



DIVERSE APPROACHES TO MODELLING THE ASSIMILATIVE CAPACITY OF A POLLUTED WATER BODY

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ABSTRACT

The assimilative capacity of a water body requires knowledge of the river characteristics, pattern of effluent loading, river hydraulic resilience and far-field mixing to effectively determine it, hence it is considered appropriate that diverse models and approaches are utilized in its determination". This study evaluated the assimilative capacity of Ikpoba River using different approaches namely: homogeneous differential equation, ANOVA/Duncan Multiple range test, first and second order differential equations, correlation analysis, Eigen values and eigenvectors, multiple linear regression, bootstrapping and far-field mixing Analytics. In the rainy, dry, and harmattan seasons the values of assimilative capacity for Guinness, Ewa road and UBTH point sources of effluent discharge are as follows: 1.07×10^{10} kg/day, 1.07×10^{10} kg/day and 1.29×10^{10} kg/day; 8.1×10^9 kg/day, 9.07×10^9 kg/day and 1.05×10^{10} kg/day; 1.28×10^{10} kg/day, 1.03×10^9 kg/day and 1.95×10^{10} kg/day in that order. It was also discovered that by model tricking, assimilative capacity had a range of values that varied from 231.09mg/l upper limit to -222.27mg/l lower limit at 95% significance level and beyond this range, the model developed will breakdown, and its robustness weakened by poor predictive capacity. The upper control limit for each of the various pollutants considered indicates the saturated value of the dissolved oxygen (DO), and the negative lower control limits depict dissolved oxygen debt whereby the DO falls below critical (anoxia) to harsh critical level. At this level any reaeration or reoxygenation effort is not practically palpable until recovery is made up to critical level. The eclectic approaches adopted have crossed the stream where it is shallowest.

Keywords: assimilative capacity, differential treatments, 3-D mixing zone, oxygen debt, Homogenous Differential Equation

1. INTRODUCTION

The assimilative capacity of a water body may be defined in different ways as:

- i. The ability of a water body to take in effluent that causes diminution of dissolved oxygen and over time, is able to recover from the diminution.
- ii. The first definition sees assimilative capacity from the angle of dissolved oxygen deficiency. Assimilative capacity can also be evaluated in terms of the ability of water body to adsorb effluents rich in different pollutants and over time, is able to bounce back to its original purity level.
- iii. It is also a measure of the capacity of the water

body to recover from any form of pollution abuse within a reasonable length of time and distance downstream.

- iv. It can also be said to be the natural ability of a river to readily whittle away the mass concentration distribution of wastes in far-field mixing zone of river to almost nothing.
- v. It is also the ability of a water body to counter the combined effects of the actions and reactions of the various water quality determining parameter in order to regain its natural water quality.

Taken together, the assimilative capacity of a river refers to the hydraulic resilience of the water body to restore its polluted condition to natural purity state

with due consideration to time and distance downstream from the source of pollution.

Since assimilative capacity can be defined in different ways, it may also be modeled by different approaches hence this paper attempts to use various approaches to model its different aspects.

The paper [1] generally addressed the effects of heavy metal pollution in some Nigerian rivers, [2] studied the effects of heavy metal pollution on the reproductive abilities of some dominant fish population in Ikpoba River. Further, [3] explored the heavy metal concentrations in the water, sediments and selected fish fauna in Warri River and its tributaries, while [4] researched on the prediction of the response of a water body to deoxygenation due to BOD loading and its DO level. Incidentally, the studies under reference had hardly modelled assimilative capacity. This study, to the best of our knowledge, appear to be a seminal work that uses diverse approaches to model assimilative capacity.

The paper [5] studied the heavy metal concentrations in the sediments upstream of Ikpoba River and reservoir in Benin City, using atomic absorption spectrophotometric technique (AAS). The result of the study revealed higher concentrations of heavy metals upstream than in the reservoir downstream sediments [6] noted that the levels of Lead (Pb) in the fish species in Ikpoba River dam, Benin City exceeded the limit recommended in fish and fishery products by the Food and Agricultural Organization (FAO) of the United Nations, while the levels of cadmium (Cd) and iron (Fe) in the water body exceeded the recommended levels for potable water by the World Health Organization (WHO).

The study [7] determined the reaeration coefficient for polluted water body as a function of depth, temperature, hydraulic radius and velocity using multiple regression analysis. The papers [8, 9] separately studied the self-purification potential of two different tropical urban streams respectively, and noted that the reaeration and deoxygenation rates of any water body determine its self-purification potential. The later further pointed out that the rate of self-purification is also influenced by temperature, nature and size of organic pollutants, hydraulic features and its algal content. Furthermore, [4] deployed Streeter-Phelps model to predict the response of River Illo, Ota, Nigeria to deoxygenation due to BOD loading and its DO level. The study concluded that there is slow reaeration of the river and hence poor pollution recovery potential.

The aim of this study is to adopt diverse modelling approaches to predict the assimilative capacity of Ikpoba River, by surveying its pollution status, modelling the far-field mixing of industrial plume into it, analyzing the mathematical distribution of brewery effluent plume into the river, undertaking the eigenvalue, eigenvector, correlation and ANOVA analysis of all effluent discharges and runoffs into the river and then embarking on an eudiometric theoretic- approach to modelling the assimilative capacity of the river by incorporation of bootstrapping needful for sensitivity analysis of the model used.

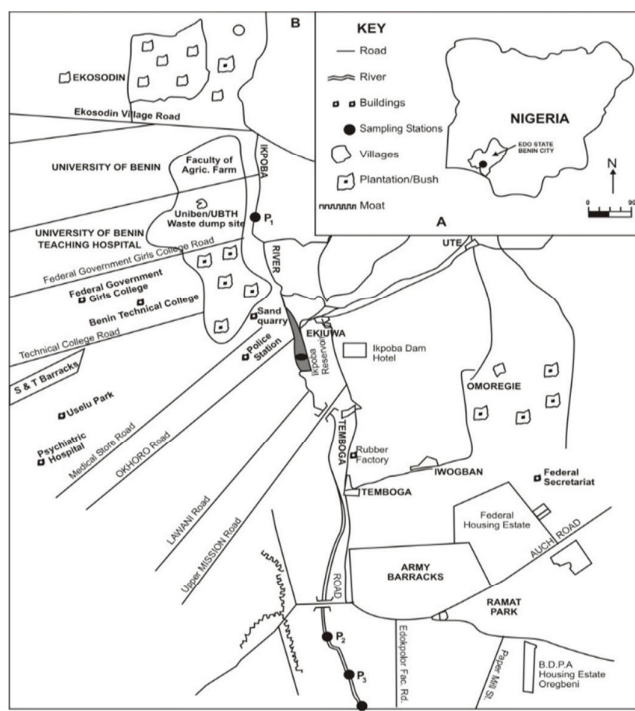


Figure 1: (A) Map of Nigeria showing Edo State, (B) Map of Ikpoba River showing Sampling Stations

2. MATERIALS AND METHODS

2.1 Study Site

Figure 1 depicts the map of relevant portion of Ikpoba River studied. Ikpoba River is located in Benin City, Nigeria (Figure 1). As shown in the figure, the river flows from top to bottom of the map, passing through the points P₁, P₂, P₃ and P₄ which are the effluent discharge points, where samples were taken for the study. It traverses the city, flowing from North to South. P₁ is the point where the wastewater from the Teaching Hospital is believed to enter the river. P₂ which is at Ewa Road junction is where runoffs and wastes from the abattoir discharge into the river. P₃ is the point where the effluents from a brewery were

channeled into the river and P₄ is 200m downstream of P₃.

