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DETERMINATION OF REAERATION COEFFICIENT K_2 FOR POLLUTED STREAM AS A FUNCTION OF DEPTH, HYDRAULIC RADIUS, TEMPERATURE AND VELOCITY

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Abstract

A polluted stream was studied to determine the effect of geometric parameters like velocity, depth, hydraulic radius and temperature on water quality and its modeling. The data from sample stations include velocity of flow, average depth of flow, DO, self purification ratio (f) and reaeration constant (K_2). The reaeration coefficient ranged from 0.01d^{-1} to 0.19d^{-1}. Three new empirical models for K_{21}, K_{22} and K_{23} were derived from field data by multiple regression analysis. The coefficient of correlation between K_2 observed and K_{21} predicted; K_{22} derived from hydraulic radius and K_{23} predicted from model constructed with effect of temperature are 0.789, 0.808 and 0.914 respectively. The predictions indicated significant relationship with the observed values at 95% level of significance.

Keywords: stream/lake, modelling, reaeration coefficient, standard error, self purication factor

1. Introduction

Recently, water quality studies are considered very important because of the need to comply with guidelines and standards for environmental pollution. It is envisaged that if proper monitoring and enforcement of the guidelines for permissible water pollution limits are carried out on all rivers, stream and lakes, a great deal of compliance on the part of the individuals, communities and industry will be achieved.

The study of pollution in natural ecosystem has many aspects, physical, chemical and biological [1]. Pollutants affect marine organisms to different degree. Most physiological functions can be affected without lethal effects and sub lethal physiological responses[2],[3],[4] and [5]. It may reduce the fitness of an organism through such processes as reproduction impairment, suppressed immune responsiveness [6], diseases, energy deficits and untimely death.

In many studies of water quality, emphasis is placed on the source and path of the pollution spreads. This is enhanced through data collection from field survey. These data are analyzed statistically, tabulated and plotted on seasonal or monthly variation, which gives rise to estimates of water pollution level and forms the basis for water quality forecasting at selected locations beyond point of generation.

Oxygen transfer at the surface of the stream as the water flows gradually is an effective process for environmental quality of the aquatic ecosystems [7]. If dissolved oxygen (DO) drops below recommended limits, aquatic ecosystem health could be seriously impaired and desirable uses of resources could be precluded [8].

As the stream water flows downstream it is influenced by dilution and other factors such as physical (gravity, sunlight and aeration), chemical (oxidation, reduction, and coagulation); biological (bacteria attack, algae, protozoa, rolifera and aquatic plants); hydrological (precipitation, landforms, flora, topography); and climatic conditions (wind, temperature, humidity) and general weather conditions of receiving environment. These geometric factors affect self purification capacity of the lake/stream. The purification capacity of a lake/stream is directly proportional to discharge. Therefore factors that normally influence discharge such as wind, velocity, precipitation, landforms and vegetative cover, topography and temperature affect its assimilation capacity [9].

Changes in temperature can have a significant impact on DO depletion. The reactions governing BOD

Table 1: Values of re-oxygenation co-efficient f at 20°C [12].

Types of water body	**Value of f (day^{-1})**
Small pond	0.05 - 1.0
Sluggish streams/lakes	1.0 - 1.5
Large stream with low velocity	1.5 - 2.0
Large streams with moderate velocity	2.0 - 3.0
Swift Streams	3.0 - 5.0
Rapids	> 5.0

exertion typically have a temperature dependence of rate doubling for each 10°C increase in temperature. This means that there will be a significantly increased rate of BOD exertion when the temperature is the highest [10].

Temperature also affects the rate of organic waste assimilation. Increase in temperature means increase in rate of oxidation, decrease in oxygen saturation capacity and rate of diffusion. Increase temperature may destroy aquatic life [11].

1.1. *Reoxygenation coefficient (K_2)*

[12] developed standard tables on range of acceptable values for values of reo-oxygenation coefficient f at 20°C as follows:

The above values of re-oxygenation co-efficient f implies that the higher the flow velocity, the higher the re-oxygenation coefficient.

