Nigerian Journal of Technology (NIJOTECH)
Vol. 32. No. 2. July 2013, pp. 211-216

ISSN 1115-8443
www.nijotech.com

PREDICTION OF CONCRETE MIX COST USING MODIFIED REGRESSION THEORY

D. O. Onwuka+, C. E. Okere+* , S. U. Onwuka++
+ DEPARTMENT OF CIVIL ENGINEERING, FEDERAL UNIVERSITY OF TECHNOLOGY, OWERRI, NIGERIA.
++DEPT. OF PROJECT MANAGEMENT TECHNOLOGY, FEDERAL UNIV. OF TECHNOLOGY, OWERRI, NIGERIA.

*(corresponding author) chinenyeeokere@gmail.com, +daviesonwuka@yahoo.com, ++dasyldavis@yahoo.com

Abstract
The cost of concrete production which largely depends on the cost of the constituent materials, affects the overall cost of construction. In this paper, a model based on modified regression theory is formulated to optimise concrete mix cost (in Naira). Using the model, one can predict the cost per cubic meter of concrete if the mix ratios are given. The model can also give possible mix ratios for a specified cost. Statistical tool was used to verify the adequacy of this model. The concrete cost analysis is based on the current market prices of concrete constituent materials. A Price Fluctuation Factor (PFF) can be used to take care of price fluctuations over time.

Keywords: *concrete; regression theory; optimisation; cost; model*

1. Introduction

Concrete, which is a man- made geology is widely used in construction. Communities around the world rely on concrete as a safe and simple building/construction material. It is made by mixing cement, water, fine and coarse aggregates and sometimes admixtures in their right proportions to obtain the specified property. The cost of producing concrete affects the overall cost of construction. The cost of concrete depends on the cost of the constituent materials. In other words, if the constituent materials are chosen in their correct proportions, the optimum cost of concrete production can be obtained. Various mix design methods have been used. These methods are based on the conventional approach of selecting arbitrary mix proportions, subjecting concrete samples to laboratory and then adjusting the mix proportions in subsequent tests. Apparently, these methods are tedious, time consuming and expensive.

To minimize some of these limitations an optimization procedure has been proposed. It is a process that seeks for the maximum or minimum value of a function of several variables while at the same time, satisfying a number of other imposed requirements [1]. This process involves using statistical techniques to fit empirical models to the data for each performance criterion.

In this paper, a mathematical model based on modified regression theory of statistics by Osadebe [2] is formulated for the optimisation of concrete mix cost. Osadebe assumed that the response function, $F(z)$, is continuous and differentiable with respect to its predictors, Z_i. By making use of Taylor's series, the response function could be expanded in the neighbourhood of a chosen point. The formulation of the regression equation is done from first principles using the so-called absolute volume (mass) as a necessary condition. This principle assumes that the volume (mass) of a mixture is equal to the sum of the absolute volume (mass) of all the constituent components. If the total quantity of concrete is designated s, then $\sum s_i = s$. For concrete of four components, $1 \le i \le 4$ and so in keeping with the principle of absolute volume (mass), $s_1 + s_2 + s_3 + s_4 = s$ or $s_1/s + s_2/s + s_3/s + s_4/s = s/s$ where $z_i = s_i/s$ which is the fractional portion. This fractional portion or predictor is the ratio of the actual portions to the quantity of concrete. Some researchers have applied this method of optimisation and they came up with interesting results. Ogah used this method to study the shear modulus of rice husk ash concrete. The concrete components used are water, rice husk ash-45% slaked lime mix, river sand and crushed rock [3]. Mama and Osadebe [4] formulated models for prediction of compressive strength of sandcrete blocks using Scheffe's and Osadebe's optimization theories. The results of the predictions were comparatively analysed and it was found that the two models are acceptable. Onwuka et.al. [5] applied this method in prediction of concrete mix ratios for desired strength of concrete and vice versa. Okere et al.[6] generated a model for optimisation of modulus of rupture of concrete using this method. The details of this theory can be found in these references.

