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A MATHEMATICAL MODEL FOR PREDICTING THE FLEXURAL STRENGTH CHARACTERISTICS OF CONCRETE MIXES MADE WITH GRANITE CHIPPINGS

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Abstract
Abakaliki, Ebonyi state of Nigeria produces most of the crushed-granite chippings used in the South-eastern part of the country. In this research work, these granite-chippings and fine aggregates from Amansea River in Anambra State of Nigeria were tested for their physical and mechanical properties based on BS 812: Parts 1&2:1975. Using these aggregates, sixty concrete beams of dimensions 600 mm X 150mm X 150 mm were made, cured and tested based on BS 1881:1983. Scheffe's (4, 2) lattice polynomial with regression equation was used to develop a mathematical model for predicting the flexural strength characteristics of concretes made with these aggregates. The mathematical model developed was $\hat{Y} = 4.28x_1 + 4.42x_2 + 3.4x_3 + 2.71x_4 + 0.2x_1x_2 + 0.04x_1x_3 - 0.14x_1x_4 - 0.08x_2x_3 - 0.3x_2x_4 + 1.22x_3x_4$. Finally, the student's t-test and the Fisher test were used to test the model's validity.

***Keywords**:* Concrete, Flexural Strength, Scheffe, Granite-Chippings, Model

1. Introduction

1.1 Actual and Pseudo-Components

The requirement of the simplex that $x_1 + x_2 + x_3 + x_4 = 1$ makes it impossible to use the normal mix ratios such as 1:1:2, etc., at a given water/cement ratio. Hence, a transformation of the actual components (normal mix ratios) to meet this condition is unavoidable. The design matrix is shown in Table 1. $x^{(i)}_1$, $x^{(i)}_2$, $x^{(i)}_3$ and $x^{(i)}_4$ are the pseudo-components for the *ith* experimental points. For any actual component Z, the pseudo-component (x) is given by

$$X = AZ \quad (1)$$

Where A is the inverse of Z matrix and

$$Z = BX^T \quad (2)$$

Where B is the inverse of Z matrix and X^T is the transpose of the matrix.

1.2 The Scheffe's (4, 2) Lattice Polynomial

Simplex is the structural representation of the line or planes joining the assumed positions of the constituent materials (atoms) of a mixture [1]. Scheffe [2] considered experiments with mixtures of which the property studied depended on the proportions of the components present but not on the quantity of the mixture. If a mixture has a total of q components and x_i be the proportion of the ith component in the mixture such that $x_i \geq 0$ (i = 1, 2… q), then

$$x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots\ldots + x_q = 1 \quad (3)$$

Scheffe described mixture properties by reduced polynomials obtainable from eqn (4):

$$\hat{Y} = b_0 + \Sigma b_i x_i + \Sigma b_{ij} x_i x_j + \Sigma b_{ijk} x_i x_j x_k + \Sigma b_{i1,i2\ldots i_n} x_{i1} x_{i2} x_i n \quad (4)$$

Where ($1 \leq i \leq q$, $1 \leq i \leq j \leq q$, $1 \leq i \leq k \leq q$) respectively and b is constant coefficient.

Multiplying eqn. (3) by b_0 and multiplying the outcome by x_1, x_2, x_3 and x_4 in turn and substituting into eqn. (4), we have:

$$\hat{Y} = b_0 x_1+b_0 x_2+ b_0 x_3+ b_0 x_3+ b_0 x_4+ b_1 x_1 + b_2 x_2+ b_3x_3+ b_4x_4 + b_{12} x_1 x_2+ b_{13} x_1 x_3+b_{14} x_1 x_4+b_{23}x_2x_3 + b_{24}x_2x_4+b_{34}x_3x_4+b_{11}(x_1- x_1x_2 - x_1x_3 - x_1x_4) + b_{22}(x_2- x_1x_2 - x_2x_3 - x_2x_4) + b_{33}(x_3- x_1x_3 - x_2x_3 - x_3x_4) + b_{44}(x_4- x_1x_4 - x_2x_4 - x_3x_4) \quad (5)$$