Determination of K_2

Many researches have been carried out on mechanism of reaeration as influenced by temperature, river geometry and hydrodynamics factors [13] and [11]. Generally, it is agreed that Streeter - Phelps equation is not a (true) representative of stream self purification model as it suffer a number of limitation. Apart from questions of stream geometry and flow regime, it ignores a number of mechanisms where BOD and DO concentrations are raised or lowered [14]. Some authors have included other parameters in the formulation of their models [15] and [14].

1.2. *Equations for determination of coefficient of reaeration* K_2

Many attempts have been made to relate empirically the reaeration rate constant to key stream parameters [16]. The most commonly used equation for determing K_2 is that of [17] which states that

$$K_2 = \frac{3.9V^{1/2}}{H^{3/2}} \tag{1}$$

Where K_2 = reaeration coefficient at 20°C (day^{-1}); V = average stream velocity (m/s); H = average stream depth (m).

[17] came up with the equation of reaeration constant for slight slope river to be

$$K_2 = \frac{10.046V^{2.696}}{H^{3.902}} \tag{2}$$

[18] also produced a model for the determination of reaeration constant of any medium slope river to be

$$K_2 = \frac{1.923V^{1.325}}{H^{2.006}} \tag{3}$$

[18] later improved the equation by considering temperature as a factor which influences reaeration constant as follows.

$$K_2 = \frac{5.06V^{0.919}}{H^{1.673}}(1.024)^{T-20} \tag{4}$$

[19] showed that

$$K_2 = \frac{3.93V^{0.5}}{H^{1/5}} \tag{5}$$

[20] analyzed the polluted status of Amadi creek, considering the effect of hydraulic radius in place of the depth of the river at different location, and obtained

$$K_2 = \frac{11.635V^{1.0954}}{R^{0.016}} \tag{6}$$

Where R is hydraulic radius.

[16] confirmed in his work that reaeration rate constants (K_2) depend on the condition of the river. A fast moving shallow stream will have a higher reaeration rate constant than a sluggish stream of stagnant pond or lake. Also the rate of reaeration affects it purification capacity. Rate of reaeration may be enhanced by physical and hydrologic characteristics of the lake, depth of water temperature and rate of flow of lake during rainy season. Purification or assimilation ratio (f) can be computed by the following equation:

$$F = K_2/K_1$$

Where K_2 = is the reaeration constant; K_1 = is the BOD rate constant.

1.3. *Significance of water quality modelling*

[21] stated that empirical relationships between pollutant concentration and stream flow or other hydrologic variables in water quality modeling can be used to describe water quality loading mechanisms that cannot be obtained on a more theoretical basis. It is often stressed that a model is as good as the data and theory on which it is based. The result therefore, of study will help to confirm the influence of stream geometric parameters on stream pollution which will in-turn form basis of any recommendations and will further restrict waste discharge within the lake self purification capacity and prevent the occurrence of

pollution [11]. The required parameters for determination of water quality in this study include temperature, velocity of flow at sample station, depth and hydraulic radius of stream.

To predict the DO levels in streams, some mathematical model have been proposed. The rate of BOD degradation K1 is expressed as

$$\ln\frac{L}{L_A} = -K_1 t \tag{7}$$

$$K_1 = \frac{1}{t}\ln\left(\frac{L_A}{L}\right) \tag{8}$$

Where L_A and L are the BOD at time zero and t respectively.

To get L_A (ultimate) = 1.46BOD$_5$ [14] at temp 20°C. Therefore to get vary field or stream temperature, at stream conditions.

$$L_{A(T)} = L_{A(20)}[1 + 0.002(T - 20)] \tag{9}$$

Where $L_{A(T)}$ and $L_{A(20)}$ are the ultimate BOD at T°C and 20°C. The combination of the effects of temperature changes can cause considerable changes on the total load of the receiving water.

1.4. Study area

The area under study is the Oshika stream/lake. The stream is close to Oshika village located between Ahoada and Mbiama in Ahoada West Local Government Area, at a distance of 75km from Port Harcourt along East-West Road. The geographic location is a tropical rain forest with Niger flood plains and seasonal swamp forest. It is a fresh water swamp with farm land on dryland and patches of bush fallow with disperse oil palm, oranges, pear and coconut trees. The soil around the area is loam or clay loam and is mixed with clay and silts in some areas. During the rainy seasons the Lake flows and empties into a foodplain about 8km from Oshika village. During the dry periods of the year, when there is no surface water contribution, the water lies relatively quiescent in the lake. In the dry season, the lake is stagnant and dry land containing isolated bodies of water mainly in evacuated ponds traditionally used for dry season fish harvest [22].