2.2 Sampling Design and Measurements

In the first approach, samples of polluted water was obtained from the various point sources, along the river course represented with (P₁-P₃)The river water samples for physiochemical, microbiological and heavy metal analysis were collected from four predetermined points. At each point, upstream and downstream, the samples for physiochemical and microbiological analyses were collected with 50ml reagent bottles which were washed with distilled water and further rinsed with water collected at the sample point before the collection of samples. Oxygen fixation for the specimens was done with Winkler solutions A and B. Distilled-water-diluted nitric acid was required to maintain the oxidation state of the elements and also to prevent materials from adhering to the walls of the container. The samples meant for heavy metal analyses were collected with 1 litre plastic container.

Upon handing over the samples to the personnel of the laboratory where the analyses were carried out they removed the samples from the ice chest and had them frozen prior to subsequent chemical analysis in line with fundamentals of field studies. All the water quality parameters were analyzed using appropriate standard methods namely APHA, HATCH Digital titration and Determination Bacteriology as specified in [10-12].Effluent parameters namely temperature, pH, turbidity, DO, BOD and COD were determined in the laboratory.

All the laboratory analyses were done in *Earth Quest International Limited, Warri, Nigeria*.

In addition, measurement of the depth and width of the river were taken at several points with the aid of a graduated aluminum pole in order to obtain average values. The diameter of the culvert through which the effluent flows into the river was measured with a metric tape in order to determine the flow rate. The speed of the river was determined by float method which involves the use of cork made to traverse a known distance along the river cause and the time taken to cover a known distance was observed. The speed of the river was calculated by dividing the measured distance by the time taken to traverse the distance. This procedure was repeated ten times and the average speed computed. The replication serves as a correction factor.

2.3 Method of Data Analysis

The t-test statistics was used to determine significant differences between the samples obtained from upstream and downstream respectively. The means of the samples were compared with National Environmental Standards and Regulations Enforcement Agency’s (NESREA’s) maximum contaminant level (MCL).

The next approach comprises a mathematical analysis of brewery effluent distribution in the river studied. Partial differential equation (PDE), second order differential equation, ANOVA model and Latin Square model were deployed to model effluent concentration distribution in the river. Ten-day consecutive observations were taken to evaluate spatial concentration distribution downstream of pollution sources up to far-field. Boundary conditions governing effluent concentration dilution were identified and applied. ANOVA and Duncan multiple range tests were used to determine the differential effects of treatments and the merit order sequentiality of the various pollutants under reference.

For the correlation analysis, 13 by 13 matrix of pollutant variables were fitted into correlation coefficient formular given as:

$$r_k = \frac{\sum xy}{\sqrt{(\sum x^2).(\sum y^2)}} \tag{1}$$

$$x = X_{ij} - \bar{X}_j$$

$$y = Y_{ij} - \bar{Y}_j$$

Where \bar{X}_j, \bar{Y}_j are column means

X and Y are a pair of column vectors of pollutant variable, taken two at a time in line with

$${}^n C_2 = \frac{n!}{(n-2)! 2!} \tag{2}$$

Equation (2) gives the number of correlation coefficients obtainable, and in this case we have:

$${}^{13} C_2 = \frac{13!}{(13-2)! 2!} = 78$$

The purpose of this correlation analysis is to ascertain the interplay that exist among the variables studied Eigen value and eigenvectors were used as analytics to investigate the interplay of various pollutants and to relativize the influence of pollution parameters in water quality of the water body. In this situation, bacteriological and physiochemical analysis were conducted on samples of polluted water from different point sources. Further, statistical analysis were conducted in order to ascertain whether there was differential treatment of the parameters with due

regards to the various stations. Part of the aim of the study was to ascertain the contribution of each of the point sources to river pollution.

Another aspect of the diverse investigation was to model mixing of the industrial effluent plume in the river under investigation. The affected section of the river (the section sandwiched between P₁ and P₄) was sectioned into three segments. And beginning from the point source, a 3-D mixing zone, 2-D mixing zone, and 1-D mixing zone along the river course were isolated and investigated. A first order homogenous differential equation was used to show that mass concentration distribution of effluent plume morphed into drifting circles that echoes Doppler shift run.

Moreover, a specific study focused on modelling of effluent assimilative capacity of the river under reference was carried out. The approach incorporated hydraulic factors such as velocity, depth, temperature, turbidity and discharge among others as veritable factors that influence assimilative capacity of water bodies. Basic Streeter-Phelps equation including monograph of allowable BOD loading was incorporated in the analysis. Also assortment of Streeter-Phelps equations and their modifications were adopted to determine the assimilative capacity. Furthermore, water quality determinants for Ikpoba River by seasons were computed using the method of [13]

Finally, the eudiometric concept was used to model the assimilative capacity of the river in question. In this setting, ordinary least squares (OLS) of the multivariate linear regression incorporating bootstrapping was used to undertake sensitivity analysis of the assimilative capacity determined. The method focused on how specific changes in some of the model parameters can affect the assimilative capacity of the river without affecting the robustness of the multivariate linear regression.

Attempt was also made to develop the theory that underpins geometric projection of assimilative capacity. Model tricking methodology employed

helped to illustrate sensitivity analysis determination. This eclectic approaches provide a world view of assimilative capacity determination and associated sensitivity analysis.

3. RESULTS AND DISCUSSION

The results of the effluent parameter determination are shown in Table 1. It was observed that the pollutant parameters showed a general tendency to increase with distance along the river course from P₁ to P₄. The upstream point source (P₁) is a government owned hospital while the second point source (P₂) is a government owned abattoir, and the third point source (P₃) is a conflation of effluents from two large breweries that are adjacent to each other, while the fourth point is 200m away from the brewery point source. The stretch of river basin from P₁ to P₄ is slanting towards the river course and this incline made leachates to accumulate longitudinally along the river course. The nature of topology made the vector of distribution of pollutant parameters to increase along the river course as shown in Table 1 where it is evident that the column vectors of the parameters show a general tendency to increase with distance along the river course. Moreover, as the river traverses the point sources, the various effluents introduced from the point sources tend to add averagely to the pollution level of the river.

