [N]\{\psi\} \qquad (14)$$

The interpolation functions $N_i(r,z)$ are the same as those developed for linear rectangular elements, with $x = r$ and $y = z$ [13]. This will enable us to evaluate the integrals of the K_{ij}^e and f_{ij}^e. Let's consider an approximation of the form:

$$N(r,z) = c_1 + c_2 r + C_3 + C_4 rz \qquad (15)$$

and use a rectangular element with sides a and b (Figure 1a).

We choose a local coordinate system (a,b) to derive the interpolation function. Thus equation (15) becomes

$$N(r,z) = c_1 + c_2 a + c_3 b + c_4 ab \qquad (16)$$

and require

$$N_1 = N(0,0) = c_1;\; N_2 = N(a,0) = c_1 + c_2 a N_3 = N(a,b) = c_1 + c_2 a + c_3 b + c_4 ab N_4 = N(0,b) = c_1 + c_3 b \qquad (17)$$

Solving for $c_i\,(i=1,\ldots,4)$, in equations (17) we obtained the following

$$c_1 = N_1;\quad c_2 = \frac{N_2 - N_1}{a};\quad c_3 = \frac{N_4 - N_1}{b}$$
$$c_4 = \frac{N_3 - N_4 + N_1 - N_1}{ab} \qquad (18)$$

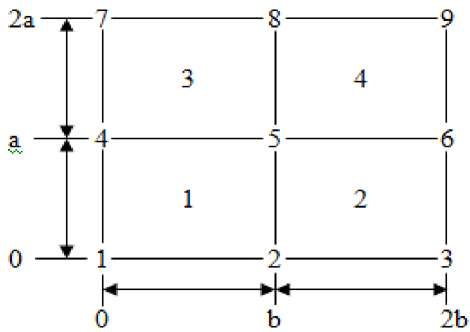

Figure 1b: Four Linear Rectangular Elements

We substituted equation (18) into equation (16) and noting that $a = r$and $b = z$, we obtained

$$N(r,z) = \left(1-\frac{r}{a}\right)\left(1-\frac{z}{b}\right)N_1 + \left(1-\frac{z}{b}\right)N_2 + \frac{rz}{ab}N_3 + \frac{z}{b}\left(1-\frac{r}{a}\right)N_4 = \phi_1 N_1 + \phi_2 N_2 + \phi_3 N_3 + \phi_4 N_4 \quad (19)$$

$$\phi_1\left(1-\frac{r}{a}\right)\left(1-\frac{z}{b}\right) \qquad \phi_2 = \frac{r}{a}\left(1-\frac{z}{b}\right) \qquad \phi_3 = \frac{rz}{ab} \qquad \phi_4 = \frac{z}{b}\left(1-\frac{r}{a}\right) \quad (20)$$

We differentiated equations (20) with respect to $r$and z

$$\frac{d\phi_1}{dr} = \left(-\frac{1}{a}\right)\left(1-\frac{z}{b}\right) = \left(\frac{-1}{a}+\frac{z}{ab}\right);\ \frac{d\phi_1}{dz} = \left(-\frac{1}{b}\right)\left(1-\frac{r}{a}\right) = \left(\frac{-1}{b}+\frac{r}{ab}\right);\ \frac{d\phi_2}{dr} = \frac{1}{a}\left(1-\frac{z}{b}\right) = \frac{1}{a}-\frac{z}{ab}$$

$$\frac{d\phi_2}{dz} = \frac{r}{ab};\ \frac{d\phi_3}{dr} = \frac{z}{ab};\ \frac{d\phi_3}{dz} = \frac{r}{ab};\ \frac{d\phi_4}{dr} = -\frac{z}{ab};\ \frac{d\phi_4}{dz} = \frac{1}{b}\left(1-\frac{r}{a}\right) = -\frac{r}{ab}+\frac{1}{b} \quad (21)$$