2. Methodology

2.1. Measurement of time and velocity

Twelve sampling stations were established, along Oshika stream/lake from Oshika downstream. The study distance is 2,400metres and samples will be collected on monthly basis. The analysis covered the following parameter: DO, velocity, temperature, depth and hydraulic radius. The time at which readings were taken at every sample station was recorded using an electronic clock. The time between two readings was determined from the difference in time between the two readings. Velocity of the lake/Stream at each location was determined using a current meter.

2.2. Measurement of temperature

A laboratory thermometer was tied to a rope and lowered into the lake/stream such that half of the stem thermometer was submerged. The temperature of the lake/stream was read off the stem of the thermometer. Temperature readings were taken at every station simultaneously with the other readings.

2.3. Determination of distance, depth and width

A scaled map of study area was obtained from the Rivers State land and Survey Bureau. The position of Oshika lake/Stream was located using GPS indicating landmarks and measurement. For the purpose of this research, samples were collected at varying interval downstream. The depths were measured by dropping a loaded tape to the bottom of the lake and width were measured using a tape across the lake/stream.

2.4. Determination of dissolved oxygen

The following procedures were followed for dissolved oxygen test by Winkler method using the digital titrator. First, samples were collected into BOD bottle along the lake/stream. Sample was allowed to overflow the bottle for 2-3 minutes to ensure air bubble were not trapped. Immediately trapper was inserted so that air was not trapped in the bottle and inverted several times to mix. A floc precipitate was formed which settled out after five minutes. Then the stopper was removed and the contents of one dissolved oxygen reagent powder pillow was added. The stopper was replaced without trapping air in the bottle and inverted several times to mix. The floc dissolved, leaving a yellow color if oxygen is present. 20ml of prepared sample was measured accurately and transfered to 250ml Erlenmeyer flask. Attached was a clean straight stem delivery tube to a 0.2 sodium Thio-sulphate titration cartridge. It was twisted into the titrator body. The delivery tube was flushed and the counter was reset to zero and the tip wiped. The prepared solution was titrated with 0.2 sodium Thiosuphate until the samples changed from yellow to colorless. The number of digits was recorded. Calculate digits required x 0.1 = mg/l dissolved oxygen.

