EXPERIMENTAL AND ANALYTICAL COMPARISON OF TORSION, BENDING MOMENT AND SHEAR FORCES IN REINFORCED CONCRETE BEAMS USING BS 8110, EURO CODE 2 AND ACI 318 PROVISIONS

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ABSTRACT

This study investigated the effect of combined actions of torsional moments, bending moments and shear forces in reinforced concrete beams with concrete compressive strength of 30N/mm². The ultimate torsional moments, bending moments, and shear forces of the beams were determined experimentally, through a simple test arrangement set-up on fifteen beam specimens grouped from BC1 to BC5, three beam specimens in each group. The combined loads were induced by loading the test beams at an eccentricity of 65mm from the beam's principal axis at the mid-span, using Computerized Universal Testing Machine TUE-C-100. BS 8110, Euro code 2 and ACI 318 were used to calculate the ultimate torsional moments provided by both longitudinal and transverse reinforcements, bending moments and shear forces induced. The values obtained from the codes were compared with those of experimental results for validation. It was observed that Eurocode 2 predicted the highest bending moment of 21.1530kNm, the highest torsional moments of 9.8470kNm and 12.6193kNm, for torsional resistance provided by longitudinal and transverse reinforcements respectively, at an angle crack of 45°, while BS 8110 predicted the least values. ACI 318 predicted the highest value of internal shear forces that the beams possessed before yielding to the applied loads.

Key words: *bending moment, shear force, torsional moment, codes, reinforcement*

1. INTRODUCTION

In structural analysis and design, the effects of torsion are usually neglected and only bending, shear and axial forces are taken into consideration. This is because torsion was considered to be a secondary effect that will be covered in the factor of safety [1] or be taken care of by shear design. Less attention was paid in the past by reinforced concrete designers to these effects of torsion in reinforced concrete members. When torsion is encountered in reinforced concrete members, it usually occurs in combination with flexure and transverse shear. The interactive behaviour of torsion with bending moment and flexural shear in reinforced concrete beam is fairly complex, owing to non-homogeneous, non-linear and composite nature of the material and presence of cracks [2].

On several situations, beams and slabs are subjected to torsion in addition to bending moment and shear force. Combined torsional moment, bending moment and shear force may be induced in a reinforced concrete beam in various ways during the process of load transfer in a structural system. When a beam is subjected to

transverse loading such that the resultant force passes through the longitudinal shear axis, the beam will not twist, but bends. However, when the resultant force acts away from the shear center axis, moment is induced into the system [3]. This moment causes a body to rotate and if the structural system tries to resist such rotational tendency, bending and /or torsion results [4]. This implies that if applied loads are "eccentric" from the centroid and the resultant forces do not pass through the member's centroid, then torsion occurs [5]. It is therefore imperative to take into cognizance the combined effect of torsion, bending and shear stresses while designing members subjected to these forces.

There are well established procedures for dimensioning reinforced concrete beams subjected to axial load or moment, or combined axial load and moment. These procedures are based on rational, simple, general design models which can be embodied in a few paragraphs of code or specification documents. Such failure models provide the designer with means to evaluate the ultimate moment capacity of quite irregular sections in reinforced concrete. The same basic models, in addition can be used

to study the interaction between axial load and moment, making the related design process relatively simple and straightforward.

However, design provisions in codes in the areas of combined torsion, bending and shear stresses are not of the same level of rationality and general applicability. The absence of rational models has resulted in highly empirical design procedures characterized by large differences in values evaluated from different design codes when compared to experimental test results.

Structural members curved in plan, members of a space frame, eccentrically loaded beams, inverted L-beams as in supporting sunshades and canopies, curved box girders in bridges, edged beams of slabs or spandrel beams in buildings and spiral stair-cases are typical examples of structural elements subjected to combined torsion, bending and shear stresses and their effect cannot be neglected while designing such members [6, 7].

This paper investigates the effects of combined torsional moment, bending moment and shear force induced in reinforced concrete beams at a given compressive strength of concrete. It examines the design procedures for torsion, bending and shear in BS 8110, Euro code 2, and ACI 318 building code; evaluates the torsional, bending and shear stresses induced in reinforced concrete beams subjected to combined actions of shear, bending and torsion, using these codes and experimental work; and validates the provisions of the codes by comparing the calculated code values with experimental results.