The results of the water quality determining parameters obtained from samples at different points, as shown in Table 1, evidently reveal that there are variations in the values of PO₄ obtained from the various sample points in this study. However, the ones that are outstandingly significant are those obtained from downstream of P₁, that is, the one obtained from the downstream of the government owned hospital as well as the one obtained from downstream of the brewery which are 0.489 and 0.868 mg/l respectively. This implies that the government owned Hospital and the brewery contribute more to the addition of PO₄ in the river.

Table 1: Data Matrix of Ikpoba River Quality Parameters

	PO ₄	NO ₃	Cd	Cu	Fe	Pb	Turbidity	Temp.	Feecal col.	Ph	DO	BOD	COD
P1 (ups)	0.317	0.491	0.346	0.259	1.552	0.194	2.5	26.467	16.333	7.443	8	3.267	25
P1 (dns)	0.489	0.724	0.316	0.191	1.425	0.137	4.333	25.066	26.667	7.827	7.567	5.767	39.667
P2 (ups)	0.246	0.629	0.641	0.245	1.467	0.155	2.367	27.033	20.667	7.457	6.8	0.7	28.4
P2 (dns)	0.334	0.707	0.732	0.267	2.737	0.126	4.233	27.467	25	7.27	6.433	4.1	28.133
P3 (ups)	0.432	0.499	0.4	0.137	2.097	0.135	2.633	27.8	12	7.41	7	1.267	27.933
P3 (dns)	0.868	0.933	0.77	0.503	5.062	0.717	30	27.433	126.33	6.16	0.267	16.467	277.633
200M dns	0.133	0.487	0.399	0.049	1.108	0.039	2.933	26.7	13.333	7.52	6.667	1.733	12.933

The NO₃ profile of the water samples varies significantly and ranged from 0.487 to 0.933 mg/l. Again, the brewery downstream (P3) was found to have the highest contribution to the nitrate profile of the section of the river studied with 0.933mg/l, followed by the hospital (P1) downstream) with 0.487mg/l.

The Cadmium (Cd) regime of the samples ranged from 0.316- 0.770mg/l with the highest value of 0.77mg/l obtained from the brewery (P3) downstream followed by the government owned hospital P1 (downstream) with a value of 0.732mg/l.

The values of water quality determining parameters obtained from P3 and P1 are consistently higher than values obtained from the other stations studied as can be seen in Table 1. It can therefore be inferred that effluents from these two stations contribute more to the degradation of the water quality of the section of the river studied.

Besides, the t-test statistics conducted on the effluent parameters inclined us to reject the null hypothesis of lack of differential treatment before and after the point sources towards downstream for DO, COD and BOD, except for temperature and turbidity.

What we can infer from all this is that Ikpoba River is actually being loaded with assortment of industrial and municipal effluents having characteristics value beyond NESREA standards.

In this study, effluent concentration distribution $C(x)$ along the river course measured from point source is modelled as follows:

3.1 Model Building with Partial Differential Equation (PDE)

Q is the discharge rate g/sec., C is the Concentration in mg/L or ppm, U is the river speed in metres/sec, A is the cross sectional area of river in m² and $C = C(x)$. This variable represents effluent concentration; it varies along downstream only but not across river bank or down the river bed. $C(x)$ is twice differentiable in x . In other words it is assumed that:

$$c''(x) = \frac{dc^2}{dx^2} \text{ exists} \tag{3}$$

$\beta = 0.0447\text{g/s/g}$ (the values for the chemical decay constant can be obtained from the literature, these values are peculiar to the type of chemical plant in question, from literature $\beta = 0.0447\text{g/s/g}$ for wastewater effluent associated with brewery industries) [14 and15].

$0 < x < L$, see Figure 2a.

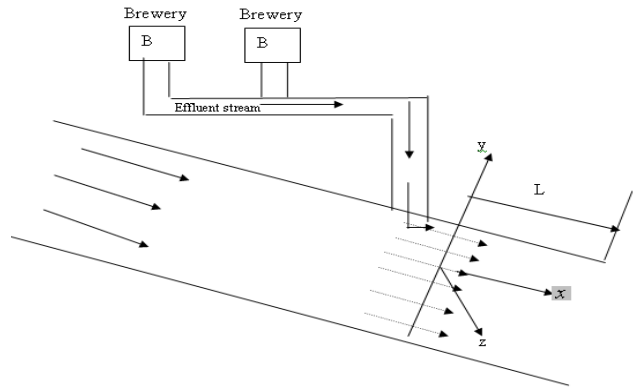


Figure-2a. Sketch of effluent stream discharge into Ikpoba River.

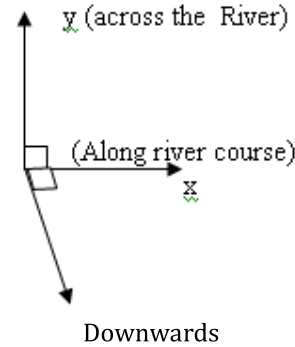


Figure 2b: coordinate system for river flow
In conducting a mass balance, we notice that:

y is measured along the width of the river and x is measured along the length L , of the river (longitudinal), z measures the depth of the river'. The mass balance equation is as depicted in equation (4)

$$\left(\begin{array}{c} \text{Effluent} \\ \text{concentration} \\ \text{input into river} \end{array} \right) = \left(\begin{array}{c} \text{Transport of} \\ \text{Pollutant by} \\ \text{diffusion} \end{array} \right) + \left(\begin{array}{c} \text{Transport of} \\ \text{pollutant by} \\ \text{convection} \\ \text{with moving} \\ \text{stream} \end{array} \right) + \left(\begin{array}{c} \text{Transport of} \\ \text{pollutant by} \\ \text{Chemical} \\ \text{decay} \end{array} \right) \tag{4}$$

$$\frac{-Q(x)}{A} = kc'' - UC' - \beta C$$

$$\therefore -\frac{Q(x)}{A} = kc'' - UC'' - \beta C \tag{5}$$

Where $-\infty < x < \infty$

U is the River is speed in ms^{-1} , A is the Cross sectional area of river in m^2 , $C(x)$ is the Effluent concentration by distance along the river, $Q(x)$ is the Discharge rate

of effluent, and k is the Diffusion Coefficient (diffusivity)

$$kC'' = k\nabla^2 C = k \frac{\partial^2 C}{\partial x^2} \tag{6}$$

$$UC' = U\nabla C = U \frac{\partial C}{\partial x} \tag{7}$$

The general Solution of (2) when the diffusion component kC'' is ignored i.e.

$$UC' + \beta C = \frac{Q(x)}{A} \tag{8}$$

However if the diffusion component is considered, the general solution of (5) becomes:

If we ignore the diffusion component $kC'' = k\nabla^2 C = k \frac{\partial^2 C}{\partial x^2}$, equation (5) reduces to

$$UC' + \beta C = \frac{Q(x)}{A} \tag{9}$$

Condition for the solution of the differential equation is to solve for $C(x)$

Assuming that U, Q, A and β are constants. For $C(x)$, C is dependent variable while x is independent variable.