2.5 Evaluation of Finite Elements to obtain the Global System of Algebraic Equations

We rewrote [K^e] in equation (11) as the sum of four basic matrices and using the interpolation function of (21) evaluated the several K_{ij}^e of each matrix using Figures 1a and 1b

$$K_{ij}^e = (K^1 + K^2 + K^3 + K^4\ ) \quad (22)$$

$$K^1 = \begin{bmatrix} K_{11}^1 & K_{12}^1 & K_{13}^1 & K_{14}^1 \\ K_{21}^1 & K_{22}^1 & K_{23}^1 & K_{24}^1 \\ K_{31}^1 & K_{32}^1 & K_{33}^1 & K_{34}^1 \\ K_{41}^1 & K_{42}^1 & K_{43}^1 & K_{44}^1 \end{bmatrix} \quad K^2 = \begin{bmatrix} K_{11}^2 & K_{12}^2 & K_{13}^2 & K_{14}^2 \\ K_{21}^2 & K_{22}^2 & K_{23}^2 & K_{24}^2 \\ K_{31}^2 & K_{32}^2 & K_{33}^2 & K_{34}^2 \\ K_{41}^2 & K_{42}^2 & K_{43}^2 & K_{44}^2 \end{bmatrix} similarly\ for K^3 and K^4$$

$$K_{11}^1 = \int_0^b \int_0^a \left[\left(\left(-\frac{1}{a}\right)\left(-\frac{z}{b}\right)\right)^2 + \left(\left(-\frac{1}{b}\right)\left(-\frac{r}{a}\right)\right)^2\right] drdz = \frac{a}{3b} + \frac{b}{3a} \quad (23)$$

$$K_{12}^1 = \int_0^b \int_0^a \left[\left(-\frac{1}{a}+\frac{z}{ab}\right)^2 + \left(-\frac{1}{a}+\frac{r}{ab}\right)\left(-\frac{r}{ab}\right)\right] drdz = \frac{a}{6b} + \frac{b}{3a} \quad (24)$$

Similarly we evaluated all elements of K^1 matrix

$$K_{11}^2 = \int_0^b \int_a^{2a} \left[\left(\left(-\frac{1}{a}\right)\left(-\frac{z}{b}\right)\right)^2 + \left(\left(-\frac{1}{b}\right)\left(-\frac{r}{a}\right)\right)^2\right] drdz = \frac{a}{3b} + \frac{b}{3a} \quad (25)$$

$$K_{12}^2 = \int_0^b \int_a^{2a} \left[\left(-\frac{1}{a}+\frac{z}{ab}\right)^2 + \left(-\frac{1}{a}+\frac{r}{ab}\right)\left(-\frac{r}{ab}\right)\right] drdz = \frac{b}{3a} + \frac{5a}{6b} \quad (26)$$

Similarly we evaluated all elements of K^2matrix

$$K_{11}^3 = \int_0^{2b} \int_a^{a} \left[\left(\left(-\frac{1}{a}\right)\left(1-\frac{z}{b}\right)\right)^2 + \left(\left(-\frac{1}{b}\right)\left(1-\frac{r}{a}\right)\right)^2\right] drdz = \frac{a}{3b} + \frac{b}{3a} \quad (27)$$

$$K_{12}^3 = \int_b^{2b} \int_0^a \left[\left(-\frac{1}{a}+\frac{z}{ab}\right)^2 + \left(-\frac{1}{b}+\frac{r}{ab}\right)\left(-\frac{r}{ab}\right)\right] drdz = \frac{a}{6b} + \frac{b}{3a} \quad (28)$$

Similarly we evaluated all elements of K^3matrix

$$K_{11}^4 = \int_b^{2b} \int_a^{2a} \left[\left(\left(-\frac{1}{a}\right)\left(1-\frac{z}{b}\right)\right)^2 + \left(\left(-\frac{1}{b}\right)\left(1-\frac{r}{a}\right)\right)^2\right] drdz = \frac{a}{3b} + \frac{b}{3a} \quad (29)$$

$$K_{12}^4 = \int_b^{2b} \int_a^{2a} \left[\left(-\frac{1}{a}+\frac{z}{ab}\right)^2 + \left(-\frac{1}{b}+\frac{r}{ab}\right)\left(-\frac{r}{ab}\right)\right] drdz = \frac{-5a}{6b} + \frac{b}{3a} \quad (30)$$