2.5. Methods of obtaining field values of reaeration coefficients (K_2)

The reaeration coefficient from field values were obtained using the equation

$$K_2 = \frac{\ln(DO_{initial}/D_{deficit})}{time(days)} \tag{10}$$

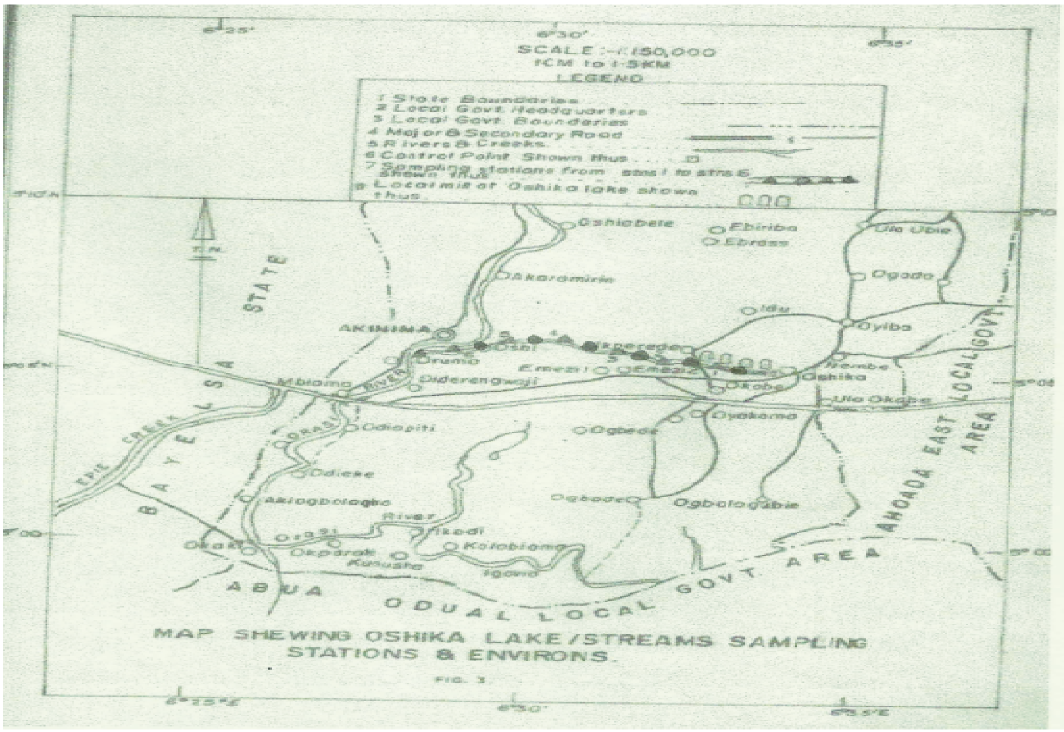

Figure 1: Map showing Oshika Lake/Streams sampling stations.

Where K_2 is the reaeration rate constant; $DO_{initial}$ is the initial dissolved oxygen; $D_{deficit}$ is the defference between saturation dissolved oxygen and the observed dissolved oxygen.

2.6. Description of steps for obtaining model K_{21}, K_{22} and K_{23}

The equations 11, 15 and 19 were constructed and derived by multiple regression analysis based on measured data and its original equations are stated as follows:

$$K_{21} = \frac{aV_{a_1}}{H^{a_3}} \tag{11}$$

Where V = velocity (m/s);H = depth (m) and a, a_1 and a_3 are constants.

$$K_{22} = \frac{aV^{a_1}}{R^{a_3}} \tag{12}$$

Where V = velocity (m/s); R = Hydraulic radius; a, a_1 and a_3 are constants

$$K_{23} = \frac{aV^{a_1} b^{-(T-20)}}{R^{a_4}} \tag{13}$$

Where V = velocity (m/s); T = Temperature (°C); R = Hydraulic radius; a, a_1, b and a_4 are constants.

3. Results and Discussions

3.1. Prediction of K_2 from stream data

The resulting equations for models K_{21}, K_{22} and K_{23} obtained by solving the regression equations are as follows:

$$K_{21} = \frac{5.77 \times 10^5 V^{1.6899}}{H^{4.2733}} \tag{14}$$

Where K_{21} indicates the relationship between velocity and depth to reaeration coefficient of stream.

$$K_{22} = \frac{56974V^{1.463228}}{R^{4.26585}} \tag{15}$$

Where K_{22} indicates the relationship between velocity aand hydraulic radius to stream reaeration coefficient.

$$K_{23} = \frac{2.213E6 \times V^{1.216} 1.8355^{-(T-20)}}{R^{4.847}} \tag{16}$$

Where K_{23} indicates the relationship between velocity, temperature and hydraulic radius to reaeration coefficient of stream.

The coefficient of correlations of observed and predicted K_2 for the various models were found to be 0.789, 0.808 and 0.914 respectively and with standard errors of 4.2%, 4% and 3.6% respectively. This indicated that the models K_{23} represents the stream better than K_{21} and K_{22}. Also, the different between K_2 observed and K_{23} may be caused by some natural conditions of the stream.

The study indicated that reaeration coefficient, K_2 of Oshika stream/lake varied from 0.01 day^{-1} to 0.19 day^{-1}. The purification factor of Oshika stream/lake is 0.44. The purification factor indicated that the lake/stream is polluted.

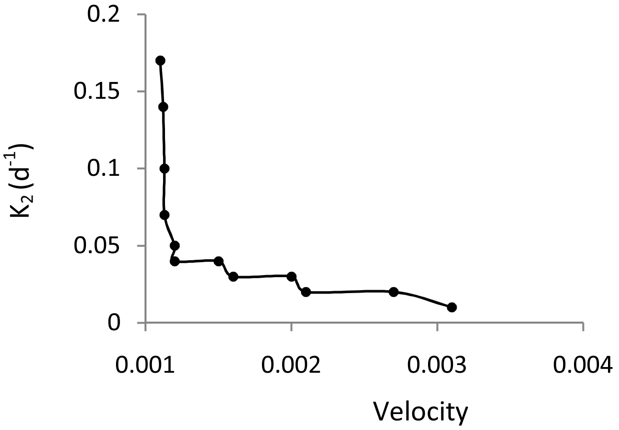

Figure 2: Variation of K_2 with velocity for wet season.