2. Methodology

The cement used for the analysis was eagle cement brand of Ordinary Portland Cement conforming to BS 12 [7]. The fine aggregate used was river sand free from deleterious matters such as dirts, clay and organic matter. The fine aggregate falls into zone 3 of the grading curve. The coarse aggregate was normal weight and irregular shaped with a maximum size of 20mm. Both the fine and coarse aggregate were hard and durable, and conform to the specifications of BS 882 [8].
Portable drinking water was used for the analysis.

Here, optimization method is used in formulating a mathematical model for predicting the cost per cubic metre of concrete. The model is based on the modified regression theory.

2.1 Formulation of cost optimisation model based on modified regression theory

The polynomial equation is given by Osadebe [2].

$$Y = \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 + \alpha_4 z_4 + \alpha_{12} z_1 z_2 + \alpha_{13} z_1 z_3 + \alpha_{14} z_1 z_4 + \alpha_{23} z_2 z_3 + \alpha_{24} z_2 z_4 + \alpha_{34} z_3 z_4 \quad (1)$$

In general, Eq.(1) is given as

$$Y = \sum \alpha_i z_i + \sum \alpha_{ij} z_i z_j \quad (2)$$

where $1 \le i \le j \le 4$

Eqs. (1) and (2) are the optimization model equations.
Y is the response function at any point of observation, z_i is the predictor (ratio of the actual portions to the quantity of concrete) and α_i is the coefficient of the optimization model equations.
Eq. (2) can be put in matrix form as

$$[Y^{(n)}] = [Z^{(n)}]\{\alpha\} \quad (3)$$

Rearranging Eq. (3) gives:

$$\{\alpha\} = [Z^{(n)}]^{-1}[Y^{(n)}] \quad (4)$$

An existing optimisation method proposed by Scheffe [9] was used in obtaining the actual portion or component which is the starting point for the experimental part of this modified regression theory. In order to satisfy the requirement that the sum of all the components of a mixture must produce a unit product, a component transformation was carried out using the following relationship:

$$[Z] = [A]\,[X] \quad (5)$$

where [Z] = matrix of actual component proportions
[X] = matrix of pseudo components proportions
[A] = matrix of coefficients
The pseudo and actual components of concrete were determined and presented in Table 1 [10].
The actual mix proportions, $\mathbf{s_i^{(n)}}$ and the corresponding fractional portions, $z_i^{(n)}$ are presented in Table 2.
The actual components S_i (from Table 2) were converted to actual mix ratios in kg per cubic metre of concrete as shown in Table 4.
The values of $Y^{(n)}$ matrix are determined from the cost analysis shown in Table 5 and presented in Table 6.
With the values of the matrices $Y^{(n)}$ and $Z^{(n)}$ known, it is easy to determine the values of the constant coefficients of Eq. (4). These values of the fractional portions $Z^{(n)}$ were used to develop $Z^{(n)}$ matrix presented in Table 3.

Table 1. Pseudo Components with their corresponding Actual Component Values.

