



# MODELLING FLOW OVER STEPPED SPILLWAY WITH VARYING CHUTE GEOMETRY

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## Abstract

*This study has modeled some characteristics of the flows over stepped spillway with varying chute geometry through a laboratory investigation. Using six physically built stepped spillway models, with each having six horizontal plain steps at 4cm constant height, 30 cm width and respective chute slope angles at 31°, 32°, 33°, 34°, 35° or 36° placed respectively within a tilting flume to obtain some varying flow data in 36 different experiments. These obtained flow data were used as inputs in a simple regression analysis to derive the parameters for each of the three mathematical relationships developed through dimensional analysis to respectively predict the flow rate, the sequent depth ratio and the total energy dissipated in the non-aerated flow region over each of the stepped spillway models. The percentage difference between the values predicted by each of these modeled equations and their respective actual values ranges between 0.10% and 1.28%.*

**Keywords:** stepped Spillway, chute Slope, energy dissipated, hydraulic models

## 1. Introduction

Stepped spillways (cascades) are commonly used for river training, debris dam structures, storm water systems and aeration cascades [1]. Stepped cascade flows are characterized by the strong kinetic energy of flow that requires clear understanding of its nature and characteristics. In recent times, practicing engineers have resorted to the use of mathematical modeling in order to obtain the details of such complex hydraulic problems. In developing nations such as Nigeria, where there is scanty information on hydraulic parameters required for designing stepped spillways, the resolve to development of mathematical models for providing these data becomes very obvious and important.

It is with this view in mind that this study plan to use some measured flow parameters obtained through the laboratory model investigations to establish some mathematical relationships for predicting some hydraulic parameters for flow over stepped spillways. Specifically, these derived equations will be used to determine the sequent depth ratio, hydraulic jump length and the total energy dissipated over the non-aerated region of the stepped spillway with varying chute geometry model and applied in prototype development study.

## 2. Hydraulic Modeling of Stepped Spillway

A basic dimensional analysis of the flow over the chute of the stepped spillway, assuming that the dominant feature is the momentum exchange between the free stream and the cavity flow within the steps of the spillway [2,3,4], is as presented in equation (1):

$$f_1(H_o, H_n, U_m, h_w, L_i, L_s, K_s, tg(\theta), \mu_w, \rho_w, g) = 0 \quad (1)$$

where  $H_o$  in (m) is the maximum energy with respect to the model base,  $H_n$ (m) is the head loss at the onset of the uniform flow,  $U_m$  (m/s) is the mean velocity,  $h_w$ (m) is the equivalent clear water depth,  $L_i$  (m) is the non-aerated water length,  $L_s$ (m) is the cavity length,  $K_s$  (m) is the step roughness height,  $tg(\theta)$  (radian) is the slope of the model,  $\mu_w$  (kg/s.m) is the dynamic viscosity of water,  $\rho_w$ (kg/m<sup>3</sup>) is the density of water and  $g$ (m/s<sup>2</sup>) is the acceleration due to gravity [3,5]. The configuration of a stepped spillway is as shown in Figure 1.

The dimensional analysis concept can be used to re-arranged equation (1) into Equation (2) as

$$\frac{H_n}{H_o} = \phi \left( R_e, Fr, \frac{K_s}{h_w}, \frac{L_i}{L_s}, tg(\theta) \right) \quad (2)$$

where,  $\frac{H_n}{H_o}$  is the rate of energy dissipation in percentage, for the non-aerated region.  $H_o$  is as defined in

