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FORMULATION AND IMPLEMENTATION OF MATHEMATICAL MODELS SUITABLE FOR DEFORMATION ANALYSIS OF STRUCTURES

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ABSTRACT
Deformation monitoring is the systematic measurement and tracking of the alteration in the shape or dimensions of an object as a result of stresses induced by applied loads. Deformation monitoring is a major component of logging measured values that may be used for further computation, deformation analysis, predictive maintenance and alarming. The main purpose of structural deformation monitoring scheme and analysis is to detect any significant movements of the structure. An effective approach is to model the structure by using well-chosen discrete points located on the surface of the structure which, when situated correctly, accurately depict the characteristics of the structure. The problem of Engineering Geodesy is the formulation of the Mathematical model that will reveal the characteristic of the structure being monitored. In this paper, the mathematical model that reveals the changes in shape of a circular reservoir structure is formulated and applied numerically. A numerical solution was carried out using the Mathematical model formulated.

Keywords: *Accuracy, deformation, diameter, Monitoring, Mathematical model.*

1. INTRODUCTION

Technical structures are subject to natural causes and to human interaction that may lead to collapse or structural failure. Deformation monitoring is primarily related to the field of Engineering Geodesy, but may be also related to other Engineering fields like civil, mechanical, construction, and Geosciences like geology, Geophysics etc. The measuring devices used for deformation monitoring depend on the application, the chosen method and the preferred measurement interval. Deformation analysis is concerned with determining if the displacement between two epochs is significant enough to trigger alarm. Deformation data must be subjected to rigorous mathematical and statistical analysis before any significant decision can be taken. A thorough check against specified limits is carried out, and reviewed to see if movements below specified limits imply potential risks.

Analysis of deformations of any type of a deformable body includes geometrical analysis and physical interpretation [1]. The ultimate goal of the geometrical analysis is to determine in the whole deformable object the displacement and strain fields in the space and time domains [1].

For numerical validation and implementation of our Mathematical model for shape object, circular reservoir storage tank was used. It is necessary to model the structure using well-chosen discrete monitoring points located on the surface of the structure at different monitoring points around the circumference of the Reservoir which, when situated correctly, accurately depict the characteristics of the structure.

Any movements of the monitoring point locations (and thus deformations of the structure) can be detected by maintaining the same point locations over time and by performing measurements to them at specified time intervals. This enables direct point displacement comparisons to be made [2]. Common approach for this method is to place physical targets on each chosen discrete point to which measurements can be made. However, there are certain situations in which monitoring the deformations of a large structure using direct displacement measurements of targeted points is uneconomical, unsafe, inefficient, or simply impossible. The reasons for this limitation vary, but it may be as

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simple as placement of permanent target prisms on the structure is too difficult or costly [2].

1.1 An Overview of Forcados and Environs

The Forcados Yokri field is located in OMLs 43 and 45 in Burutu Local Government Area of Delta State of Nigeria. It is bounded approximately by the co-ordinates 319453mE to 335236mE and 148355mN to 141626mN with area coverage of 244.64sq. Km (24464.0 hectares). The entire Forcados Yorkri area features meandering creeks and mangrove swamp. The land terrain is covered by mangrove forest. The area has a humid tropical climate characterized by high rainfall and high temperature [8].

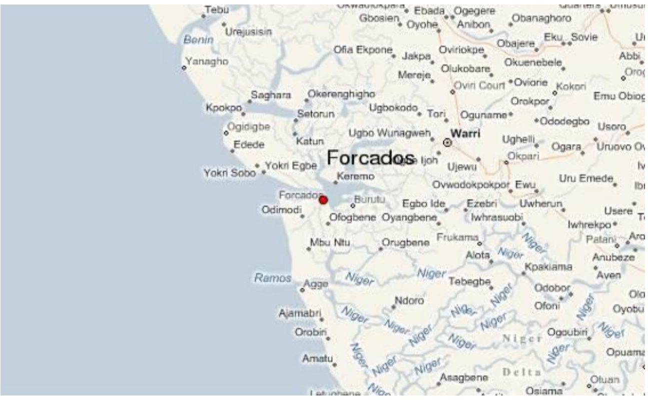

Figure 1.0: Niger Delta Area of Nigeria

The area is influenced by two climatic periods, the dry and wet seasons. The dry season is experienced between Novembers and March, while the wet season begins in April and ends in October and is typified by southwest trade winds laden with moisture blowing across the Atlantic Ocean [8].