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Aim of the Study: To determine the effects of combined torsional moment, bending moment and shear force induced in reinforced concrete beams at a given compressive strength.

Objectives of Study: The objectives of this study include:

1) To examine and compare the torsional, bending and shear design procedures in BS 8110, Euro code 2, and ACI 318 building code.

2) To evaluate the torsional stresses, bending stresses and shear stresses induced in reinforced concrete beams subjected to combined actions of shear, bending and torsion using standard design codes and experimental work.

3) To validate the provisions of the codes by comparing the calculated design code values with that of experimental results.

2. PROVISIONS FOR CURRENT STRUCTURAL STANDARD CODES OF PRACTICE FOR SHEAR, TORSION AND BENDING

2.1 ACI building code Provision (ACI 318)

The first provision for torsion appeared in ACI 318-63 [8]. It consisted of one sentence, which prescribed the use of closed stirrups in edge and spandrel beams and one longitudinal bar in each corner of those closed stirrups. Comprehensive design provisions for torsion were introduced through a series of papers by ACI committee 438, in 1968 and 1969 and later adopted in the 1971 ACI Building code. These design requirements remained essentially unchanged through the 1989 and 1992 provisions [1] and [9]. The method of design for torsion and for combined torsion, shear and flexure in beams were revised in the ACI 318-95 Code of 1995 and remain essentially unchanged since then. This design procedure for solid and hollow members is based on a thin-walled tube, space truss analogy.

The basic truss equation relating the torsional strength to the quantity of longitudinal reinforcement is

$$T_n = 2A_o \frac{A_l f_{yl}}{P_h} tan\theta \qquad (1)$$

The basic truss equation relating the torsional strength to quantity of hoop reinforcement is

$$T_n = 2A_o \frac{A_t f_{yv}}{s} Cot\theta \qquad (2)$$

Where: A_o is the gross area enclosed by perimeter of the wall within the shear flow path, P_h is the outside perimeter of concrete cross section, A_l is cross sectional and of longitudinal bars, A_t is the cross sectional area in one leg of stirrup bars, f_{yl} is the yield strength of longitudinal bars, fyv is the yield strength of transverse bars, θ is the angle of inclination, and s is the spacing of links

The ACI provisions permit A_o to be taken as $0.85A_{oh}$. Section 11.6.3.6 of the ACI Code states that the angle of inclination θ of the compression diagonals "shall not be taken smaller than 30 degrees." but then goes on to suggest that θ be taken equal to 45 degrees for non-pre stressed members and 37.5 degrees for pre stressed members. The code allows any value between 30 and 60 degrees (ACI 11.5.3.6).The commentaries suggest that θ can be obtained by analysis [10].

2.2 Euro Code Provision

The EC2 equations for torsional design are developed from a structural model where it is assumed that the concrete beam in torsion behaves in a similar fashion to a

thin-walled box section. The box is reinforced with longitudinal bars in each corner with closed loop stirrups as transverse tension ties and the concrete providing diagonal compression struts [11]. Euro code 2 gives the principles and some limited design equations for a generalized shape of a hollow box section.

The basic truss equation relating the torsional strength to the quantity of longitudinal reinforcement is

$$T_{Rd1} = 2A_k \frac{A_{sl}f_{yl}}{U_k} tan\theta \qquad (3)$$

The basic truss equation relating the torsional strength to quantity of hoop reinforcement is

$$T_{Rd2} = 2A_k \left(\frac{A_{sw}}{S}\right) f_{yt} Cot\theta \qquad (4)$$

Where: A_k is the area enclosed with in the centre line of the hollow box section, s is the spacing of links, A_{gw} is the cross section area of links, A_{gl} is the area of longitudinal bars, f_yt is yield strength of transverse bars, f_{yl} is yield strength of longitudinal bars and θ is the angle of inclination.

There are practical limitations on the values of θ that can be used and EC2 recommends that $1.0 \leq Cot\theta \leq 2.5$ representing limiting values of θ to 45° and 22° respectively, but can be taken as θ =45°.