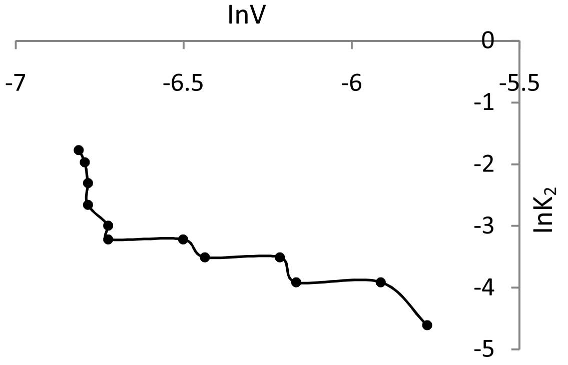

Figure 3: Variation of $\ln K_2$ with $\ln V$ for for wet season.

3.2. *Variation of K_2 with velocity*

Figure 2 and 3 indicate variation of natural logarithm on K_2 ($\ln K_2$) with natural logarithm of velocity ($\ln V$). K_2 seems to be independent of velocity in the dry season because the stream is very slow flowing, but picks up speed in the wet season because of increase in rainfall. The coefficient of correlation between $\ln K_2$ and ($\ln V$ is 0.534. This shows that K_2 increases with increase in velocity.

3.3. *Variation of K_2 with hydraulic radius*

Figures 4 and 5 indicates the variation of K_2 with hydraulic radius. The coefficient of regression between K_2 and R is 0.71. This high correlation coefficient shows that the lower the K_2 of stream, the higher the hydraulic radius.

3.4. *Variation of velocity with hydraulic radius*

Figures 6 shows the variation of velocity with hydraulic radius. The coefficient of regression between the natural logarithm of velocity and the natural logarithm of hydraulic radius is 0.881. The high correlation coefficient shows that the lower the speed of stream flow the higher the hydraulic radius.

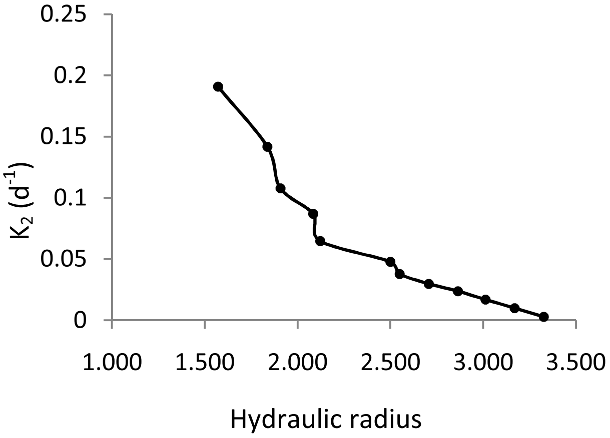

Figure 4: Variation of K_2 with the hydraulic radius in dry season.

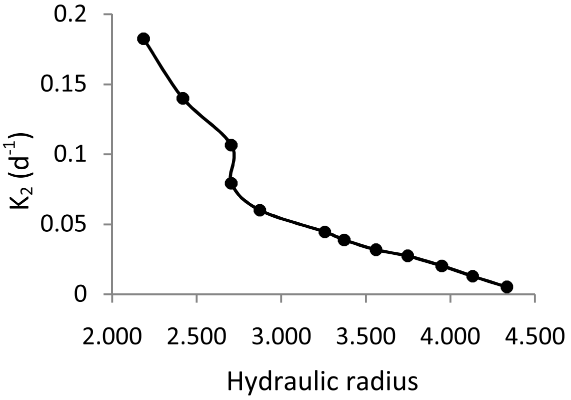

Figure 5: Variation of K_2 with the hydraulic radius in the wet season.