N	X_1	X_2	X_3	X_4	Response	Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.549	1	2	4
2	0	1	0	0	Y_2	0.501	1	2.5	6
3	0	0	1	0	Y_3	0.45	1	3	5.5
4	0	0	0	1	Y_4	0.6	1	1.5	3.5
5	0.5	0.5	0	0	Y_{12}	0.525	1	2.25	5
6	0.5	0	0.5	0	Y_{13}	0.449	1	2.5	4.75
7	0.5	0	0	0.5	Y_{14}	0.575	1	1.75	3.75
8	0	0.5	0.5	0	Y_{23}	0.475	1	2.75	5.75
9	0	0.5	0	0.5	Y_{24}	0.551	1	2	4.75
10	0	0	0.5	0.5	Y_{34}	0.525	1	2.25	4.5
Control points									
11	0.5	0.25	0.25	0	C_1	0.5125	1	2.375	4.875
12	0.25	0.25	0.25	0.25	C_2	0.525	1	2.25	4.75
13	0	0.25	0.25	0.5	C_3	0.5375	1	2.125	4.625
14	0	0.25	0	0.75	C_4	0.575	1	1.75	4.125
15	0.75	0	0.25	0	C_5	0.525	1	2.25	4.375
16	0	0.5	0.25	0.25	C_6	0.5125	1	2.375	5.25
17	0.25	0	0.5	0.25	C_7	0.5125	1	2.375	4.625
18	0.75	0.25	0	0	C_8	0.5375	1	2.125	4.5
19	0	0.75	0.25	0	C_9	0.4875	1	2.625	5.875
20	0	0.4	0.4	0.2	C_{10}	0.5	1	2.5	5.3

Table 2. Values of actual mix proportions and the corresponding fractional portions

N	S1	S2	S3	S4	S	RESPONSE	Z1	Z2	Z3	Z4
1	0.549	1	2	4	7.549	Y_1	0.072725	0.132468	0.264936	0.529872
2	0.501	1	2.5	6	10.001	Y_2	0.050095	0.09999	0.249975	0.59994
3	0.45	1	3	5.5	9.95	Y_3	0.045226	0.100503	0.301508	0.552764
4	0.6	1	1.5	3.5	6.6	Y_4	0.090909	0.151515	0.227273	0.530303
5	0.525	1	2.25	5	8.775	Y_{12}	0.059829	0.11396	0.25641	0.569801
6	0.499	1	2.5	4.75	8.749	Y_{13}	0.057035	0.114299	0.285747	0.542919
7	0.575	1	1.75	3.75	7.075	Y_{14}	0.081272	0.141343	0.24735	0.530035
8	0.475	1	2.75	5.75	9.975	Y_{23}	0.047619	0.100251	0.275689	0.576441
9	0.551	1	2	4.75	8.301	Y_{24}	0.066378	0.120467	0.240935	0.57222
10	0.525	1	2.25	4.5	8.275	Y_{34}	0.063444	0.120846	0.271903	0.543807

Table 3. $Z^{(n)}$ matrix

Z1	Z2	Z3	**Z4**	Z1Z2	Z1Z3	Z1Z4	Z2Z3	Z2Z4	Z3Z4
0.072725	0.132468	0.264936	0.529872	0.009634	0.019267	0.038535	0.035095	0.070191	0.140382
0.050095	0.09999	0.249975	0.59994	0.005009	0.012522	0.030054	0.024995	0.059988	0.14997
0.045226	0.100503	0.301508	0.552764	0.004545	0.013636	0.024999	0.030302	0.055554	0.166662
0.090909	0.151515	0.227273	0.530303	0.013774	0.020661	0.048209	0.034435	0.080349	0.120523
0.059829	0.11396	0.25641	0.569801	0.006818	0.015341	0.034091	0.029221	0.064935	0.146103
0.057035	0.114299	0.285747	0.542919	0.006519	0.016298	0.030965	0.032661	0.062055	0.155137
0.081272	0.141343	0.24735	0.530035	0.011487	0.020103	0.043077	0.034961	0.074917	0.131104
0.047619	0.100251	0.275689	0.576441	0.004774	0.013128	0.02745	0.027638	0.057789	0.158919
0.066378	0.120467	0.240935	0.57222	0.007996	0.015993	0.037983	0.029025	0.068934	0.137868
0.063444	0.120846	0.271903	0.543807	0.007667	0.017251	0.034501	0.032858	0.065717	0.147863

Table 4. Mix ratios in kg per cubic metre of concrete

S/n	**Water**	**Cement**	**Fine aggregates**	**Coarse aggregates**
1	174.83	317.88	635.76	1271.52
2	120	240	600	1440
3	108.54	241.2	723.6	1326.6
4	218.2	363.6	545.4	1272.6
5	143.59	273.5	615.37	1367.5