2.3 British Standard (BS 8110)

If a section is subjected to bending moment M, shear force V, and torsion T, it is necessary to design the transverse and longitudinal steel. It should be separately designed for shear force, bending moment and torsion. In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement.

If the design relies on the torsional resistance of a beam, further consideration should be given using the following sections (BS 8110-2:85 3.4.5.13).

The torsional resistance of all links crossing the cracks is calculated as:

$$\frac{A_{sv,t}}{S_v} = \frac{T}{0.8x_1y_1(0.87f_{yv})} \qquad (5)$$

And the required longitudinal reinforcement, is calculated from

$$A_l = \frac{A_{sv,t}f_{yv}(x_1 + y_1)}{S_v f_y} \qquad (6)$$

Rearranging Equation (6), the torsional moment provided by longitudinal reinforcement is determined as:

$$T_{US1} = \frac{A_l(0.8x_1y_1)(0.87f_{yl})}{(x_1+y_1)} \qquad (7)$$

The angle of cracks is fixed at $\theta = 45^0$

Where, x_1 and y_1 is the length of arms of stirrups, s_v is spacing of links, $A_{sv}t$ is the cross sectional area of links, A_l is area of longitudinal bars, f_{yv} is yield strength of transverse bars, f_{yl} = yield strength of longitudinal bars,

3. MATERIALS AND METHOD
3.1 Materials

The concrete beam specimens were cast to attain 28-day concrete cube strength of about $30MPa$, with design mix ratio of cement, fine and coarse aggregates of 1:1.5:3, batched by volume, water/cement ratio of 0.52. The concrete was placed in two layers, each layer being deposited continuously around the beam formwork and uniformly tamped with a steel rod. The top surface of concrete is struck with a wooden float and neatly finished with a steel trowel. The arrangements of reinforcement in beam specimens are grouped into five as detailed in Table 1.The beam specimens have the same cross-section of 150*150*1500mm, with beams marked BC2, BC3, BC4 and BC5 reinforced longitudinally and transversely with varying number of bars and spacing of stirrups. While beam marked BC1 has no reinforcement. The cast beam specimens were cured using wet burlap sack to cover and water being sprinkle on it at least twice a day while the concrete cubes were wholly immersed in water tank for the 28 days.

3.2 Method

The confirmatory tests carried out in this study include; sieve analysis of fine and course aggregates, uniaxial tensile strength of reinforcement bars, and slump test of wet concrete, the compressive strength of concrete cubes at 7 and 28 days respectively, the combined actions of bending moment, torsional moment and shear force test (loaded at an eccentricity of 65mm from the shear centre axis at mid-span of the beam specimens).

The methodology used in this study was based on experimental design of fifteen rectangular reinforced concrete beams. Each beam specimen was loaded eccentrically from its principal axis by a combined action of bending, shear and torsional load, applied through a system of Computerized Universal Testing machine TUE-C-100. The experimental results were compared with design provisions of three international standard codes namely: BS 8110, Euro Code 2 and ACI 318 building code. Choice of test assembles: The objective of the design of rectangular beam was to provide a simplified test arrangement and to ensure a failure of the test beams in combined actions of bending, shear and torsion without causing a failure in other elements. The test beams were seated on two steel supports resting on the laboratory floor. The test beam is simply supported but restricted at both ends on the steel support in other to avoid rotation, then could be loaded at any desired eccentricity from the beam principal axis, through a system of Computerized Universal Testing machine TUE-C-100 with a point-loading head.