1.2 Structural Integrity and Reservoir Characteristics

The structural integrity of bulk liquid storage tanks has been a major area of concern for the petrochemical industry for a number of years. Although API 653 remains the oil industry standard relative to tank inspection and maintenance, the frequency of testing and inspection can also be affected by various state and local regulations. The schedule of this inspection process may depend on a number of factors including the age of the tank, its proximity to ground water, the leak record of the tank, the date of the tank's last integrity test, the construction material of the tank, the product stored, soil conditions, previous corrosion rate calculation and a number of other factors.

Although most of the API 653 inspection elements can be addressed from the outside of the tank, accessing the tank floor from the outside of the tank is a major barrier in service inspection. Traditional methods of bottom inspection have required emptying the tank's product and thoroughly clearing and degassing prior to the allowance of personnel entry for a floor inspection. There are significant advantages to the tank owner if proper bottom inspection can be completed while the tank is in service. Robotic technology is now available as a method for determining the integrity of the above ground storage tank floors while the tank remains on line and in service.

Since API 653 now allows the use of robotics as an alternative method for assessing the condition of a tank floor as long as certain conditions are met, quantitative remaining tank floor life data can now be integrated with the balance of the external inspection element in order to provide a broader assessment of the tank's condition without taking the tank out of service.

Storage tanks for crude oil are common on farms and rural areas globally. These above- ground storage tanks are constructed of steel, and over the years, tens and thousands of the tanks have corroded and leaked petroleum products into the soil contaminating groundwater and our environment.

The tanks which were designed with floating roof plate of thickness 6.0mm were constructed in the 70s with the following properties [8].

1. Nominal Diameter:	76.2m
2. Temperature:	58ºf
3. Nominal Volume	100,000m^3
4. Height	22m
5. Liquid Gravity	0.85 to 0.9

6. API 650 @ atmospheric temperature

7. The hydrostatic pressure is 2 bars

I) Thickness: we have ten segments at vary thickness

i.	Bottom plate Thickness		6.0mm
II)	1st	plate thickness	34.5mm
III)	2nd	plate thickness	30.6mm
IV)	3rd	plate thickness	26.7mm
V)	4th	plate thickness	22.9mm
VI)	5th	plate thickness	19.0mm
VII)	6th	plate thickness	15.0mm
VIII)	7th	plate thickness	11.3mm
IX)	8th	plate thickness	10.0mm
X)	9th	plate thickness	10.0mm

Obviously, farm tank owners need to pay close attention to the installation and maintenance of crude oil storage tanks. What is needed to be known include: -:

- The health, environmental, safety and liability issues surrounding storage tanks
- State regulations affecting storage tank replacement and upgrading

- Tank replacement/ upgrade options.
- Considerations when buying and selling property.
- Additional sources of information.

Figure 2.0: - Forcados Reservoir

2. FORMULATION OF MATHEMATICAL MODEL

Mathematical model is the method of simulating real-life situations with mathematical equations to forecast their future behavior. Mathematical modeling uses tools such as decision-theory, queuing theory, and linear programming, and requires large amounts of number crunching. Analysis of the Geomatics Engineering component measurements before the project is actually undertaken is key in any deformation studies. The aim is to study the determination of the accuracy of the Geomatics techniques used for measurements and its availability as a monitoring technique for structural deformations, which can be also called "Pre-specified accuracy". We shall assume here that all components of the Geomatics measurements are free of bias caused by systematic errors. This means that variances, or standard deviation, can be used as measures of accuracy. We shall further assume that all measurement components are independent. We shall formulate and implement the Mathematical models for 2D and 1D deformation analysis in the sections presented below.