Similarly we evaluated all elements of K^4matrix

Equation (22) became

$$[K^e] = \begin{bmatrix} \begin{bmatrix} \frac{a}{3b}+\frac{b}{3a} & \frac{a}{6b}+\frac{b}{3a} & \frac{-a}{6b}-\frac{b}{6a} & \frac{a}{3b}+\frac{b}{6a} \\ \frac{a}{6b}+\frac{b}{6a} & \frac{a}{3b}+\frac{b}{3a} & \frac{b}{6a}-\frac{a}{3b} & \frac{b}{6a}-\frac{a}{6b} \\ \frac{-a}{6b}-\frac{b}{6a} & \frac{b}{6a}-\frac{a}{3b} & \frac{a}{3b}+\frac{b}{3a} & \frac{a}{6b}-\frac{a}{3a} \\ \frac{a}{3b}+\frac{b}{6a} & \frac{b}{6a}-\frac{a}{6b} & \frac{a}{6b}-\frac{b}{3a} & \frac{a}{3b}+\frac{b}{3a} \end{bmatrix} + \begin{bmatrix} \frac{a}{3b}+\frac{b}{3a} & \frac{b}{3a}-\frac{5a}{6b} & \frac{5a}{6b}-\frac{b}{6a} & \frac{a}{3b}+\frac{b}{6a} \\ \frac{b}{3a}-\frac{5a}{6b} & \frac{7a}{3b}+\frac{b}{3a} & \frac{b}{6a}-\frac{7a}{3b} & \frac{b}{6a}-\frac{5a}{6b} \\ \frac{5a}{6b}-\frac{b}{6a} & \frac{b}{6a}-\frac{7a}{3b} & \frac{7a}{3b}+\frac{b}{3a} & \frac{5a}{6b}-\frac{b}{3a} \\ \frac{a}{3b}+\frac{b}{6a} & \frac{b}{6a}-\frac{5a}{6b} & \frac{5a}{6b}-\frac{b}{3a} & \frac{a}{3b}+\frac{b}{3a} \end{bmatrix} \\ + \begin{bmatrix} \frac{a}{3b}+\frac{b}{3a} & \frac{a}{6b}+\frac{b}{3a} & \frac{-a}{6b}+\frac{5b}{6a} & \frac{a}{3b}-\frac{5b}{6a} \\ \frac{a}{6b}+\frac{b}{3a} & \frac{a}{3b}+\frac{b}{3a} & \frac{-5b}{6a}-\frac{a}{3b} & \frac{5b}{6a}-\frac{a}{6b} \\ \frac{-a}{6b}+\frac{5b}{6a} & \frac{-5b}{6a}-\frac{a}{3b} & \frac{a}{3b}+\frac{7b}{3a} & \frac{a}{6b}-\frac{7b}{3a} \\ \frac{a}{3b}-\frac{5b}{6a} & \frac{5b}{6a}-\frac{a}{6b} & \frac{a}{6b}-\frac{7b}{3a} & \frac{a}{3b}+\frac{7b}{3a} \end{bmatrix} + \begin{bmatrix} \frac{a}{3b}+\frac{b}{3a} & \frac{-5a}{6b}+\frac{b}{3a} & \frac{5a}{6b}+\frac{5b}{6a} & \frac{a}{3b}-\frac{5b}{6a} \\ \frac{-5a}{6b}+\frac{b}{3a} & \frac{7a}{3b}+\frac{b}{3a} & \frac{-5b}{6a}-\frac{7a}{3b} & \frac{5b}{6a}-\frac{5a}{6b} \\ \frac{5a}{6b}+\frac{5b}{6a} & \frac{-5b}{6a}-\frac{7a}{3b} & 7\frac{a}{3b}+\frac{7b}{3a} & \frac{-5a}{6b}-\frac{7b}{3a} \\ \frac{a}{3b}-\frac{5b}{6a} & \frac{5b}{6a}-\frac{5a}{6b} & \frac{-5a}{6b}-\frac{7b}{3a} & \frac{a}{3b}+\frac{7b}{3a} \end{bmatrix} \end{bmatrix} \tag{31}$$