Five groups of specimens, each group contains three 150*150*1500mm long rectangular reinforced concretewere cast. All the samples have effective span of 1200mm.The beams were designed for stresses of 30N/mm^2in concrete and 460N/mm^2 in reinforcement. The designed test beam specimens are to carry a maximum uniform distributed load of 25kN/m at an eccentricity of65mm, required 8mm diameter of stirrups at 150mm centre to center and 10mm diameter of longitudinal reinforcement to resist calculated shear and torsion. However, the spacing of the stirrups was varied for the test beam specimens to study the effect of transverse reinforcement on the ultimate strength of the test beam. Number of bars provided as longitudinal reinforcement was also varied to determine its effect on torsional capacity of the beams tested. The choice of 8mm diameter bars for transverse reinforcements, 10mm diameter bars for longitudinal reinforcements and compressive strength of 30N/mm^2 for concrete was to ensure that load capacity of the Computerized Universal Testing Machine TUE-C-100 was not exceeded. Hence, to ensure that the beam specimens failed before the maximum load capacity of the machine is reached, 8mm and 10mm diameter bars were selected in the experimental work.

Figure 1: Experimental Set-up

4. RESULTS AND DISCUSSION

To validate the experimental results, the three standard design codes, BS 8110, Euro Code 2 and ACI 318, were used to design the beams and their values tabulated for comparison. The values determined for comparison include: the torsional moments predicted by the codes for torsional resistance provided by both transverse and longitudinal reinforcements, bending moments and shear forces.

Table 1: Summary of characteristics of Reinforced Concrete Beams Tested (all the samples are 150*150*1500mm long and compressive strength of 30N/mm^2).

Beam notation	Longitudinal bars (mm)	Areas of longitudinal bars (mm^2)	Stirrups bars (mm)	Spacing of links (mm)	Areas of transverse bars (mm^2/mm)	Eccentricity of loading (mm)
BC1	Not reinforced					65
BC2	4T10	314	R8	200	0.503	65
BC3	4T10	314	R8	100	1.006	65
BC4	6T10	471	R10	200	0.503	65
BC5	6T10	471	R10	100	1.006	65

Table 2: Bending moments predicted by the Codes and experimental result.

Beam sample	BS 8110 (kNm)	Euro Code 2 (kNm)	ACI 318 (kNm)	Experimental Result (kNm)
BC1	3.34308	3.34308	3.34308	3.34308
BC2	12.9470	14.1002	11.701	12.4416
BC3	12.9470	14.1002	11.701	15.2066
BC4	19.4198	21.1503	17.553	13.2526
BC5	19.4198	21.1503	17.553	15.4451

Table 3: Shear forces predicted by the Codes and experimental result.

Beam sample	BS 8110 (kN)	Euro Code 2 (kN)	ACI 318 (kN)	Experimental Result (kN)
BC1	5.772	5.772	5.772	5.772
BC2	13.2601	21.775	23.562	20.9628
BC3	26.5201	43.5497	47.1231	25.4045
BC4	13.2601	21.775	23.562	22.3145
BC5	26.5201	43.5497	47.1231	25.9686

Table 4: Torsional moments predicted by the Codes for torsional resistance provided by longitudinal reinforcement at angle of crack, $\theta = 45^0$ and experimental result.

Beam sample	BS 8110 (kNm)	Euro Code 2 (kNm)	ACI 318 (kNm)	Experimental Result (kNm)
BC1	-	-	-	0.6914
BC2	5.0265	6.5647	5.6097	2.6662
BC3	7.5398	6.5647	5.6097	3.2436
BC4	5.0265	9.8470	8.4151	2.8419
BC5	7.5398	9.8470	8.4151	3.31695

Table 5:Torsional moments predicted by the Codes for torsional resistance provided by transverse reinforcement at angle of crack, $\theta = 45^0$ and experimental result.

Beam sample	BS 8110 (kNm)	Euro Code 2 (kNm)	ACI 318 (kNm)	Experimental Result (kNm)
BC1	-	-	-	0.6914
BC2	1.6104	6.3096	3.652	2.6662
BC3	3.2208	12.6193	7.3047	3.2436
BC4	1.6164	6.3096	3.652	2.8419
BC5	3.2208	12.6193	7.3047	3.31695

Calculations on how the ultimate torsional resistances provided by longitudinal and transverse reinforcements were calculated using BS 8110, Euro Code 2 and ACI 318 code. Refer to Table 1 for other parameters.