Re-arranging eqn. (5), we have

$$\hat{Y} = \sum \alpha_i x_i + \sum \alpha_{ij} x_i x_j \quad (6)$$

$$\alpha_{ij} = b_{ij} - b_{ii} + b_{jj} \text{ and } \alpha_i = b_0 + b_i + b_{ii} \quad (7)$$

Let the response function to the pure components (x_i) be denoted by y_i and the response to a 1:1 binary mixture of components i and j be y_{ij}. From eqn (6), it can be written that

$$\Sigma\alpha_i x_i = \Sigma y_i x_i \quad (8)$$

Where (i = 1 … 4)

Evaluating y_i, for instance gives:

$$y_i = \alpha_I \quad (9)$$

Also evaluating y_{ij}, gives in general the equations of the form

$$\alpha_{ij} = 4y_{ij} - 2 y_i - 2y_j \quad (10)$$

For the Scheffe's (4, 2) lattice polynomial, that is eqn. (6) becomes:

$$\hat{Y} = y_1 x_1 +y_2 x_2 +y_3 x_3+y_4 x_4+ (4y_{12} - 2y_1 - 2y_2) x_1 x_2 + (4y_{13} - 2y_1 - 2y_3) x_1 x_3 + (4y_{14} - 2y_1 - 2y_4) x_1 x_4 + (4y_{23} - 2y_2 - 2y_3) x_2 x_3 + (4y_{24}- 2y_2 - 2y_4) x_2 x_4 + (4y_{34} - 2y_3 - 2y_4) x_3 x_4 \quad (11)$$

1.2 The student's t-test

The unbiased estimate of the unknown variance $S_Y{}^2$ is given by Biyi [3]

$$S_Y^2 = \frac{\sum \left(y_i - \hat{Y} \right)^2}{n-1} \quad (12)$$

If $a_i = x_i (2x_i - 1)$, $a_{ij} = 4 x_i x_j$; for ($1 \le i \le q$) and ($1 \le i \le j \le q$) respectively. Then,

$$\varepsilon = \Sigma a^2_i +\Sigma a^2_{ij} \quad (13)$$

where ε is the error of the predicted values of the response. The t-test statistic is given by Biyi [3]:

$$t = \frac{\Delta Y}{S_Y} \frac{\sqrt{n}}{\sqrt{1+\varepsilon}} \quad (14)$$

where $\Delta Y = Y_0 - Y_t$; Y_0 = observed value, Y_t = theoretical value; n = number of replicate observations at every point; ε = as defined in eqn.(13).

1.3 The fisher's test

The Fishers-test statistic is given by

$$F = S_1^2/S_2^2 \quad (15)$$

The values of S_1 (lower value) and S_2 (upper value) are calculated from eqn. (12).

2. Materials and method

2.1 Preparation, Curing and Testing of Beam Samples

The aggregates were sampled in accordance with the methods prescribed in BS 812: Part 1:1975 [4]. The test sieves were selected according to BS 410:1986 [5]. The water absorption, the apparent specific gravity and the bulk density of the coarse aggregates were determined following the procedures prescribed in BS 812: Part 2: 1975 [6]. The Los Angeles abrasion test was carried out in accordance with ASTM. Standard C131: 1976 [7]. The sieve analyses of the fine and coarse aggregate samples satisfied BS 882:1992 [8]. The sieving was performed by a sieve shaker. The water used in preparing the experimental samples satisfied the conditions prescribed in BS 3148:1980 [9]. The required concrete specimens were made in threes in accordance with the method specified in BS 1881: 109:1983 [10].These specimens were cured for 28 days in accordance with BS 1881: Part 111: 1983 [11]. The testing was done in accordance with BS 1881: Part 118:1983 [12] using flexural testing machine.

2.2 Testing the Fit of the Quadratic Polynomials

The polynomial regression equation developed was tested to see if the model agreed with the actual experimental results. The null hypothesis (that there was agreement between the experimentally-observed data and the theoretically-obtained data) was denoted by H_0 and the alternative (that there was no agreement between these two) by H_1.

3. Results and discussion

3.1 Physical and Mechanical Properties of Aggregates

The maximum aggregate size for the granite chipping was 20 mm and 2mm for the fine sand. The granite chippings had water absorption of 2.7%, moisture content of 44.2%, apparent specific gravity of 2.26, Los

Angeles abrasion value of 22% and bulk density of 2072.4 kg/m^3.