2.1 Mathematical Model for 2D Deformation Analysis

Given a figure in plane, on the edge of which

$$f(x,y) = 0 \qquad (1)$$

$$(x,y), \;\; i = 1, \ldots, n \qquad (2)$$

The edges of this figure have been monitored for n times. By assuming the shape of the figure depends on exterior parameters a_j, j = 1, …, m, we have

$$F\left(x, y; a_j\right) = 0 \qquad (3)$$

Let us call the observation (x_i, y_i), $i = 1, \ldots, n$. We construct approximate value that are sufficiently close to the observations, and for which holds

$$f\left(\Delta x_i, y_i^{(0)}; a_j^{(0)}\right) = 0 \qquad (4)$$

Recalling Taylor expansion, we have:

$$f\left(x_i, y_i; a_j\right) = f\left(x_i^{(0)}, y_i^{(0)}; a_j^{(0)}\right) + \frac{\partial f}{\partial x}\Big|_{x-x_i^{(0)}} \Delta x_i + \frac{\partial f}{\partial y}\Big|_{y-y_i^{(0)}} \Delta y_i + \sum_{j=1}^{n} \frac{\partial f}{\partial x}\Delta aj \qquad (5)$$

Where

$$\Delta x_i = x_i - x_i^{(0)}, \qquad \Delta y_i = y_i - y_i^{(0)}, ja, \Delta aj = aj - a_j^{(0)} \qquad (6)$$

$$The\ expression \;\; f\left(x_i, y_i; a_j\right) - f\left(x_i^{(0)}, y_i^{(0)}; a_j^{(0)}\right) \; (must\ vanish)$$

This is how we obtain the final observation equation as:

$$\frac{\partial f}{\partial x_i}\Delta x_i + \frac{\partial f}{\partial y_i}\Delta y_i + \sum_{j=1}^{m} \frac{\partial f}{\partial a_j}\Delta aj = 0 \qquad (7)$$

the left-hand side of equation (7) constitutes a linear combination of the edge point observation (x_i, y_i) which is computable if the partial derivatives of the edge function f (x, y; a_j) with respect to x and y can be computed. This is the same for the design matrix $\frac{\partial f}{\partial a_i}$.

The observation equation for curve in 3D surface, can be we written thus;

$$\frac{\partial f}{\partial x_i}\Delta x_i + \frac{\partial f}{\partial y_i}\Delta y_i + \frac{\partial f}{\partial z_i}\Delta z_i \sum_{j=1}^{m} \frac{\partial f}{\partial a_j}\left(a_j - a_j^0\right) = 0 \quad (9)$$

If the observation equations are of equal weight and are precise, we have that;

$$\|\nabla f\| = \sqrt{\left(\left(\frac{\partial f}{\partial x_i}\right)^2 + \left(\frac{\partial f}{\partial y_i}\right)^2 + \left(\frac{\partial f}{\partial z_i}\right)^2\right)} \qquad (10)$$

These values do not depend on the values of x_i and y_i.

The replacement observation can be written thus;

$$l_i \equiv \frac{\partial f}{\partial x_i}\Delta x_i + \frac{\partial f}{\partial y_i}\Delta y_i \frac{\partial f}{\partial z_i}\Delta z_i \qquad (11)$$

Considering a circular structure like Cylindrical Reservoir with nominal diameter of 76.2m, we write the equation of a circle as thus;

$$x^2 + y^2 = r^2 \qquad (12)$$

Where; r is the radius, x and y are the planimetry initial coordinates of the monitoring stations. The equation for a freely positioned circle is given as;

$$(x - X)^2 + (y - Y)^2 = r^2 \qquad (13)$$

Where X and Y are the coordinates of the monitoring station for the 2nd epoch of observation, x and y and the initial (first) epoch of observation.

The function f is given as;

$$f\left(x, y, a_j\right) = (x - X)^2 + (y - Y)^2 = r^2 \qquad (14)$$

$$\text{The vector } a_j \qquad a = \begin{bmatrix} X \\ Y \\ r \end{bmatrix} \qquad (15)$$

Partial derivatives of equation (14) gives

$$\begin{bmatrix} \frac{\partial f}{\partial x} = 2(x - X), & \frac{\partial f}{\partial y} = 2(y - Y) \\ \frac{\partial f}{\partial a_1} = -2(x - X), \frac{\partial f}{\partial a_2} = -2(y - Y), & \frac{\partial f}{\partial a_2} = -2r \end{bmatrix} \quad (16)$$

These partial derivatives are evaluated at suitable approximate values: $X^{(1)}, Y^{(1)}, r^{(1)}$