ACI 318:

$$\theta = 45^o; f_c' = 4000\ ps; f_y = 60000\ psi;$$
$$A_{cp} = 6 * 6 = 36in^2, P_h = 2_{yo} + 2_{xo} = 2 \times 4 + 2 \times 4 = 16in$$
$$A_o = 0.85A_{oh} = 0.85 \times 4 \times 4 = 13.6in^2$$

BC 2: $T_{ACI,Al} = 2 \times \frac{13.6 \times 0.4867 \times 60 \times 1}{16} = 49.6434 in - kips\ (5.6097kNm\ SIU)$

$$T_{ACI,At} = 2 \times 13.6 \times 0.019803 \times 60 \times 1$$
$$= 32.3185 in - kips\ (3.652kNm\ SIU)$$

BC 5: $T_{ACI,Al} = 2 \times \frac{13.6 \times 0.7301 \times 60 \times 1}{16} = 74.4702in - kips\ (8.4151kNm\ SIU)$

$$T_{ACI,At} = 2 \times 13.6 \times 0.03961 \times 60 \times 1$$
$$= 64.6435 in - kips\ (7.3047kNm\ SIU)$$

Euro Code 2:

$$\boldsymbol{\theta = 45^o}, f_{yz} = 500N/mm^2, f_{ck} = 30N/mm^2$$
$$A_k = (150 - 38)(150 - 38) = 12544mm$$

BC 2: $T_{EC2,AL} = 500 \times \left(\frac{314}{600}\right) \times 2 \times 12544 \times 1 \times 10^{-6} = 6.5647kNm$

$$T_{EC2,At} = 500 \times (0.503) \times 2 \times 12544 \times 1 \times 10^{-6}$$
$$= 6.3096kNm$$

BC 5: $T_{EC2,AL} = 500 \times \left(\frac{471}{600}\right) \times 2 \times 12544 \times 1 \times 10^{-6} = 9.8470kNm$

$$T_{EC2,At} = 500 \times (1.006) \times 2 \times 12544 \times 1 \times 10^{-6}$$
$$= 12.6193kNm$$

BS 8110:

$$f_{cu} = 30N/mm^2, f_y = 460N/mm^2, d = 111mm, f_{yv} = 250N/mm^2$$
$$y_1 = 150 - 2 \times 25 = 100mm, x_1 = 150 - 2 \times 25 = 100mm$$

BC 2: $T_{BS,AL} = 0.503 \times (0.8 \times 100 \times 100)(0.87 \times 460) \times 10^{-6} = 1.6104\ kNm$

$$T_{BS,At} = \frac{314 \times (0.8 \times 100 \times 100)(0.87 \times 460)}{(100 + 100)} \times 10^{-6}$$
$$= 5.0265kNm$$

$$T_{BS,Conc} = \frac{1}{2} \times 0.2\sqrt{30} \times 150^2 \times \left(150 - \frac{150}{3}\right) \times 10^{-6}$$
$$= 1.2324kNm$$

BC 5: $T_{BS,AL} = 1.006 \times (0.8 \times 100 \times 100)(0.87 \times 460) \times 10^{-6} = 3.2208\ kNm$

$$T_{BS,At} = \frac{471 \times (0.8 \times 100 \times 100)(0.87 \times 460)}{(100 + 100)} \times 10^{-6}$$
$$= 7.5398kNm$$

Table 2 presents the bending moments predicted by the codes and experimental results. From this table, it was clear that values predicted by the codes matched with each other in analytical and experimental results for the control beam, BC1. But for other beams, the predictions of the codes differ, both analytically and experimentally. For instance, Euro code 2 predicted the highest value of internal ultimate bending moment that the tested beams possessed before failure/yielding occurred, while ACI 318 predicted the least value. This is evident from the table where, for beam group BC5, Euro code's value was 21.1503kNm, BS 8110 value, 19.4198kNm, and ACI 318 value, 17.553kNm, while the experimental result was 15.4451kNm.While ACI 318 predicted the least value 17.553kNm. However, when the values were subjected to ANOVA Test at 5% level of significance, there was a significant difference statistically ($F_{critical}$ (3.2388) > $F_{calculated}$ (0.11) .From this table, beams are said to have failed/ yielded by bending moment if values observed from experiment are greater than calculated values predicted by each code. This implies that beams in ACI 318 yielded/ failed in group beams BC2 and BC3; only beam group BC3 failed/ yielded in Euro code 2 while in BS 8110, beam group BC3 failed/ yielded.