Typical riser height is twice riser diameter (*H=2D*) for top risers opened to atmospheric pressure [14]. The height (*z*) to diameter (*d*) ratio used in this work is *2:1*. Therefore, *b=z=2mm* and *a=r=0.5mm* and equation (31) became

$$[K^e] = \begin{bmatrix} \begin{bmatrix} 1.4167 & 1.375 & -0.70833 & 0.75 \\ 1.375 & 1.4167 & 0.5833 & 0.625 \\ -0.70833 & 0.5833 & 1.4167 & -1.2917 \\ 0.75 & 0.625 & -1.2917 & 1.4167 \end{bmatrix} + \begin{bmatrix} 1.4167 & 1.125 & -0.4583 & 0.75 \\ 1.125 & 1.9167 & 0.0833 & 0.4583 \\ -0.4583 & 0.0833 & 1.9167 & -1.125 \\ 0.75 & 0.4583 & -1.125 & 1.4167 \end{bmatrix} \\ \begin{bmatrix} 1.4167 & 1.375 & 3.2917 & -3.25 \\ 1.375 & 1.4167 & -3.4167 & 3.2917 \\ 3.2917 & -3.4167 & 9.4167 & -9.2917 \\ -3.25 & 3.2917 & -9.2917 & 9.4167 \end{bmatrix} + \begin{bmatrix} 1.4167 & 1.125 & -3.5417 & -3.25 \\ 1.125 & 1.9167 & -3.9167 & 3.5417 \\ 3.5417 & -3.9167 & 1.9167 & -9.5417 \\ -3.25 & 3.5417 & -9.5417 & 1.4167 \end{bmatrix} \end{bmatrix} \tag{33}$$

Next we evaluated the $|f^e|$ matrix using equation (13)

$$|f^e| = \int_{T_1}^{T_2} N(U_1)d\Omega = U_1 \int_{T_1}^{T_2} N_i d\Omega \tag{34}$$

The velocity of the molten metal surface entering the riser (boundary *ab*) is $U_1 = 1588.6288mm/s$ [3] and hence the vector $|f^e|$ will be nonzero only for elements 1 and 3 (Figure 1b). These nonzero vectors can be computed as follows [12]:

$$|f^e| = \int_{T_1}^{T_2} N(U_1)d\Omega = U_1 \int_{T_1}^{T_2} N_i d\Omega = \frac{U_1\Omega_{ij}}{2}[N_i] \tag{35}$$

Where Ω_{ji} denote the lengths of the edge z

$$|f^e| = \left(\frac{U_1\Omega_{ij}}{2}\right)\begin{Bmatrix} N_1^1 \\ N_2^1 \\ N_3^1 \\ N_4^1 \end{Bmatrix} = \left(\frac{U_1\Omega_{ij}}{2}\right)\begin{Bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{Bmatrix}; \quad |f^2| = |f^4| = 0; \quad |f^3| = \left(\frac{U_1\Omega_{ij}}{2}\right)\begin{Bmatrix} N_1^3 \\ N_2^3 \\ N_3^3 \\ N_4^3 \end{Bmatrix} = \left(\frac{U_1\Omega_{ij}}{2}\right)\begin{Bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \tag{36}$$

2.6 Assembly of the System Matrix

We assembled the system matrix using figure 1b and equation (10)

$$\begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 & K_{14}^1 & K_{13}^1 & 0 & 0 & 0 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^1 & K_{12}^2 & K_{24}^1 & K_{23}^1 + K_{14}^2 & K_{23}^2 & 0 & 0 & 0 \\ 0 & K_{21}^1 & K_{22}^2 & 0 & K_{24}^2 & K_{23}^2 & 0 & 0 & 0 \\ K_{41}^1 & K_{42}^2 & 0 & K_{24}^1 + K_{11}^3 & K_{43}^1 + K_{12}^3 & 0 & K_{14}^3 & K_{13}^3 & 0 \\ K_{31}^1 & K_{32}^1 + K_{41}^2 & K_{42}^2 & K_{34}^1 + K_{21}^3 & K_{33}^1 + K_{44}^2 + K_{22}^3 + K_{11}^4 & K_{43}^2 + K_{12}^4 & K_{24}^3 & K_{23}^3 + K_{14}^4 & K_{13}^4 \\ 0 & K_{31}^2 & K_{32}^2 & 0 & K_{34}^2 + K_{21}^4 & K_{33}^2 + K_{22}^4 & 0 & K_{24}^4 & K_{23}^4 \\ 0 & 0 & 0 & K_{41}^3 & K_{42}^4 & 0 & K_{44}^4 & K_{43}^4 & 0 \\ 0 & 0 & 0 & K_{31}^3 & K_{32}^3 + K_{41}^4 & K_{42}^4 & K_{34}^3 & K_{33}^3 + K_{44}^4 & K_{43}^4 \\ 0 & 0 & 0 & 0 & K_{31}^4 & K_{32}^4 & 0 & K_{34}^4 & K_{33}^4 \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \psi_6 \\ \psi_7 \\ \psi_8 \\ \psi_9 \end{Bmatrix}$$