The observation equation can be written thus;

$$\left(x_i^{(0)} - X^{(1)}\right)\Delta x_i + \left(y_i^{(0)} - Y^{(1)}\right)\Delta y_i - \left(x_i^{(0)} - X^{(1)}\right)\Delta X - \left(y_i^{(0)} - Y^{(1)}\right)\Delta Y - r^{(0)}\nabla r = 0 \quad (17)$$

From which the linearized unknowns $\Delta X, \Delta Y, \nabla r$ which are the deformation values can be solved if the number of equations exceed three. From the above, we can say that;

$$\begin{bmatrix} x_i^{(0)} = X^{(1)} + r^{(1)} \cos \varphi_i^{(0)} \\ y_i^{(0)} = Y^{(1)} + r^{(1)} \sin \varphi_i^{(0)} \end{bmatrix} \quad (18)$$

where; φ is the direction angle.

For a circular reservoir with 20 monitoring points, the directional angles are gotten by dividing 360 by numbers of studs (monitoring points) i.e. 360/20. The first monitoring point directional angle is 18^0, the second is 36^0, 3rd is 54^0 etc.

2.2 Mathematical Model for 1-D Analysis

Subsidence is the sinking or settling of the ground surface. This can be as a result of the gradual settlement or sudden sinking of the Earth surface owing to subsurface movement of earth materials. Structures on surface of the earth continue to experience settlement immediately after construction. The problem of Geodesy is how to model rate of settlement.

Assuming a two epoch of observation, we have; [5]

$$\begin{matrix} h_i(t_1) i = 1, \dots, n \\ h_i(t_2) i = 1, \dots, n \end{matrix} \quad (19)$$

and the corresponding variance matrices of the height are available

$Q(t_1)$ and $Q(t_2)$

The variance and covariance factor are given thus

$$\left.\begin{matrix} Var\left(h_i^{(k)}\right) = \frac{\sum \bar{x} - x_i}{n} \\ Cov(x, y) = \sum \frac{(x_i - x)(y_i - y)}{n - 1} \end{matrix}\right\} \quad (20)$$

By calculating the height difference between the two epochs and their variance, assuming that the measurement made at t_1 and t_2 are statistically independent of each other; [5]

$$\left.\begin{matrix} \Delta h_i^{(1)} = h_i^{(1)}(t_2) - h_i^{(1)}(t_1), i = 2, \dots, n \\ Q_{\Delta h \Delta h}^{(1)} = Q^{(1)}(t_1) + Q^{(1)}(t_2) \end{matrix}\right\} \quad (21)$$

The deformation can be computed thus;

$$Q = \left[\Delta h^{(1)}\right]^T \left[\Delta h^{(1)}\right]\left[Q_{\Delta h \Delta h}^{(1)}\right]^{-1} \quad (22)$$

3. RESULTS AND DISCUSSION

To develop a reliable and cost-effective monitoring system of any of the storage oil tanks, deformation monitoring scheme consisted of measurements made to the tank from several monitoring stations (occupied stations), which are chosen in the area around the tank, that are referred to several reference control points. The geodetic instruments are setup at these monitoring stations (occupied stations) and observations carried out to determine the coordinates of monitoring points on the tank surface. The tanks under study are located in Forcados Terminal in Delta State, Nigeria. The nominal diameter 76.2m constructed in the Seventies [2]. For the purpose of this paper, only tank 2 was examined, and the data used for measurement carried out in 2003 and 2004 and at full oil levels.

The circular cross section of the oil storage tank is divided into several monitoring points distributed to cover the perimeter of this cross section called studs. These monitoring points are situated at equal distances on the outer surface of the tank. The (studs) points are fixed, with each stud carrying an identification number and made permanent throughout the life of the tank [2]. The purpose is to maintain the same monitoring point during each epoch of observations [9]. To determine the coordinates of occupied stations around the monitored oil storage tank, traverse network was run from the control points around the vicinity of the tank to connect the bench marks used for the monitoring. Table 1 are the coordinates of monitoring station for 2003 and 2004 epochs, while table 2 are the direction angles and monitoring points coordinates.