Table 3 presents the shear forces predicted by the codes and experimental result. From this table, ACI 318 predicted the highest shear force that the tested beams possessed to resist the applied loads, while BS 8110 gave the least value. From beam group BC3, ACI 318 value is 47.1231kN, Euro code 2 value is 43.5497kN while BS 8110 value is 26.5201kN. A beam is said to fail/ yield in

shear force if the value obtained experimentally is greater than that predicted by the codes. Beam groups BC2 and BC4 failed/ yielded in BS 8110; BC3 did not fail/yield in Euro code 2; while none of the beam groups failed/ yielded in ACI 318.

Table4 presents the ultimate torsional moments predicted by codes for torsional resistance provided by longitudinal reinforcement at $\theta = 45^o$. From the table, Euro code 2 predicted the highest value of torsional moment followed by ACI318 while BS 8110 predicted the least value. In other words, in beam group BC4, Euro code 2 value is $9.847kNm$, ACI 318value is $8.4154kNm$ while BS 8110 value is $5.0265kNm$. In this case, a beam is said to fail/ yield if the torsional moment observed from experiment is greater than that predicted by codes. This implies that none of the beams predicted by the three codes failed/ yielded in longitudinal reinforcement due to torsion, because values calculated from codes are greater than experimental results in longitudinal reinforcement resistance to torsion.

Table 5 presents the ultimate torsional moments predicted by codes for torsional resistance provided by transverse reinforcement at $\theta = 45^o$. From the table, Euro code 2 predicted the highest value of torsional moment of 12.6193kNm in beam group BC5, followed by ACI 318, with a value of 7.3047kNm, while BS 8110 predicted the least value of 3.31659kNm. In this case, a beam is said to fail/yield if the torsional moment observed from experiment is greater than that predicted by codes. This implies that none of the beams predicted by Euro code 2 and ACI 318 code failed/yielded in transverse reinforcement due to torsion, while all the beams predicted by BS 8110 yielded in transverse reinforcement. From all the tables presented, there is increase in beam's capacity to resist effect of the applied combined loads. In other words, as the areas of longitudinal reinforcement increases, together with a reduction in the spacing of the transverse reinforcement down the group, the higher the strength of the beams' capacity to resist the applied loads. This indicates the need to provide torsional reinforcements together with those calculated for bending moment and shear force design as their interaction increases the capacity of the beams to resist yielding/ failure.

5. CONCLUSION AND RECOMMENDATION

The experimental result indicated that the codes studied were able to predict quite conservative and consistent values of the internal ultimate torsional moments, bending moments and shear forces induced in beams subjected to combined torsion, bending and shear force. Hence, this study concludes that Euro code 2 gave the highest internal ultimate bending moment and torsional

strengths for torsional resistance provided by both longitudinal and transverse reinforcements at $\theta = 45^0$. ACI 318 predicted the highest internal shear forces that the beams possessed before yielding to the applied loads, while BS 8110 predicted the least values of bending moment, shear force and torsional resistance provided by both longitudinal and transverse reinforcements.

It can also be concluded from this study that the beams failed due to the combined actions of torsion, shear and bending moment effects. Therefore, increase in the capacity of the beams to resist the applied combined loads, were as a result of the increase in areas of both longitudinal and transverse reinforcements down the beam groups.

The study also concluded that the calculated amount of reinforcement obtained from torsional design must be provided in addition to the full bending and shear reinforcement at ultimate loads, as the interaction of torsion and shear force or torsion and bending or combination of the three loads led to provision of higher areas of longitudinal and transverse reinforcement. This increases the capacity of thebeams to resist the effects of combined loads.

The study recommends further research at optimizing the angles of cracks, dimensions of beam cross section and eccentricity of loading in other to determine its effects on capacity of beams to resist combined loads.

The research also recommends the inclusion of torsional design for beams that are subjected to its effects or in combined actions, as increase in reinforcement due to torsion in both longitudinal and transverse directions of the beam increases the capacity of such beam.

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