3.2 The Regression Equation for the Flexural Strength Tests Results

Applying the responses (average flexural strengths) in determining the coefficients of the (4, 2) lattice polynomial to eqns. (9) and (10), we had α_1= 4.28, α_2= 4.42, α_3 =3.4, α_4=2.71, α_{12}=0.2, α_{13} = 0.04, α_{14}=- 0.14, α_{23}= -0.08, α_{24}= -0.3, α_{34}= 1.22. Thus, from eqn.(11): $\hat{Y} = 4.28 x_1 + 4.42 x_2 + 3.4 x_3 + 2.71 x_4 + 0.2 x_1 x_2 + 0.04 x_1 x_3 - 0.14 x_1 x_4 - 0.08 x_2 x_3 - 0.3 x_2 x_4 + 1.22 x_3 x_4$. This is the mathematical model for the response prediction of the flexural strength characteristics of the granite chippings concrete, based on Scheffe's (4, 2) polynomial. $\hat{Y}$ represents the flexural strength of the concrete.

Table 1 Design Matrix for Scheffe's (4, 2) Lattice Polynomial

Legend: z_1= water/cement ratio; z_2=Cement; z_3=Fine aggregate; z_4=Coarse aggregate

Pseudo-components					Actual components			
S/N	x_1	x_2	x_3	x_4	z_1	z_2	z_3	z_4
1	1	0	0	0	0.6	1	1.5	2
2	0	1	0	0	0.5	1	1	2
3	0	0	1	0	0.55	1	2	5
4	0	0	0	1	0.65	1	3	6
5	½	½	0	0	0.55	1	1.25	2
6	½	0	½	0	0.575	1	1.75	3.5
7	½	0	0	½	0.625	1	2.25	4
8	0	½	½	0	0.525	1	1.5	3.5
9	0	½	0	½	0.575	1	2	4
10	0	0	½	½	0.6	1	2.5	5.5
Control								
11	½	¼	¼	0	0.5625	1	1.5	2.75
12	½	0	¼	¼	0.6	1	2.0	3.75
13	0	½	¼	¼	0.55	1	1.75	3.75
14	¼	¼	¼	¼	0.575	1	1.875	3.75
15	¾	¼	0	0	0.575	1	1.375	2
16	¾	0	¼	0	0.5875	1	1.625	2.75
17	¾	0	0	¼	0.6125	1	1.875	3.0
18	0	¾	¼	0	0.5125	1	1.25	2.75
19	0	¾	0	¼	0.5375	1	1.5	3.0
20	0	0	¾	¼	0.5850	1	2.25	5.25

Table 2: Flexural Strength Tests Results and Sample Variances, S_i^2, for Crushed –Granite Concrete, based on Scheffe's (4, 2) Simplex Lattices

S/NO	Replication	Responses y_i(N/mm^2)	Response symbol	Σy_i	Σy_i^2	$\check{y}$	$(\Sigma y_i)^2$	S_i^2
1	1A 1B 1C	4.15 4.35 4.34	y_1	12.84	54.98	4.28	164.87	0.012
2	2A 2B 2C	4.30 4.56 4.40	y_2	13.26	58.64	4.42	175.83	0.015