Applying integrating factor method, we have:

$$UC' + \beta C = \frac{Q(x)}{A} \tag{9}$$

Reducing equation (10) to homogenous differential equation, the right hand side becomes zero. Hence we have:

$$UC' = -\beta C, \quad C' = -\frac{\beta C}{U}, \quad \frac{C'}{C} = -\frac{\beta}{U}$$

$$\log_e C = -\int \frac{\beta}{U}, \quad C = ke^{-\int \frac{\beta}{U}}$$

Since the logarithm of a number to any base is the power to which the base must be raised to give you the number.

$$C(x) = k(x)e^{-\int \frac{\beta}{U} dx}$$

$$\frac{dc}{dx} = \frac{dk}{dx} e^{-\int \frac{\beta}{U} dx} - k \left(-\frac{\beta}{U} e^{-\int \frac{\beta}{U} dx} \right)$$

$$\frac{dc}{dx} = \frac{dk}{dx} e^{-\int \frac{\beta}{U} dx} + k \frac{\beta}{U} e^{-\int \frac{\beta}{U} dx} \tag{11}$$

Substituting (11) in (10)

$$U \left[\frac{dk}{dx} e^{-\int \frac{\beta}{U} dx} - k \frac{\beta}{U} e^{-\int \frac{\beta}{U} dx} \right] + k\beta e^{-\int \frac{\beta}{U} dx} = \frac{Q(x)}{A} \tag{12}$$

$$U \frac{dk}{dx} e^{-\int \frac{\beta}{U} dx} - k \frac{\beta U}{U} e^{-\int \frac{\beta}{U} dx} + k\beta e^{-\int \frac{\beta}{U} dx} = \frac{Q(x)}{A}$$

$$\frac{dk}{dx} e^{-\int \frac{\beta}{U} dx} - k\beta e^{-\int \frac{\beta}{U} dx} + k\beta e^{-\int \frac{\beta}{U} dx} = \frac{Q(x)}{A}$$

$$\frac{dk}{dx} = \frac{Q(x)}{A} e^{\int \frac{\beta}{U} dx}, \quad \int \frac{dk}{dx} = \int \frac{Q}{A} \cdot e^{\int \frac{\beta}{U} dx}$$

$$K(x) = \int \frac{Q}{A} \cdot \frac{U}{\beta} e^{\int \frac{\beta}{U} dx} + C_o$$

And the general solution of (5) is:

$$C(x) = \frac{Q(x)}{A\beta} \left(1 - e^{-\frac{\beta x}{\mu}} \right) + C_o e^{-\frac{\beta x}{\mu}} \tag{14}$$

However, if the diffusion coefficient is not ignored, then the general solution becomes

$$C(x) = (A + Bx)e^{mx} \tag{15}$$

Expanding the general solution: $C(x) = (A + Bx)e^{mx}$ as series approximation to a function, and noting that m is a constant,

$$e^{mx} = 1 + mx + \frac{m^2 x^2}{2!} + \frac{m^3 x^3}{3!} + \frac{m^4 x^4}{4!} + \frac{m^5 x^5}{5!} + \dots \tag{16}$$

$$(A + Bx)e^{mx} = \sum_{n=1}^{\infty} \frac{m^n x^n}{n!}$$

$$= (A + Bx) \left[1 + mx + \frac{m^2 x^2}{2!} + \frac{m^3 x^3}{3!} + \frac{m^4 x^4}{4!} + \frac{m^5 x^5}{5!} + \dots \right] \tag{17}$$

$$= A + Amx + \frac{Am^2 x^2}{2!} + \frac{Am^3 x^3}{3!} + \frac{Am^4 x^4}{4!} + \frac{Am^5 x^5}{5!} + \dots$$

$$+ Bx + Bmx^2 + \frac{Bm^2 x^3}{2!} + \frac{Bm^3 x^4}{3!} + \frac{m^4 x^5}{4!} + \frac{m^5 x^6}{5!} + \dots$$

Grouping similar terms:

$$(A + Bx)e^{mx} = A + (Am + B)x + \left(\frac{Am^2}{2} + Bm \right) x^2$$

$$+ \left(\frac{Am^3}{6} + \frac{Bm^2}{2} \right) x^3 + \left(\frac{Am^4}{24} + \frac{Bm^3}{6} \right) x^4$$

$$+ \left(\frac{Am^5}{120} + \frac{Bm^4}{24} \right) x^5 + \dots \tag{18}$$

Take partial sum of the first three terms:

$$\text{Set } A = c, \quad (Am + B) = b \quad \text{and} \quad \left(\frac{Am^2}{2} + Bm \right) = a$$

and

$$(A + Bx)e^{mx} \approx c + bx + ax^2 \tag{19}$$

$$\Rightarrow (A + Bx)e^{mx} = ax^2 + bx + c \tag{20}$$

$$\therefore C(x) = ax^2 + bx + c \tag{21}$$

With the aid of programmable calculator which relates the exponential decay curve to parabolic curve, its equivalent was given as a, b and c ; where, $a = -5.42, b = -42.34, c = 3023.85$. $y = ax^2 + bx + c$

Thus giving

$$C(x) = -5.42x^2 - 42.34x + 3023.85 \text{ in mg/l} \tag{22}$$

We can also use Fourier expansion for equation (5) as suggested by [16] as follows:

$$C(x) = F^{-1} \left\{ \frac{Q}{A} \cdot \frac{1}{Kw^2 + iuw + \beta} \right\} \tag{23}$$

Breaking the denominator of (23) into partial fractions:

$$\frac{1}{Aw^2 + iuw + \beta} = \frac{1}{Aw^2 + i\left(\frac{u}{K}\right)w + \frac{\beta}{K}} \quad (24)$$

Considering only the denominator for now,

$$w^2 + i\left(\frac{u}{K}\right)w = \left(\frac{\beta}{K}\right) \quad (25)$$

$$w^2 + i2w\frac{u}{K} + \left(\frac{u}{K}\right)^2 = a^2 + 2ac + c^2 \quad (26)$$

$$\left(w + i\frac{u}{K}\right)^2 = \left(w + i\frac{u}{K}\right)\left(w + i\frac{u}{K}\right) \quad (27)$$

$$= w^2 + i\frac{u}{K} \cdot w + i\frac{u}{K} \cdot w + \left(\frac{u}{K}\right)^2 \quad (28)$$

$$= w^2 + 2i\frac{u}{K} \cdot w - \left(\frac{u}{K}\right)^2 \quad (29)$$

Reintroducing $\left(\frac{\beta}{K}\right)$ from equation (24) along with other manipulations, we have

$$\left[w^2 + i2w\frac{u}{K} + \frac{u^2}{K^2}\right] - \frac{u^2}{K^2} + \frac{\beta}{K} - iw\frac{u}{K} \quad (30)$$

$$\left[\left(\frac{u^2}{K^2} + \frac{\beta}{K}\right)\right] - iw\frac{u}{K} \quad (31)$$

$$\left(w + \frac{iw}{K}\right)^2 + \left(\frac{u^2}{K^2} + \frac{\beta}{K}\right) - iw\frac{u}{K} \quad (32)$$

$$\left(w + \frac{iw}{K}\right)^2 - \frac{iwu}{K} + \left(\frac{u^2}{K^2} + \frac{\beta}{K}\right) \quad (33)$$