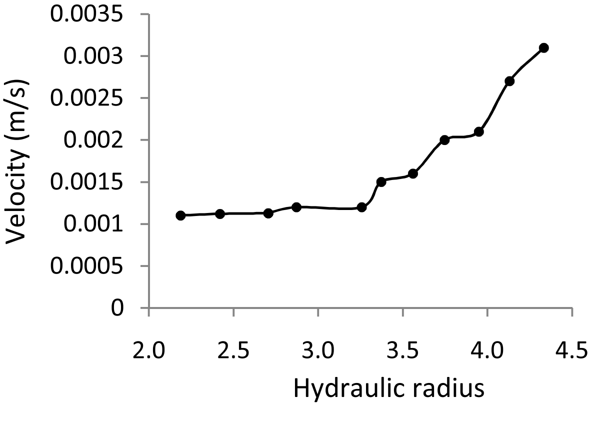

Figure 6: Variation of velocity with hydraulic radius in the wet season with K_2 observed.

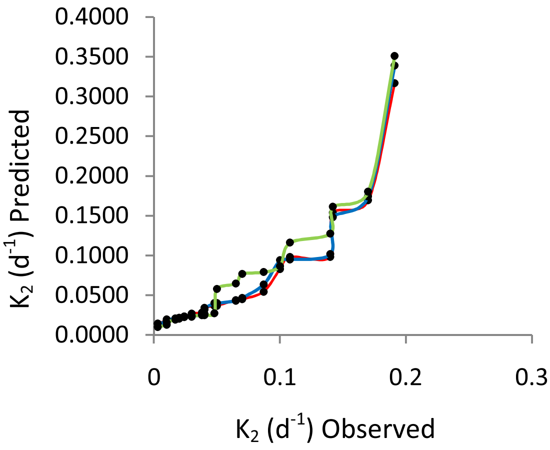

Figure 7: Variation of K_2 predicted from various models.

Table 2: Derived models, their correlation coefficients (CC) and standard errors (SE).

S/No	Dependent variable, Y	Non-linear model ($Y = \frac{aV_{a_1}}{H^{a_3}}$), ($Y = \frac{aV^{a_1}}{R^{a_3}}$), ($Y = \frac{aV^{a_1}b^{-(T-20)}}{R^{a_4}}$).
	K_{21}	$Y = \frac{5.77\times10^5 V^{1.6899}}{H^{4.2733}}$; CC = 0.786; SE = 4.2%
	K_{22}	$Y = \frac{56974V^{1.463228}}{R^{4.26585}}$; CC = 0.808; SE = 4.0%
	K_{23}	$Y = \frac{2.213E6\times V^{1.216}1.8355^{-(T-20)}}{R^{4.847}}$; CC = 0.914; SE = 3.6%

3.5. Variation of K_2 observed with K_2 predicted

The correlation coefficient of predicted K_{21}, K_{22}, and K_{23} from measured data is 0.789, 0.808, and 0.914. Figure 7 shows that they have positive relationship except for some natural conditions that may have contributed to a minor difference.

The hydraulic radius is given by $R = \frac{A}{P}$, where A is area and P is wetted perimeter.

4. Conclusion and Recommendation

The reaeration coefficient, K_2 of Oshika stream/lake varied from 0.01 to 0.19. The correlation coefficients of K_{21}, K_{22} and K_{23} are 0.789, 0.808 and 0.914 respectively. Asides from other factors, the reaeration rate values are influenced by the geometric parameters as seen in the determined correlation coefficients in this study. Increase in velocity of stream water results in increase value of K_2 as high velocity means higher assimilatry capacity of stream. Also, K_2 increases with increase in temperature. The influence of depth, hydraulic radius, temperature and velocity is however necessary in explaining the discrepancies between observed and estimated values. The results obtained can be used as a baseline for future studies. K_{23} would be recommended because it incorporates the crucial parameters that describes the stream.

Table 3: Correlation Coefficients.

	$\ln V$	$\ln H$	$\ln R$	K_{21}	K_{22}	K_{23}
$\ln K_2$	-0.35	-0.73	-0.76	-	-	
$\ln V$	-	0.76	0.881	-	-	
K_2 Observed	-	-	-	0.789	0.808	0.914

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