S/n	Water	Cement	Fine aggregates	Coarse aggregates
6	137.14	274.28	685.7	1302.83
7	195.05	339.22	593.64	1272.08
8	114.28	240.6	661.65	1383.45
9	159.04	289.16	578.32	1373.51
10	152.27	290.03	625.57	1305.14
CONTROL POINTS				
11	140.37	273.89	650.49	1335.21
12	147.8	281.52	633.42	1337.22
13	155.65	289.59	615.38	1339.35
14	185.24	322.15	563.76	1328.87
15	154.6	294.48	622.58	1288.35
16	134.61	262.65	623.79	1375.9
17	144.49	281.94	669.61	1303.97
18	158.04	294.03	624.81	1323.14
19	117.15	240.3	630.79	1411.76
20	129.03	258.06	645.15	1367.72

Table 5. Current market prices of concrete components

Components of concrete	Unit cost (Naira per kg)
Cement	36
Fine Aggregate	0.833
Coarse Aggregate	6.25
Water	3.50

Table 6. Cost estimate (in Naira) per cubic metre of concrete of the different mix ratios

S/n	Water	Cement	Fine Aggregate	Coarse Aggregate	Response Symbol	Total (response)
1	611.91	11,443.68	529.80	7,947	Y_1	20,532.39
2	420	8,640	500	9,000	Y_2	18,560
3	379.89	8,683.2	603	8,291.25	Y_3	17,957.34
4	763.7	13,089.6	454.5	7,953.75	Y_4	22,261.55
5	502.56	9,846	512.81	8,546.88	Y_{12}	19,408.25
6	479.99	9,874.08	571.42	8,142.69	Y_{13}	19,068.18
7	682.68	12,211.92	494.7	7,950.5	Y_{14}	21,339.80
8	399.98	8,661.6	551.38	8,646.56	Y_{23}	18,259.52
9	556.64	10,409.76	481.93	8,584.44	Y_{24}	20,032.77
10	532.95	10,441.08	543.81	8,157.13	Y_{34}	19,674.97
CONTROL POINTS						
11	491.30	9,860.04	542.08	8,345.06	C_1	19,238.48
12	517.3	10,134.72	527.85	8,357.63	C_2	19,537.5
13	544.78	10,425.29	512.82	8,370.94	C_3	19,853.78
14	648.34	11,597.4	469.8	8,305.44	C_4	21,020.98
15	541.1	10,601.28	552.15	8,052.19	C_5	19,746.72
16	471.13	9,455.4	519.83	8,599.38	C_6	19,045.74
17	505.72	10,149.84	558.01	8,149.81	C_7	19,363.38
18	553.14	10,585.08	520.68	8,269.63	C_8	19,928.53
19	410.03	8650.8	525.66	8,823.50	C_9	18,409.99
20	451.61	9,290.16	537.63	8,548.25	C_{10}	18,827.65

2.2 Cost analysis

The current market prices of concrete constituent materials were obtained and presented in Table 5. They were used to determine the total cost of producing one cubic meter ($1m^3$) of concrete in naira for the different mix ratios and presented in Table 6. In case of price fluctuations over time, the current prices of concrete components can be obtained by the base prices with a price fluctuation factor (PFF) given in Table 7.

Table 7. Optimal values of PFF for years 1-10

Number of years	PFF
Year 1	1.2326
Year 2	1.4638
Year 4	1.9300
Year 6	2.3939
Year 8	3.3260
Year 10	3.3260

Source: [11]

3. Results and analysis

The result of the cost analysis for one cubic meter of concrete is presented in Table 6.

3.1 Determination of the optimisation model based on modified regression theory

Substituting the values of $Y^{(n)}$ from analytical results presented in Table 6 into Eq. (4) gives the following values of the coefficients of the model developed i.e. Eq. (1).