Equation (32) is likened to $a^2 - 2ac + c^2$

Where $a^2 = \left(w + \frac{iw}{K}\right)$, $c^2 = \left(\frac{u^2}{K^2} + \frac{\beta}{K}\right)$, $2ac = \left(\frac{iwu}{K}\right)$

Hence, we factorize this as:

$$\left[\left(w + \frac{iw}{K}\right) - \left(\frac{u^2}{K^2} + \frac{\beta}{K}\right)\right]^2 \quad (34)$$

We therefore have:

$$\frac{1}{\left(w + \frac{iw}{K}\right)} = \frac{1}{\left(\frac{u^2}{K^2} + \frac{\beta}{K}\right)} \quad (35)$$

Equation (35) is of the form $(x - a)^2$. Guided by this form $(x - a)^2$, we accessed the Fourier transform table. This structural form can be used to select appropriate

solution of the Fourier differential equation from Fourier Transform Table. Thus the solution becomes

$$C(x) = \begin{cases} C_0 e^{-wx}, & x < 0 \\ C_0 e^{-wx}, & x > 0 \end{cases} \quad (36)$$

The distribution of effluent concentration can be represented by signals that resemble Fourier series. It is damped by the decay exponential function e^{-mx} . Hence the signal, i.e., concentration dies off (whittles away) downstream as $x \rightarrow +\infty$. However, without much loss of engineering accuracy, the function, as tabulated in Table 2 can be represented by a simple function within the zone of pollution $0 < x < L$, where L is a few breadths measured downstream.

An elaborate ANOVA conducted had earlier shown that significant differences exist between upstream and downstream effluent concentration about point sources.

Duncan multiple range test was conducted in order to ascertain the order in which the effluent parameters vary in intensity with distance.

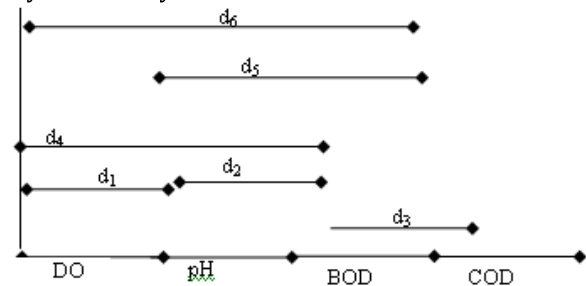


Figure 3. Duncan test DO results for pH for the effluent parameters.

All d_i 's ($i= 1, 2... 4$) are significant except d_1 . This implies that there is no significance difference between dissolved oxygen and pH concentrations but there are significant differences between any pairs of effluent parameters over time and distance.

Table-2 Distribution of Concentration by 2nd Order Differential Equation.

x(m)	0	1	2	3	4	5	6	7	8	9	10
C(x)mg/L	3000	2983	2935	2865	2776	2675	2563	2446	2324	2201	2078

Table 3: Correlation Matrix of 13 Pollution Variables of Ikpoba River Water

	PO4	NOB	Cd	Cu	Fe	Pb	Turb.	Temp	Feacal Col.	pH	DO	BOD	COD
PO4	1												
NO3	0.816	1											
Cd	0.399	0.683	1										
Cu	0.813	0.371	0.699	1									
Fe	0.793	0.809	0.731	0.998	1								
Pb	0.864	0.788	0.572	0.912	0.918	1							

	PO4	NOB	Cd	Cu	Fe	Pb	Turb.	Temp	Feacal Col.	pH	DO	BOD	COD
Turb.	0.757	0.823	0.590	0.862	0.928	0.972	1						
Temp.	0.223	0.006	0.599	0.235	0.341	0.268	0.247	1					
Feacal Col.	0.792	0.892	0.618	0.971	0.925	0.976	0.995	0.213	1				
pH	-0.70	-0.69	-0.76	-0.8	-0.95	-0.20	-0.93	-0.54	-0.85	1			
DO	-0.66	-0.81	-0.69	-0.7	-0.93	-0.93	-0.97	-0.39	-0.96	-0.959	1		
BOD	0.54	0.60	0.369	0.84	0.892	0.931	0.963	0.053	0.9694	-0.84	-0.90	1	
COD	0.57	0.56	0.49	0.6	0.91	0.97	0.99	0.24	0.99	0.93	0.97	0.9552	1

The test confirmed that all effluent parameters namely COD, BOD, pH, DO are time and distant dependent. The result highlighted that significant correlation exist among the pollutant variable studied. For instance, faecal coliform correlates positively with pH, DO, BOD and COD. The eigenvalue analysis presents the results in four observable modes as shown in Figure 4

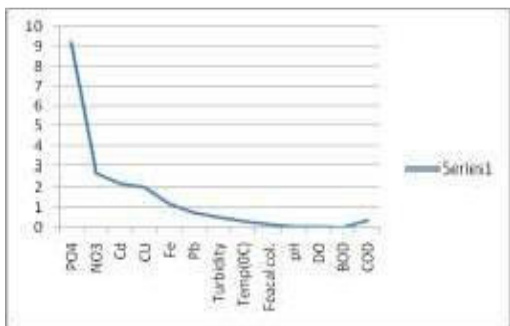


Figure 4: Eigenvalue plot by water quality parameter

The first mode comprises radical i.e. PO_4 and NO_3 with $\lambda_1=9.2151$ and $\lambda_2=2.7476$ respectively. Thus corroborating with [17 and 18] that phosphate and nitrates are the main causes of the destruction of fresh water and lake ecosystem around the world. Furthermore, point sources P1 and P3 were found to be major contributing point sources of phosphate and nitrate pollution. And these are locations where two major industries along the river course discharge their effluents into the river. This also in corroborated by [19, 20 and 21] which pointed that brewery, hospital and industrial wastes are generally are high in nitrate and phosphate components. The second mode comprises heavy metals namely cadmium, copper, iron and lead with 2.2183, 2.0063, 1.1975 and 0.7371 mg/L respectively. The effect of heavy metals on human health is, to say the least, deleterious. And according to [21], source of heavy metals into water bodies include waste water effluent containing metals from metabolic waste sludge. The third mode consists of turbidity, temperature, faecal

coliform and COD with eigenvalues of 0.5053, 0.3213, 0.1679 and 0.3662 mg/l respectively. High temperature is usually associated with bottle washing plants [22], which is a major process in the brewery industry. The pH, DO, and BOD which makes up the fourth mode are not pollutants in themselves but rather indicators of pollution hence the low eigenvalues they have.