S/NO	Replication	Responses y_i(N/mm²)	Response symbol	Σy_i	Σy_i^2	$\check{y}$	$(\Sigma y_i)^2$	S_i^2
3	3A 3B 3C	3.00 3.45 3.75	y_3	10.2	34.97	3.4	104.04	0.145
4	4A 4B 4C	2.68 2.68 2.77	y_4	8.13	22.04	2.71	66.10	0.003
5	5A 5B 5C	4.60 4.25 4.35	y_{12}	13.2	58.15	4.4	174.24	0.035
6	6A 6B 6C	3.82 3.95 3.78	y_{13}	11.55	44.48	3.85	133.40	0.007
7	7A 7B 7C	3.50 3.60 3.28	y_{14}	10.38	35.97	3.46	107.74	0.028
8	8A 8B 8C	3.80 3.79 4.08	y_{23}	11.67	45.45	3.89	136.19	0.027
9	9A 9B 9C	3.44 3.58 3.45	y_{24}	10.47	36.55	3.49	109.62	0.005
10	10A 10B 10C	3.40 3.52 3.16	y_{34}	10.08	33.94	3.36	101.61	0.035
CONTROL								
11	11A 11B 11C	4.22 4.35 4.06	C_1	12.63	53.21	4.21	159.52	0.018
12	12A 12B 12C	3.82 3.75 3.23	C_2	10.8	39.09	3.6	116.64	0.105
13	13A 13B 13C	3.85 3.90 3.86	C_3	11.61	44.93	3.87	134.79	0.00
14	14A 14B 14C	3.95 3.78 3.76	C_4	11.49	44.03	3.83	132.02	0.012
15	15A 15B 15C	4.00 4.50 4.49	C_5	12.99	56.41	4.33	168.74	0.082
16	16A 16B 16C	4.10 3.85 3.75	C_6	11.7	45.70	3.9	136.89	0.035
17	17A 17B 17C	3.90 3.57 4.14	C_7	11.61	45.09	3.87	134.79	0.08
18	18A 18B 18C	4.20 4.80 4.20	C_8	13.2	58.32	4.4	174.24	0.12
19	19A 19B 19C	4.10 3.95 3.65	C_9	11.7	45.74	3.9	136.89	0.055

S/NO	Replication	Responses $y_i(N/mm^2)$	Response symbol	Σy_i	Σy_i^2	$\check{y}$	$(\Sigma y_i)^2$	S_i^2
20	20A 20B 20C	3.25 3.60 3.47	C_{10}	10.32	35.56	3.44	106.50	0.03

Table 3a: Regression Analysis of the Flexural Strength Tests Results

Regression Statistics	
Multiple R	0.99974617
R Square	0.9994924
Adjusted R Square	0.83257193
Standard Error	0.11475943
Observations	10

Table 3b: Analysis of variance

ANOVA					
	df	SS	MS	F	Significance F
Regression	4	155.5902816	38.89757	2953.559	1.28792E-08
Residual	6	0.079018365	0.01317		
Total	10	155.6693			

Table 3c: Regression Statistics

REGRESSION STATISTICS						
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0	#N/A	#N/A	#N/A	#N/A	#N/A
x_1	4.1833	0.0844	49.541	4.54E-09	3.9766	4.3899
x_2	4.5405	0.1056	42.98	1.06E-08	4.2820	4.7989
x_3	3.7303	0.1546	24.12	3.33E-07	3.3519	4.1086
x_4	2.5525	0.2679	9.52	7.63E-05	1.8969	3.2080

Legend df = degree of freedom, SS = sum of squares, MS = mean of squares, F = F-statistic, #N/A = insignificant value, ANOVA = analysis of variance.

3.3 Regression Analysis of the Flexural Strength Tests Results for the granite-chippings Concrete

Table 3 shows the summary output of the regression analysis of the flexural strength tests results of the granite-chippings concrete. The coefficient of determination, r^2 = 0.9994 shows a very strong relationship between the independent variables (x_1, x_2, x_3, x_4) and the dependent variable, $\hat{Y}$. Since the F –observed value of 2953.559 is very high; it is extremely unlikely that an F value this high occurred by chance. From the Student's t distribution table, t critical is 3.69. The absolute values of the t stat are greater than this t critical. This shows that all the variables used in the regression equation are useful in predicting the response. The P-values being very small means that the experimentally-obtained values and the predicted values of $\hat{Y}$ have variances that are not significantly different. Thus, the regression equation for the prediction of the flexural strength characteristics of the granite-chippings concrete is valid.

3.3 Fit of the Polynomial

The polynomial regression equation developed i.e., $\hat{Y} = 4.28\,x_1 + 4.42\,x_2 + 3.4\,x_3 + 2.71\,x_4 + 0.2\,x_1x_2 + 0.04\,x_1x_3 - 0.14\,x_1x_4 - 0.08\,x_2x_3 - 0.3\,x_2x_4 + 1.22\,x_3x_4$, was tested to see if the model agreed with the actual experimental results. There was no significant difference between the experimental and the theoretically expected results. The null hypothesis, H_0 was satisfied.