$$= \left(\frac{U_1\Omega_{ij}}{2}\right)\begin{Bmatrix} N_1^1 \\ N_2^1 + N_1^2 \\ N_2^2 \\ N_4^1 + N_1^3 \\ N_3^1 + N_4^2 + N_2^3 + N_1^4 \\ N_3^2 + N_2^4 \\ N_4^3 \\ N_3^3 + N_4^4 \\ N_3^4 \end{Bmatrix} \tag{37}$$

Finally we evaluated the system matrix

$$\begin{bmatrix} 1.4167 & 1.375 & 0 & 0.75 & -0.70833 & 0 & 0 & 0 & 0 \\ 1.375 & 2.8334 & 1.125 & 0.625 & 1.3333 & 0.0833 & 0 & 0 & 0 \\ 0 & 1.125 & 1.9167 & 0 & 0.4583 & 00.0833 & 0 & 0 & 0 \\ 0.75 & 0.4583 & 0 & 2.8334 & 0.0833 & 0 & -3.25 & 3.2917 & 0 \\ -0.70833 & 1.3333 & 0.4583 & 0.0833 & 5.6668 & 3.0417 & 3.2917 & -6.6667 & 3.5417 \\ 0 & -0.4583 & 0.0833 & 0 & 0 & 3.8334 & 0 & 3.5417 & -3.9167 \\ 0 & 0 & 0 & -3.25 & 3.5417 & 0 & 9.4167 & -92917 & 0 \\ 0 & 0 & 0 & 3.2917 & -6.6667 & 3.5417 & -9.4167 & 10.8334 & -95417 \\ 0 & 0 & 0 & 0 & 3.5417 & -3.9167 & 0 & -95417 & 1.9167 \end{bmatrix}\begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \psi_6 \\ \psi_7 \\ \psi_8 \\ \psi_9 \end{Bmatrix}$$

$$= \begin{Bmatrix} 15888.6288 \\ 0 \\ 0 \\ 3177.2576 \\ 0 \\ 0 \\ 1588.6288 \\ 0 \\ 0 \end{Bmatrix} \tag{38}$$

$$\begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \psi_6 \\ \psi_7 \\ \psi_8 \\ \psi_9 \end{Bmatrix} = \begin{Bmatrix} 568.3382 \\ -898.4816 \\ 608.6242 \\ 2403.6911 \\ -305.0951 \\ -191.0951 \\ -1054.4221 \\ -59.4082 \\ -122.8500 \end{Bmatrix} \tag{39}$$

3. RESULTS AND DISCUSSION

Applying the boundary conditions with respect to figure 2 we obtained the following results as shown in Table 1:

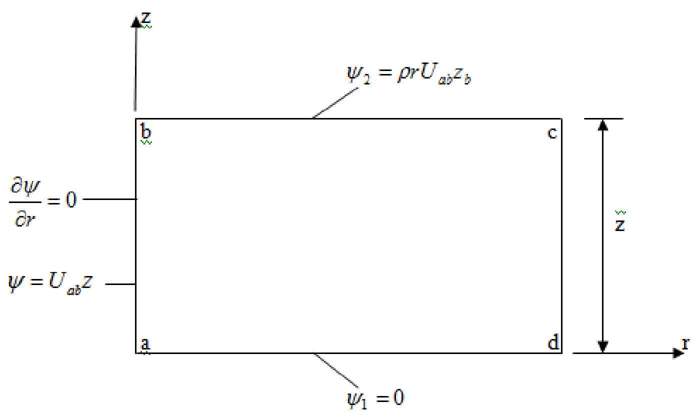

Figure 2: Computational domain and boundary conditions for the stream-function formulation.