3.2 Results of Application of Homogenous Differential Equation.

The developed model is of the form [23]:

$$x^2 + (y - A)^2 = A^2 \tag{37}$$

Which is a circle with x-axis fixed at the origin while the y-axis is steadily shifting longitudinally at the speed of the river to create Doppler shift run as shown in Figures 5 and 6. The model demonstrates the dispersion process and vividly describes the effluent matters in distribution in a waveform.

It is important to note that the effluent trajectory and degree of mixing are influenced by three important hydrodynamic attributes namely

- i. Initial jet characteristics of momentum flux
- ii. Buoyancy flux due to density differences and
- iii. Outfall geometry

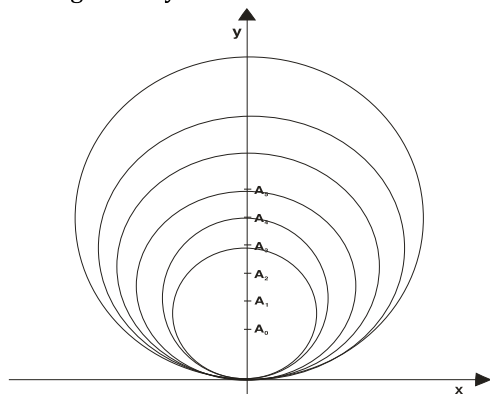


Figure 5: Drifting circle defining Doppler Shift Run

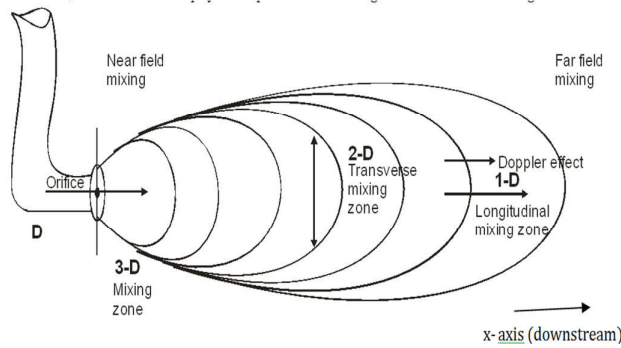


Figure 6: Plan view of effluent plume

3.3 Determination of Mass Concentration Distribution in 3-D mixing Zone.

The result of the propagation of effluent plume in Ikpoba River is in agreement with the concentration dispersion illustrated by [24] and [25]. The above observed dispersion of effluent is effected in spherical form i.e. in three dimensional pattern (3-D); vertically down towards the river bed, laterally towards the river banks and longitudinally along the river course. With respect to 3-D mixing zone, we employ the relation $Cx = \frac{1}{x}$ [26 and 27] for contaminant vector where Cx is the pollutant concentration and x is the longitudinal distance in meters measured from the orifice, the fading effluent concentration downstream up to 10 meters. The order of sequence of the contaminants is as determined by eigenvalue analytics used in earlier study by the authors [28]. Table 4 shows the complete computation of the dispersion of mass concentration of the effluent as plume from the orifice.

Table 4: Concentration dispersion in the 3-D mixing zone

A(m)	$C_x = \frac{1}{x}$	PO ₄	NO ₃	Cd	Cu	Fe	Pb	Turb.	Temp.	Feacal Coliform	pH	DO	BOD	COD
1	1	0.868	0.933	0.77	0.503	5.062	0.717	30	27.433	126.33	6.16	0.267	16.467	277.6
2	0.5	0.434	0.467	0.385	0.252	2.531	0.359	15	13.717	63.165	3.08	0.134	8.234	138.8
3	0.333	0.289	0.311	0.256	0.167	1.686	0.239	9.99	9.135	42.068	2.051	0.089	5.484	92.45
4	0.25	0.217	0.233	0.193	0.126	1.266	0.179	7.5	6.858	31.583	1.54	0.067	4.117	69.41
5	0.20	0.174	0.187	0.154	0.101	1.012	0.143	6.0	5.487	25.266	1.232	0.053	3.293	55.53
6	0.107	0.145	0.156	0.129	0.084	0.845	0.120	5.01	4.581	21.097	1.029	0.045	2.750	46.37
7	0.143	0.124	0.133	0.110	0.072	0.724	0.103	4.29	3.923	18.065	0.881	0.038	2.355	39.71
8	0.125	0.109	0.117	0.096	0.063	0.633	0.090	3.75	3.429	15.791	0.77	0.033	2.058	34.71
9	0.111	0.096	0.104	0.085	0.056	0.562	0.080	3.33	3.045	14.023	0.684	0.030	1.828	30.85
10	0.1	0.087	0.093	0.077	0.050	0.506	0.072	3.0	2.743	12.633	0.616	0.027	1.647	27.76

3.4 Determination of Location of Complete Vertical Mixing.

In determining the location of complete vertical mixing of the plume (L_{mv}) we employ $L_{mv} = 0.4 \frac{Uh^2}{E_z}$ [26] where U is the average velocity, h is the depth of the river and $E_z = \alpha_z U_* h$ is the vertical eddy diffusivity [29].

$$\alpha_z = 0.07 \pm 50$$

From [26], frictional velocity U^* is given by:

$$U^* = (0.05 \text{ to } 0.10)U$$

From our measurements

$$U = 1.35m/s, h = 4m$$

$$\beta = \text{Riverwidth} = 30metres$$

$$E_z = \alpha_z U_* h$$

$$E_z = 0.07 \times (0.1 \times 1.35) \times 4 = 0.0378$$

$$\therefore L_{mv} = \frac{0.4 \times 1.35 \times 4^2}{0.0378} = 229metres$$

The complete vertical mixing is therefore located at 229 meters from the point of discharge; and this is the point when the concentration of the bed become, 90% of the surface concentration.

3.5 Determination of Mass Concentration Distribution in 2-D mixing zone.

Shown in Table 5 is the fitting of the model:

$$Cx = \frac{1}{\sqrt{x}} \tag{38}$$

into our effluent sample data. The concentration distribution along the course of the river from 1 meter up to 2000meters is presented in Table 5.