$\alpha_1 = 114714543.6$ $\alpha_2 = 64270735.28$
$\alpha_3 = 1191745.736$ $\alpha_4 = 15118.84211$
$\alpha_5 = -350499099.3$ $\alpha_6 = -92526209.43$
$\alpha_7 = -114967885$ $\alpha_8 = -82854529.25$
$\alpha_9 = -63989091.39$ $\alpha_{10} = -1216198.133$

Substituting the values of these coefficients into Eq.(4) yields:

$$Y = 114714543.6Z_1 + 64270735.28Z_2 + 1191745.736Z_3 + 15118.84211Z_4 - 350499099.3Z_1Z_2 - 92526209.43Z_1Z_3 - 114967885Z_1Z_4 - 82854529.25Z_2Z_3 - 63989091.39Z_2Z_4 - 1216198.133Z_3Z_4 \quad (6)$$

Eq. (6) is the modified mathematical model for cost per cubic meter of concrete

3.2 Test of the adequacy of the model

The model equation was tested for adequacy against the controlled results. The statistical hypothesis for this mathematical model is as follows:

Null Hypothesis (H_0): There is no significant difference between the analytical and the predicted results at an α-level of 0.5.

Alternative Hypothesis (H_1): There is a significant difference between the analytical and predicted results at an α-level of 0.05.

The fisher test statistic was used for this test [12]. The predicted values ($Y_{predicted}$) for the test control points were obtained by substituting the values of Z_1 from (Table 3) into the model equation i.e. Eq. (6). These values were compared with the analytical result ($Y_{observed}$) given in (Table 6).

3.3 Fisher Test

For this test, the parameter y, is evaluated using the following equation:

$$y = \frac{\sum Y}{n} \quad (7)$$

where Y is the response and n the number of responses.

Using variance,

$$S^2 = \left[\frac{1}{n-1}\right][\sum(Y-y)^2] \quad and \quad y = \frac{\sum Y}{n} \quad for\ 1 \le i \le n \quad (8)$$

The Fisher statistics test computations for the controlled points are presented on Table 8.

Table 8. F-Statistics for the controlled points

Response Symbol	$Y_{(observed)}$	$Y_{(predicted)}$	$Y_{(obs)}-y_{(obs)}$	$Y_{(pre)}-y_{(pre)}$	$Y_{(obs)}-y_{(obs)}^2$	$(Y_{(pre)}-y_{(pre)})^2$
C_1	19,238.48	19238.89	-258.795	-260.295	66974.852	67753.49
C_2	19,537.50	19537.79	40.225	38.605	1618.0506	1490.346
C_3	19,853.78	19853.72	356.505	354.535	127095.815	125695.1
C_4	21,020.98	21019.21	1523.705	1520.025	2321676.93	2310476
C_5	19,746.72	19747.46	249.445	248.275	62222.808	61640.48
C_6	19,045.74	19064.67	-451.535	-434.515	203883.856	188803.3
C_7	19,363.38	19363.84	-133.895	-135.345	17927.8710	18318.27
C_8	19,928.53	19928.3	431.255	429.115	185980.875	184139.7
C_9	18,409.99	18410.1	-1087.285	-1089.09	1182188.67	1186106
C_{10}	18,827.65	18827.87	-669.625	-671.315	448397.641	450663.8
Sum	194972.75	194991.9			4617967.37	4595087
Mean	$y_{(obs)}$= 19,497.27	$y_{(pre)}$= 19,499.19				

Therefore from (Table 8), $S^2_{(obs)} = \frac{4617967.37}{9} = 513{,}107.485$ and $S^2_{(pre)} = \frac{4595087}{9} = 510{,}565.22$

But the fisher test statistics is given by

$$F = \frac{S_1^2}{S_2^2} \qquad (9)$$

where S_1^2 is the larger variance

Hence $S_2^2 = 510{,}565.22$ and $S_1^2 = 513{,}107.485$

Therefore, $F = \frac{513{,}107.485}{510{,}565.22} = 1.00$

From standard Fisher Table, $F_{0.95}(9,9) = 3.18$, we accept the Null hypothesis. Hence the model is adequate.