Table 1: Finite Element Results from the Analysis of Axisymmetric Flow of Air through the top riser of Casting using the Stream Function Model

Nodes	r (mm)	z (mm)	Stream function ψ_{FEA}	Stream function ψ_{EXACT}	Velocity (mm/s) u_{FEA}	Velocity (km/hr)	Velocity (mm/s) u_{EXACT}
9	0	0	568.3382	582.2872	0	0	0
8	0	1000	-898.4816	- 920.2290	0	0	0
7	0	2000	608.6242	622.6530	0	0	0
6	250	0	2403.6911	2448.0007	2775.9873	9.9936	2823.4303
5	250	1000	-305.0951	- 310.6024	8378.6391	30.1631	8533.9811
4	250	2000	-191.2748	- 194.8008	352.1123	1.2676	358.2416
3	500	0	1054.4221	1080.3505	963.4160	3.4683	986.1959
2	500	1000	-59.4082	-59.3844	1722.8620	6.2023	1579.2206
1	500	2000	-122.8500	-122.6702	98.1312	0.3533	97.8900

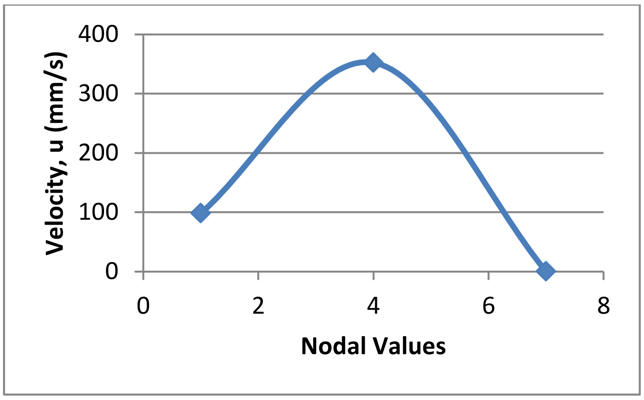

Figure 3(a)

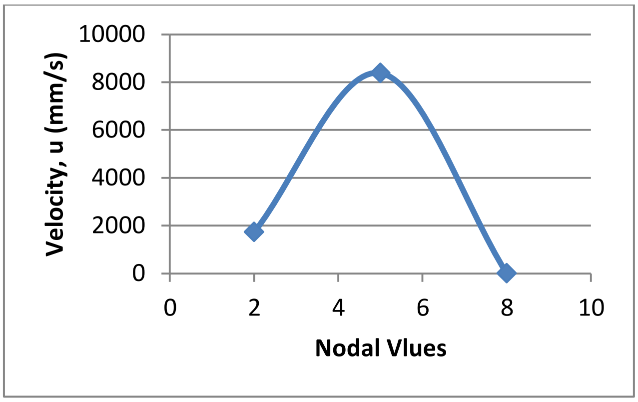

Figure 3(b)

Figure 3: Graph of velocity against nodal values showing (a) the Velocity profile at a cross section along the riser (b) the Velocity profile at a different cross section along the riser

Figures 3 of this work show that the velocity profile at any cross section is parabolic in shape with the maximum velocity at the centre. Figure 4 compares the finite element solution of velocity distribution along the top riser of casting mould at different locations and the exact solution. The result from the finite element analysis converges towards the exact solution.

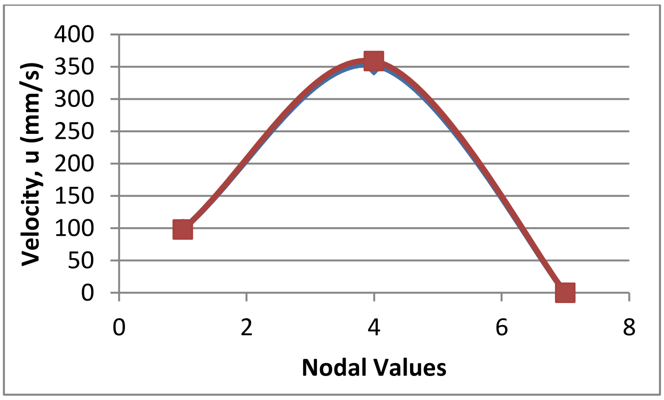

Figure 4: graph of velocity against nodal values showing the Velocity profile at different cross section of the riser this work and Exact solutions

4. CONCLUSION

In this work we have used the finite element method and the stream function model to analyze the axisymmetric flow of air through the top risers of casting. Results show that the velocity profile at any cross section is parabolic in shape with the maximum velocity at the centre. Comparing results with exact solution shows that the finite element result converged towards the exact solution. Before now all researches do was to develop empirical equations and optimized molten metal flow in casting mould. This work has gone further to analyze the behaviour of air flow in casting mould.

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