Table 5: Concentration Dispersion in 2-D Mixing zone

A(m)	$\frac{1}{\sqrt{x}}$	PO ₄	NO ₃	C _d	C _u	F _e	P _b	Turb.	Temp	PH	DO	BOD	COD	
1	1	0.868	0.933	0.77	0.503	5.062	0.717	30	27.433	126.33	6.16	0.267	16.467	277.633
2	0.707	0.614	0.660	0.54	0.356	3.579	0.507	21.21	19.395	89.315	4.355	0.189	11.642	196.287
3	0.577	0.501	0.538	0.44	0.29	2.92	0.413	17.31	15.829	72.892	3.554	0.154	9.501	160.194
4	0.5	0.434	0.467	0.38	0.25	2.53	0.35	15.0	13.717	63.165	3.08	0.134	8.234	138.817
5	0.447	0.388	0.417	0.34	0.22	2.26	0.32	13.41	12.263	56.470	2.754	0.119	7.361	124.102
6	0.408	0.354	0.381	0.31	0.20	2.06	0.29	12.24	11.193	51.543	2.513	0.109	6.719	113.274
7	0.378	0.328	0.353	0.29	0.19	1.91	0.27	11.34	10.370	47.753	2.328	0.101	6.225	104.945
8	0.354	0.307	0.330	0.27	0.17	1.79	0.25	10.62	9.711	44.721	2.181	0.095	5.829	98.282
9	0.333	0.289	0.311	0.25	0.16	1.68	0.23	9.99	9.135	42.068	2.051	0.089	5.484	92.452
10	0.316	0.274	0.295	0.24	0.15	1.60	0.22	9.48	8.669	39.920	1.947	0.084	5.204	87.732
500	0.045	0.039	0.042	0.03	0.02	0.22	0.032	1.35	1.234	5.685	0.277	0.012	0.741	12.493
1000	0.032	0.028	0.030	0.02	0.01	0.16	0.019	0.96	0.878	4.043	0.197	0.009	0.527	8.884
1500	0.026	0.023	0.024	0.02	0.01	0.13	0.019	0.78	0.713	3.285	0.160	0.007	0.428	7.218
1800	0.024	0.021	0.022	0.01	0.01	0.121	0.017	0.72	0.658	3.032	0.148	0.006	0.395	6.663
2000	0.022	0.019	0.021	0.01	0.01	0.111	0.016	0.66	0.604	2.779	0.136	0.006	0.362	6.108

Table 6: Water Quality Determinants for Ikpoba River by Seasons

Input/output data	symbol	Unit	UBTH			EWAH RD.JUNCTION			GUINNESS		
			Dry Season	Harmattan season	Rainy Season	Dry season	Harmattan season	Rainy season	Dry season	Harmattan season	Rainy season
Temperature of river	T	°C	23.6	25.2	22.6	26.8	27.5	25.9	28.2	27.0	25.9
Initial DO prior to discharge point	C _a	mg/l	7.9	7.6	8.5	7.0	6.6	7.4	7.1	6.8	7.8
DO after discharge	DO	mg/l	7.8	7.6	7.8	6.7	6.2	6.2	0.3	0.2	0.6
Critical DO	C _c	mg/l	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
Saturated DO	C _s	mg/l	8.47	8.23	8.63	8.0	7.9	8.1	7.8	8.0	8.1
DC=C _s -C _c	DC	mg/l	3.47	3.32	3.36	3.0	2.9	3.1	2.8	3.0	3.1
D _a = C _s - C _a	D _a	mg/l	0.57	0.63	0.13	1.0	1.3	0.7	0.7	1.2	0.3
Decay rate constant at T°C	K _{1(T)}	day ⁻¹	0.785	0.727	0.718	0.785	0.810	0.753	0.839	0.790	0.753
Reaeration rate T°C	K _{2(T)}	day ⁻¹	0.671	0.645	0.641	0.671	0.680	0.656	0.693	0.67	0.656
BOD ₅ (river water)		mg/l	4.1	3.10	3.4	0.6	0.8	1.3	1.1	1.3	1.5
BOD _u	L _p	mg/l	5.3	6.7	5.9	3.8	4.9	4.2	16.2	17.4	13.1
River flow rate	Q _r	m ³ /day	13996800	25920000	15246100	13996800	25920000	15246100	13996800	13996800	15246100
Discharge flow rate	Q _w	m ³ /day	2160000	1728000	2160000	1120000	1120000	6200000	4160000	1728000	2160000
Self -purification factor	f		0.85	0.89	0.89	0.85	0.84	0.87	0.83	0.85	0.87
	$\frac{K_2}{K_1}$										
D_a/D_c	D_a/D_c		0.16	0.2	0.04	0.33	0.45	0.23	0.25	0.4	0.1
L_a/D_c	L_a/D_c		2.5	2.3	2.7	2.5	1.9	2.6	2.4	1.9	2.6
L_a	L_a	mg/l	8.7	8.7	9.8	7.5	4.6	8.1	6.7	5.7	8.1
Maximum allowable BOD _u	B _u	Kg/day	5.74x10 ⁹	4.48x10 ⁹	6.55x10 ⁹	5.87x10 ⁹	2.21x10 ⁹	6.27x10 ⁹	7.39x10 ⁹	5.6x10 ⁹	1.05x10 ¹⁰
Assimilative capacity (AC)	AC	Kg/day	1.05x10 ¹⁰	1.95x10 ¹⁰	1.29x10 ¹⁰	9.07x10 ⁹	1.03x10 ⁹	1.07x10 ¹⁰	8.1x10 ⁹	1.28x10 ¹⁰	1.07x10 ¹⁰

It is evident from the table that the dispersion in 2-D zone occurs more gradually in the transverse mixing zone i.e. far-field. Figure 4 is instructive. It shows the steepness of dispersion at near field and the gradual dispersion at far-field even beyond 2km.

We employ the relation: $Cx = \frac{1}{\sqrt{x}}$

3.6 Determination of Location of Complete Transverse and Longitudinal Mixing

Far-field mixing refer to the distance to the location where complete transverse and longitudinal mixing occur and it is determined with following relation

$$L_{mh} = 0.4 \frac{UB^2}{E_y} \quad [26 \text{ and } 30] \quad (39)$$

$$E_y = \alpha_y U_* h, \alpha_y = 0.5 \pm 5 \%, \text{ so}$$

$$E_y = \alpha_y U_* h, \alpha_y = 0.5 \pm 50\% , \text{ so}$$

Table 5 shows the decay of each of the pollutant parameters along the river course. Thus about 1km downstream from point source of pollution, complete transverse and longitudinal mixing seem to have occurred.

Table 6 depicts the tableau of results of computation of several parameters from which the assimilative capacity was derived.

As can be seen from Table 6, the assimilative capacity is the last row and its value for different seasons and three different locations appear as last row under various columns. The results show that the deoxygenation rate in the river dominates the oxygenation rate, thus portraying the river as highly polluted with poor assimilative capacity potential.

Furthermore, for each of the segment studied, rainy season shows a higher tendency for minimum acceptable dissolved oxygen concentration than both the dry and harmattan seasons.

In these variety of modeling, the concept of eudiometry was applied. The level of DO, y, at any time was viewed as a projection of that value into a column vector space spanned by the pollutant variables

($x_1 \dots, x_2 \dots, x_3 \dots, x_n$). The parameters of the regression namely $\beta_0, \beta_1, \beta_2 \dots, \beta_1$ was used to establish a projection of DO unto the column vector which is expressed as y.

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n + \varepsilon_i \quad (40)$$

The β_i represents the propensity of the pollutant variables to adsorb dissolved oxygen. The values of β_i were estimated using OLS methodology and verified by MATLAB software computation. Each of the values are depicted in Tables 7 and 8

The multivariate coefficient of determination R^2 was computed as 0.71 indicating a substantial fit.