Table 1: Monitoring points coordinate for year 2003 and 2004

	Year 2004	
Studs	Easting	Northing
1	325103.304	148734.500
2	325114.486	148729.925
3	325123.840	148721.794
4	325129.769	148711.957
5	325132.586	148700.395
6	325131.674	148688.458
7	325127.104	148677.404
8	325119.604	148668.559
9	325109.164	148662.061
10	325097.484	148659.285
11	325085.603	148660.238
12	325074.539	148664.812
13	325065.475	148672.537
14	325059.236	148682.723
15	325056.462	148694.306
16	325057.347	148706.000
17	325061.934	148717.250
18	325069.714	148726.376
19	325079.833	148732.569
20	325091.491	148735.395

	Year 2003	
Studs	Easting	Northing
1	325103.320	148734.490
2	325114.440	148729.930
3	325123.860	148721.810
4	325129.770	148711.960
5	325132.580	148700.380
6	325131.690	148688.500
7	325127.200	148677.460
8	325119.590	148668.560
9	325109.190	148662.080
10	325097.470	148659.290
11	325085.590	148660.240
12	325074.560	148664.790
13	325065.490	148672.530
14	325059.250	148682.720
15	325056.450	148694.290
16	325057.320	148705.980
17	325061.920	148717.250
18	325069.690	148726.360
19	325079.860	148732.580
20	325091.460	148735.390

Table 2: Direction angles and monitoring points coordinates

Studs	$\varphi^{(0)}$	x_i	y_i	$x_i^{(0)}$	$y_i^{(0)}$	Δx_i	Δy_i	$x_i^{(0)} - X^{(1)} = (r\cos\varphi)$	$y_i^{(0)} - y^{(1)} = (r\sin\varphi)$
1	18	325103.304	148734.500	325103.320	148734.490	-0.016	0.010	0.113574474	-0.164285707
2	36	325114.486	148729.925	325114.440	148729.930	0.046	-0.005	-0.022009755	-0.080500814
3	54	325123.840	148721.794	325123.860	148721.810	-0.020	-0.016	-0.142641291	0.069550082
4	72	325129.769	148711.957	325129.770	148711.960	-0.001	-0.003	-0.166367101	0.010673424
5	90	325132.586	148700.395	325132.580	148700.380	0.006	0.015	-0.077068662	0.044632927
6	108	325131.674	148688.458	325131.690	148688.500	-0.016	-0.042	0.064587651	0.158669341
7	126	325127.104	148677.404	325127.200	148677.460	-0.096	-0.056	0.162365272	-0.088237663
8	144	325119.604	148668.559	325119.590	148668.560	0.014	-0.001	0.149837353	0.107901758
9	162	325109.164	148662.061	325109.190	148662.080	-0.026	-0.019	0.035514943	-0.17070996
10	180	325097.484	148659.285	325097.470	148659.290	0.014	-0.005	-0.102935132	-0.124218528
11	198	325085.603	148660.238	325085.590	148660.240	0.013	-0.002	-0.171454518	-0.020865088
12	216	325074.539	148664.812	325074.560	148664.790	-0.021	0.022	-0.123493434	0.168903628
13	234	325065.475	148672.537	325065.490	148672.530	-0.015	0.007	0.008364962	-0.169498881
14	252	325059.236	148682.723	325059.250	148682.720	-0.014	0.003	0.134540483	-0.16804253
15	270	325056.462	148694.306	325056.450	148694.290	0.012	0.016	0.169313696	0.14363339
16	288	325057.347	148706.000	325057.320	148705.980	0.027	0.020	0.089060841	0.017731618
17	306	325061.934	148717.250	325061.920	148717.250	0.014	0.000	-0.051696972	0.167522887
18	324	325069.714	148726.376	325069.690	148726.360	0.024	0.016	-0.157333591	0.051263901
19	342	325079.833	148732.569	325079.860	148732.580	-0.027	-0.011	-0.156083025	0.065643225
20	0	325091.491	148735.395	325091.460	148735.390	0.031	0.005	0.172	-0.084984323