3.4 t -value from table

The t-student's test had a significance level, $\alpha = 0.05$ and $t_{\alpha/l(ve)} = t_{0.005(9)} = 3.69$. This was greater than any of the t values calculated in table 4. Therefore, the regression equation for the crushed granite chippings concrete was adequate.

3.5 F-statistic analysis

Table 5 shows the F – statistic for the controlled points. The sample variances S_1^2 and S_2^2 for the two sets of data were not significantly different. It implied that the error(s) from experimental procedure were similar and that the sample variances tested were estimates of the same population variance. Based on eqn. (12), we had that $S_K^2 = 0.82705/9 = 0.0919$, $S_E^2 = 0.604959/9 = 0.06722$ & $F = 0.0919/0.06722 = 1.367$. From Fisher's table, $F_{0.95(9,9)} = 3.3$, hence the regression equation for the flexural strength of the crushed-granite concrete was adequate.

Table 4: t –Statistic for the controlled Points, granite-chippings concrete flexural test, based on Scheffe's (4, 2) polynomial

Response symbol	Y_K(N/mm²)	Y_E(N/mm²)	Y_K- $\check{Y}_K$	Y_E-$\check{Y}_E$	(Y_K- $\check{Y}_K$)²	(Y_E-$\check{Y}_E$)²
C_1	4.21	4.12	0.275	0.19975	0.075625	0.0399
C_2	3.6	3.73125	-0.335	-0.189	0.112225	0.035721
C_3	3.87	3.76625	-0.065	-0.154	0.004225	0.023716
C_4	3.83	3.76125	-0.105	-0.159	0.011025	0.025281
C_5	4.33	4.3525	0.395	0.43225	0.156025	0.18684
C_6	3.9	4.0675	-0.035	0.14725	0.001225	0.021683
C_7	3.87	3.86125	-0.065	-0.059	0.004225	0.003481
C_8	4.4	4.15	0.465	0.22975	0.216225	0.052785
C_9	3.9	3.93625	-0.035	0.016	0.001225	0.000256
C_{10}	3.44	3.45625	-0.495	-0.464	0.245025	0.215296
Σ	39.35	39.2025			0.82705	0.604959

Legend: C_i =response; $a_i = x_i\ (2x_i - 1)$; $a_{ij} = 4\ x_i\ x_j$; $\varepsilon = \Sigma a^2_i + \Sigma a^2_{ij}$; y̌ = experimentally-observed value; Ŷ= theoretical value; t = t-test statistic.

Table 5: F –statistic for the controlled points, granite-chipping concrete flexural test, based on Scheffe's (4, 2) polynomial

Table 5a: Response symbol for C_1: $\varepsilon = 0.6093$, $\check{Y}$= 4.21N/mm², $\hat{Y}$ = 4.12N/mm² and t=0.456923

i	j	a_i	a_{ij}	a_i^2	a_{ij}^2
1	2	0	0.5	0	0.25
1	3	0	0.5	0	0.25
1	4	0	0	0	0
2	3	-0.12	0.25	0.0156	0.0625
2	4	-0.12	0	0.0156	0
3	4	-0.12	0	0.0156	0
4	—	0	—	0	—
			∑	0.0468	0.5625

Table YY: Response symbol for C_2-C_{10}

RESPONSE SYMBOL	ε	Ў(N/mm²)	Ŷ(N/mm²)	t
C_2	0.4842	3.6	3.73125	-0.72251
C_3	0.7343	3.87	3.76625	0.488766
C_4	0.5939	3.83	3.76125	0.35241
C_5	0.2893	4.33	4.3525	-0.14258
C_6	0.8593	3.9	4.0675	-0.73604
C_7	0.5937	3.87	3.86125	0.044858
C_8	0.4833	4.4	4.15	1.377045
C_9	0.6405	3.9	3.93625	-0.18054
C_{10}	0.4697	3.44	3.45625	-0.09034

Legend: Ў=Σy/n where y is the response and n, the number of observed data (responses), Y_k is the experimental value (response), Y_E is the expected or theoretically calculated value (response)

4. Conclusion

The strengths (responses) of concrete were a function of the proportions of its ingredients: water, cement, fine aggregate and coarse aggregates. Since the predicted strengths by the model were in total agreement with the corresponding experimentally -observed values, the null hypothesis was satisfied. This meant that the model equation was valid.

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