Bootstrapping was used to establish the upper control limit (UCL) and the lower control limit(LCL). The mean values as determined through point estimates by ordinary least squares (OLS) are shown against the upper and lower control limits. See Table 9.

The oxygen depth atmosphere is the region below the compensation depth, where, due to diminished light, the rate of photosynthesis and hence, reaeration decreases with increasing depth. At this depth, oxygen produced by photosynthesis will be equal to oxygen used in respiration. Thus there will be more dissolved oxygen used in respiration than produced by photosynthesis hence resulting to "oxygen debt"[31].

Table 7: Manually Computed Results of Regression Analysis

β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
-263.95	78.327	-127.2	3.8202	3.645	25.7316	-3.0779	-1.2954

Table 8: MATLAB computed results of Regression analysis

β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
-279.54	80.2411	-131.07	4.0714	3.7552	27.0139	-3.1380	-1.3322

Table 9: Depiction of Confidence Limits across Some Significance Levels

90%	UCL	193.6897	193.0304	192.3905	191.8902	195.5626	184.3446	192.967
	LCL	-177.391	-178.05	-178.69	-179.19	-175.518	-186.736	-178.113
	PNT. EST.	8.149565	7.490211	6.850315	6.350073	10.02246	-1.19555	7.426784
95%	UCL	229.2187	228.5594	227.9195	227.4192	231.0916	219.8736	228.4959
	LCL	-213.579	-213.579	-214.219	-214.719	-211.047	-222.265	-213.642
	PNT. EST.	8.149565	7.490211	6.850315	6.850315	10.02246	-1.19555	7.426784
99%	UCL	299.1487	298.4894	297.8495	297.3493	301.0216	289.8036	298.426
	LCL	-282.85	-283.509	-284.149	-284.649	-280.977	-292.195	-283.572
	PNT. EST.	8.149565	7.490211	6.850315	6.350073	10.02246	-1.19555	7.426784

Gloss: UCL: Upper Control Limit, LCL: Lower Control Limit, PNT. EST.: Point Estimate, Figure 7 refers.

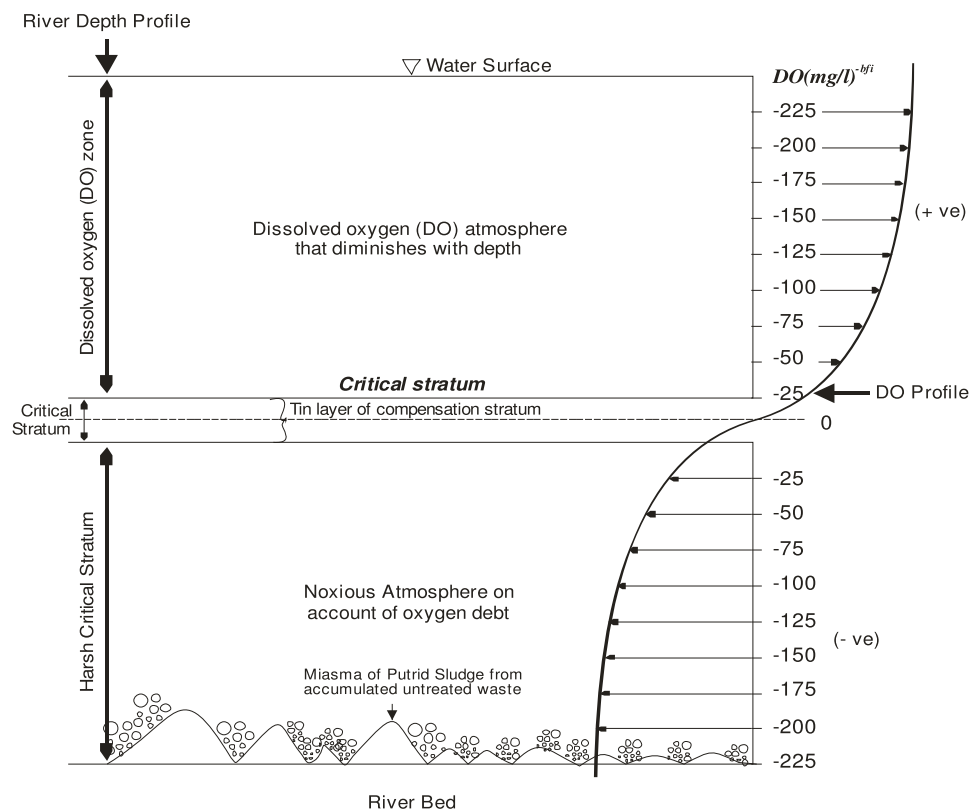


Figure 7: Relativizing the Profile of River Depth and DO

Oxygen debt theoretically denotes a condition whereby the oxygen sag zone of a water body falls below critical to harsh critical level. At this level, any reaeration effort is not practically palpable until recovery is made up to the critical level. This accounts for the negative value obtained in the lower control limit of the assimilative capacity of the river. The negative value is the amount of DO required to bring it to equilibrium as in the compensation depth.

This study has therefore established the knowledge that there could be obscure part of a river, depending on the long profile or cross profile of the river channel, where accumulation of pollutants and influence of blackening that gives rise to photosynthesis dysfunction, can create oxygen debt, (a region of severe DO privation) requiring delay manifestation of DO in any process of reaeration or reoxygenation until the dissolved oxygen deprivation is overcome.

It is akin to latent heat, say for ice, where heat supplied is initially used to melt ice, and afterwards, specific heat becomes palpable. In this situation, accumulated sludge in obscure position, starts adsorbing dissolved oxygen until the DO privation ceases. Therefore, reaeration phase starts from the compensation stratum to plume to the surface of the water body.

Taking together, we see that the sensitivity analysis conducted show that substantial changes in the sensitive model parameters did not results in wild fluctuations of DO beyond what the model could readily accommodate. In other words the model is robust and remains robust in the face of perturbation in key regression parameters.

5. CONCLUSION

In the light of the foregoing oodles of results from the models drawn from the eclectic applications we can notice that assimilative capacity concept of a river is dependent upon several factors, such as hydraulic resilience, the intensity of industrial and municipal loading, the season of the year, the spatial distribution of point sources, the mechanics of mixing at near and far-field. Other factor include the interactions between pollutant variables, the eigenvalue evaluation of the pollutant variables, and the chemical diffusivity of the chemical compounds in the pollutants, among other things.

It is interesting to note that this model can be applied to stratified water body like the lake ecosystem where the worst case scenario of negative DO level (oxygen debt) exist at the hypolimnion layer.

Our world view of assimilative capacity has been enhanced by the application of diverse models in the investigation. We have gone beyond wondering to the stage of confirming that there is truth in the old chestnut to the effect that eclecticism brings with it knowledge from different sources. We have fixed the jigsaw, and are now better able to understand assimilation capacity. Also it has been confirmed that our regression model acts as a virtual eudiometer which can easily predict assimilative capacity and determine its supremum and infimum beyond which limits our model will breakdown and cease to be robust.

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