Thus, we obtain

$$\left(x_i^{(0)} - X^{(1)}\right)\Delta x_i + \left(y_i^{(0)} - Y^{(1)}\right)\Delta y_i \begin{bmatrix} -0.0035 \\ -0.0006 \\ 0.0017 \\ 0.0001 \\ 0.0002 \\ -0.0077 \\ -0.0106 \\ 0.002 \\ 0.0023 \\ -0.0008 \\ -0.0022 \\ 0.0063 \\ -0.0013 \\ -0.0024 \\ 0.0043 \\ 0.0028 \\ -0.0007 \\ -0.003 \\ 0.0035 \\ 0.0049 \end{bmatrix}$$

$$\left(x_i^{(0)} - X^{(1)}\right)\Delta x_i + \left(y_i^{(0)} - Y^{(1)}\right)\Delta y_i = x_i^{(0)} - X^{(1)}, y_i^{(0)} - Y^{(1)} - r_i^{(0)}, r^{(1)}$$
$$= \left[x_i^{(0)} - X^{(1)}, y_i^{(0)} - Y^{(1)} - r_i^{(0)}, r^{(1)}\right]^T \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta r \end{bmatrix}$$

Where; $r_i^{(0)}$, $r^{(1)}$ are the nominal radius and observed radius at 2nd epoch

(rcosφ)	(rsinφ)	$(x_i^{(0)} - X^{(1)})\Delta x_i + (yi^{(0)} - Y^{(1)})\Delta yi$
0.113574474	-0.164285707	-0.003460049
-0.022009755	-0.080500814	-0.000609945
-0.142641291	0.069550082	0.001740025
-0.166367101	0.010673424	0.000134347
-0.077068662	0.044632927	0.000207082
0.064587651	0.158669341	-0.007697515
0.162365272	-0.088237663	-0.010645757
0.149837353	0.107901758	0.001989821
0.035514943	-0.17070996	0.002320101
-0.102935132	-0.124218528	-0.000819999
-0.171454518	-0.020865088	-0.002187179
-0.123493434	0.168903628	0.006309242
0.008364962	-0.169498881	-0.001311967
0.134540483	-0.16804253	-0.002387694
0.169313696	0.14363339	0.004329899
0.089060841	0.017731618	0.002759275
-0.051696972	0.167522887	-0.000723758
-0.157333591	0.051263901	-0.002955784
-0.156083025	0.065643225	0.003492166
0.172	-0.084984323	0.004907078

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta r \end{bmatrix} = \begin{bmatrix} -0.0035 \\ -0.0006 \\ 0.0017 \\ 0.0001 \\ 0.0002 \\ -0.0077 \\ -0.0106 \\ 0.002 \\ 0.0023 \\ -0.0008 \\ -0.0022 \\ 0.0063 \\ -0.0013 \\ -0.0024 \\ 0.0043 \\ 0.0028 \\ -0.0007 \\ -0.003 \\ 0.0035 \\ 0.0049 \end{bmatrix} \times \begin{bmatrix} 0.1136 & -0.1643 & 0.186 \\ -0.0220 & -0.0805 & 0.186 \\ -0.1426 & 0.0696 & 0.186 \\ -0.1664 & 0.0107 & 0.186 \\ -0.0771 & 0.0446 & 0.186 \\ 0.0646 & 0.1587 & 0.186 \\ 0.1624 & -0.0882 & 0.186 \\ 0.1498 & 0.1079 & 0.186 \\ 0.0355 & -0.1707 & 0.186 \\ -0.1029 & -0.1242 & 0.186 \\ -0.1715 & -0.0209 & 0.186 \\ -0.1235 & 0.2689 & 0.186 \\ 0.0084 & -0.1695 & 0.186 \\ 0.1345 & -0.1680 & 0.186 \\ 0.1693 & 0.1436 & 0.186 \\ 0.0891 & 0.0177 & 0.186 \\ -0.0517 & 0.1675 & 0.186 \\ -0.1573 & 0.0513 & 0.186 \\ -0.1561 & 0.0656 & 0.186 \\ 0.1720 & -0.0850 & 0.186 \end{bmatrix}$$

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta r \end{bmatrix} = [A]^T \times [B] \quad = \quad [\nabla X \quad \Delta Y \quad \Delta r]$$
$$= [0.0014 \quad 0.0023 \quad -0.1761]^T$$

The total displacement in x, y and r is given as

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta r \end{bmatrix} = \begin{bmatrix} -0.0014 \\ 0.0023 \\ -0.1761 \end{bmatrix}$$