3.4 Comparison of results

The results obtained from the model were compared with those obtained from the analysis as presented in Table 9.

Table 9: Comparison of Predicted Cost with Analytical Cost

S/N	Analytical Cost (in Naira)	Predicted Result (in Naira)	Percentage Difference
1	19,238.48	19,238.89	0.002
2	19537.50	19,537.79	0.001
3	19853.78	19,853.72	0.000
4	21,020.98	21,019.21	0.008
5	19,746.72	19,747.46	0.004
6	19,045.74	19,064.67	0.099
7	19,363.38	19,363.84	0.005
8	19,928.53	19,928.30	0.001
9	18.409.99	18,410.10	0.000
10	18,827.65	18,827.87	0.001

A comparison of the predicted results with the analytical results shows that the percentage difference ranges from a minimum of 0.001% to a maximum of 0.099%, which is insignificant.

4. Conclusion

1. The modified regression theory has been applied and used successfully to develop mathematical model for optimisation of cost per cubic meter of concrete.
2. The cost of concrete mix is a function of the proportions of the ingredients (cement, water, sand and coarse aggregate) of the concrete.
3. The fisher test used in the statistical hypothesis showed that the model developed is adequate.
4. Since the maximum percentage difference between the analytical result and the predicted result is insignificant (i.e. 0.099), the optimisation model will predict values of concrete mix cost if given the mix proportions and vice versa.

References

[1] Majid, K.I. *Optimum Design of Structures*, Butterworth & Co. Ltd, London, 1974.

[2] Osadebe, N.N. "Generalised mathematical modeling of compressive strength of normal concrete as a multi-variant function of the properties of its constituent components", A paper delivered at the college of Engineering, University of Nigeria, Nsukka, 2003

[3] Ogah, O.S. "A mathematical model for optimisation of strength of concrete: A case study for shear modulus of Rice Husk Ash Concrete", *Journal of Industrial Engineering International,* Vol. 5, Number 9, 2009, pp76-84.

[4] Mama, B.O. and Osadebe, N.N. "Comparative analysis of two mathematical models for prediction of compressive strength of sandcrete blocks using alluvial deposit", *Nigerian Journal of Technology*, Vol. 30, Number 3, 2011, pp 82-89.

[5] Onwuka, D.O., Okere, C.E., Arimanwa, J.I., Onwuka, S.U. "Prediction of concrete mix ratios using modified regression theory", *Computational Methods in Civil Engineering*, Vol. 2, Number 1, 2011, pp 95-107.

[6] Okere, C.E., Onwuka, D.O., Osadebe, N.N. "Mathematical model for optimisation of modulus of rupture of concrete using Osadebe's regression theory", *Research Inventy: International Journal of Engineering and Science,*Vol. 2, Issue 5, 2013,pp 01-12.

[7] British Standards Institution, BS 12: "Specification for Portland Cement", London, 1978.

[8] British Standards Institution, BS 882: "Specification for aggregates from natural sources for concrete," London, 1992

[9] Scheffe, H. "Experiments with Mixtures", *Royal Statistical Society Journal*, Vol. 20,Series B, 1958, pp 344-360.

[10] Okere, C.E. "Mathematical Models for Optimisation of Modulus of Rupture of Concrete" Thesis (M. Eng), Federal University of Technology Owerri, 2006.

[11] Nworuh, G.E. and Unaeze, G.O. "Optimisation of price fluctuation calculations", *Journal of Project Management Technology*, Volume 1, Number 1, 1997.

[12] Ogomaka, P.M.C. *Inferential Statistics for Research in Education and Social Science*, Peace wise Systems and Prints, Owerri, 2004.