The deformation at each monitoring point can be deduced using equation (18a)

$$\delta_{xyr} = A^T \times A \times B \qquad (18a)$$

$$\begin{bmatrix}-0.0035\\-0.0006\\0.0017\\0.0001\\0.0002\\-0.0077\\-0.0106\\0.002\\0.0023\\-0.0008\\-0.0022\\0.0063\\-0.0013\\-0.0024\\0.0043\\0.0028\\-0.0007\\-0.003\\0.0035\\0.0049\end{bmatrix}^T \times \begin{bmatrix}-0.0035\\-0.0006\\0.0017\\0.0001\\0.0002\\-0.0077\\-0.0106\\0.002\\0.0023\\-0.0008\\-0.0022\\0.0063\\-0.0013\\-0.0024\\0.0043\\0.0028\\-0.0007\\-0.003\\0.0035\\0.0049\end{bmatrix} \times \begin{bmatrix}0.1136 & -0.1643 & 38.1860\\-0.0220 & -0.0805 & 38.1860\\-0.1426 & 0.0696 & 38.1860\\-0.1664 & 0.0107 & 38.1860\\-0.0771 & 0.0446 & 38.1860\\0.0646 & 0.1587 & 38.1860\\0.1624 & -0.0882 & 38.1860\\0.1498 & 0.1079 & 38.1860\\0.0355 & -0.1707 & 38.1860\\-0.1029 & -0.1242 & 38.1860\\-0.1715 & -0.0209 & 38.1860\\-0.1235 & 0.2689 & 38.1860\\0.0084 & -0.1695 & 38.1860\\0.1345 & -0.1680 & 38.1860\\0.1693 & 0.1436 & 38.1860\\0.0891 & 0.0177 & 38.1860\\-0.0517 & 0.1675 & 38.1860\\-0.1573 & 0.0513 & 38.1860\\-0.1561 & 0.0656 & 38.1860\\0.1720 & -0.0850 & 38.1860\end{bmatrix}$$

$$\delta_{xyr} = \begin{bmatrix}0.0000 & -0.0001 & 0.0123\\-0.0000 & -0.0000 & 0.0123\\-0.0000 & 0.0000 & 0.0123\\-0.0001 & 0.0000 & 0.0123\\-0.0000 & 0.0000 & 0.0123\\0.0000 & 0.0001 & 0.0123\\0.0001 & -0.0000 & 0.0123\\0.0000 & 0.0000 & 0.0123\\0.0000 & -0.0001 & 0.0123\\-0.0000 & -0.0000 & 0.0123\\-0.0001 & -0.0000 & 0.0123\\-0.0000 & -0.0000 & 0.0123\\-0.0000 & 0.0001 & 0.0123\\0.0000 & -0.0001 & 0.0123\\0.0000 & -0.0001 & 0.0123\\0.0001 & 0.0000 & 0.0123\\0.0000 & 0.0000 & 0.0123\\-0.0000 & 0.0001 & 0.0123\\-0.0001 & 0.0000 & 0.0123\\-0.0001 & 0.0000 & 0.0123\\0.0001 & -0.0000 & 0.0123\end{bmatrix}$$

Equations 19 to 22 were carefully implemented using excel sheet and results are presented in Tables 3 and 4 below.

Table 3: Numerical solution

STUDS	$h_1^{(t1)}$	$h_2^{(t2)}$	$\Sigma(h_1\text{-xmean})^2$	$\Sigma(h_2\text{-xmean})^2$	$\Delta h=(h_2-h_1)$	$\Sigma(h_{1i}-h_{1i}\text{mean})(h_{2i}-h_{2i}\text{mean})/n-1$	$Q=qh_1+qh_2$	M*O*M
STUD1	3.6102	3.59685	6.0516E-06	1.92238E-06	-0.01335	4.76055E-06	0.000174291	3.1063E-08
STUD2	3.60757	3.59457	3.6766E-05	1.34432E-05	-0.013	2.22318E-05	0.000124125	2.0977E-08
STUD3	3.61249	3.59854	1.30759E-06	9.21122E-08	-0.01395	-3.47052E-07	0.000124125	2.4155E-08
STUD4	3.61532	3.60084	2.84428E-06	6.77821E-06	-0.01448	4.3908E-06	0.000124125	2.6025E-08
STUD5	3.61538	3.60325	3.05026E-06	2.51352E-05	-0.01213	8.75608E-06	0.000124125	1.8263E-08
STUD6	3.6179	3.61339	1.8203E-05	0.000229629	-0.00451	6.46524E-05	0.000124125	2.5247E-09
STUD7	3.62905	3.6166	0.000237668	0.000337218	-0.01245	0.000283101	0.000124125	1.924E-08
STUD8	3.6313	3.60884	0.000312105	0.000112434	-0.02246	0.000187327	0.000124125	6.2615E-08
STUD9	3.62481	3.60301	0.000124914	2.27863E-05	-0.0218	5.3351E-05	0.000124125	5.8989E-08
STUD10	3.61993	3.59802	3.96459E-05	4.68723E-08	-0.02191	-1.36319E-06	0.000124125	5.9586E-08
STUD11	3.6165	3.58834	8.21682E-06	9.79407E-05	-0.02816	-2.83683E-05	0.000124125	9.8429E-08
STUD12	3.60548	3.58973	6.64796E-05	7.23605E-05	-0.01575	6.93577E-05	0.000124125	3.0791E-08
STUD13	3.6079	3.58607	3.2873E-05	0.000148024	-0.02183	6.97566E-05	0.000124125	5.9151E-08
STUD14	3.60329	3.58672	0.000106988	0.00013263	-0.01657	0.000119121	0.000124125	3.408E-08
STUD15	3.60355	3.59913	0.000101677	7.98342E-07	-0.00442	-9.00961E-06	0.000124125	2.425E-09
STUD16	3.61403	3.59857	1.57212E-07	1.11222E-07	-0.01546	1.32233E-07	0.000124125	2.9667E-08
STUD17	3.61292	3.59228	5.09082E-07	3.54799E-05	-0.02064	4.24996E-06	0.000124125	5.2878E-08
STUD18	3.606	3.59289	5.82703E-05	2.85851E-05	-0.01311	4.08125E-05	0.000124125	2.1334E-08
STUD19	3.60639	3.60024	5.24683E-05	4.01401E-06	-0.00615	-1.45124E-05	0.000124125	4.6947E-09
STUD20	3.61266	3.59685	9.47702E-07	1.92238E-06	-0.01581	1.34976E-06	0.000124125	3.1026E-08
Mean =	3.613634	3.598237	**8.36561E-05**	**9.06346E-05**	**-0.0154**	**4.39875E-05**	0.000126633	3.4396E-08

4. CONCLUSION

This paper developed and implemented mathematical models suitable for researchers working on structural deformation analysis. The use of the mathematical model formulated in Excel spread sheet made the analysis suitable for researcher working on deformation analysis.

For deformation values to be visible, time and season of data acquisition must be obeyed and continuous throughout the life time of the structure. The suggested technique of deformation analysis can be used to identify and determine the values of deformation for any structure between any two epochs of observations.

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Table 4: Subsidence values at two epochs of observation

$$\delta = \left[\Delta h^{(1)}\right]^T \left[\Delta h^{(1)}\right]\left[Q^{(1)}_{\Delta\mathrm{h}\Delta\mathrm{h}}\right]^{-1} =$$

$\delta = \left[\Delta h^{(1)}\right]^T \left[\Delta h^{(1)}\right]\left[Q^{(1)}_{\Delta\mathrm{h}\Delta\mathrm{h}}\right]^{-1}$	
STUDS	(m)
STUD1	3.10625E-08
STUD2	2.09771E-08
STUD3	2.4155E-08
STUD4	2.60253E-08
STUD5	1.82633E--08
STUD6	2.52471E-08
STUD7	1.92396E-08
STUD8	6.26149E-08
STUD9	5.8989 E-08
STUD10	5.95858 E-08
STUD11	9.84291 E-08
STUD12	3.07907 E-08
STUD13	5.91515 E-08
STUD14	3.40803 E-08
STUD15	2.42495 E-08
STUD16	2.96672 E-08
STUD17	5.28783 E-08
STUD18	2.13336 E-08
STUD19	4.69471 E-08
STUD20	3.10257 E-08

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