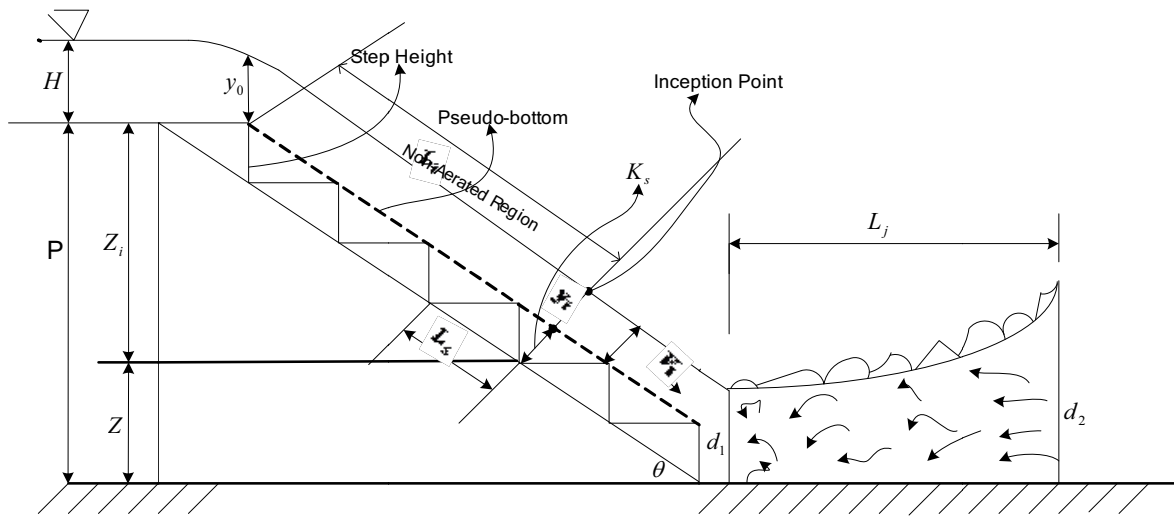


Figure 1: Schematic representation of a model stepped spillway configurations.



Figure 2: Pictorial view of the tilting flume used for the study.



Figure 3: **Left:** Tilting flume showing approach flow to the model. **Right:** Flow over the stepped spillway laboratory model

Table 1: Geometry of the developed stepped spillway models.

Model	$P$ (cm)	$S_h$ (cm)	$S_L$ (cm)	$L_S$ (cm)	$\theta$	$K_S$ (cm)	$L$ (cm)
A	24	4	6.67	7.77	31	3.43	46.60
B	24	4	6.40	7.55	32	3.39	45.29
C	24	4	6.16	7.34	33	3.35	44.07
D	24	4	5.94	7.15	34	3.32	42.92
E	24	4	5.71	6.97	35	3.28	41.84
F	24	4	5.51	6.81	36	3.24	40.83

equation (3):

$$H_o = P + h_o \quad (3)$$

$P$  is the height of spillway model,  $h_o = y_o$  the total head at spillway models' crest,  $y_o$  is the head at the crest of model;  $H_n$  and  $H_1$  are respectively defined in Equations (4) and (5).

$$H_n = H_o + H_1 \quad (4)$$

and;

$$H_1 = Z + d_1 \cos \theta + V_1^2/2g; \quad (5)$$

where  $H_1$  is the total head before the onset of aeration region;  $Z = P - Z_i$  is the vertical distance measured from the base of a model to the inception point,  $Z_i = L_i \sin \theta$  is the vertical distances measured from the crest of models to the inception point;  $d_i$  is the measured water depth, assuming a uniform flow regime over the aeration region,  $V_1^2/2g$  is the velocity head at that point;  $\phi$  is a function which is to be determined experimentally [6,7]. According to [3,7], Reynolds number,  $Re$  and Froude number,  $Fr$  are respectively defined in Equations (6) and (7).

$$Re = \frac{U_m R}{\nu} \quad (6)$$

$$Fr = \frac{U_m}{\sqrt{gh_w}} \quad (7)$$

where  $U_m$  is the mean velocity of flow,  $R$  is the hydraulic radius,  $\nu$  is the kinematic viscosity, as  $1.13 \times 10^{-6} \text{m}^2/\text{s}$  given at  $15.6^\circ\text{C}$ , the clear water depth,  $h_w$  is as defined in Equation (8)

$$h_w = (1 - C_{mean})h_{90} \quad (8)$$

Where  $V$  is depth where the local air concentration is 90 percent and can be determined according to [3] by the relationship in Equation (9)

$$h_{90} = \frac{1}{2} S_h \times F_s^{(0.1 \tan \theta + 0.5)} \quad (9)$$

where  $F_s$  is the step Froude number of flow given in Equation (10)

$$F_s = q_w / \sqrt{g \sin \theta \times S_h^3} \quad (10)$$

where  $q_w$  is the water discharge per unit width and  $S_h$  is the model step height.

$C_{mean}$  in Equation (8) is the mean air concentration and the empirical relation proposed by [8] for mean air concentration in fully developed skimming flow over stepped spillways is given in equation (11)

$$C_{mean} = 0.23 + 0.017 \left( \frac{L - L_i}{y_i} \right)^{0.46} \quad (11)$$

where  $L$  is the length of the chute;  $y_i$  is the depth of flow at the point of inception as defined in Equation (12) and  $L_i$  the cavity length, according to [3,6,12] is defined in Equation (13);

$$y_i = 0.4034 K_s / (\sin \theta)^{0.04} F_*^{0.592} \quad (12)$$

$$L_i = 9.719 K_s / (\sin \theta)^{0.0796} F_*^{0.713} \quad (13)$$

and  $K_s$  is the step roughness height,  $F_*$  is the Froude number at the point of inception,  $\frac{L_i}{L_S}$  is the dimensionless length of inception with respect to the cavity length; and  $\frac{K_s}{h_w}$  is the dimensionless step roughness height with respect to the clear water depth.

### 2.1. Percentage of energy dissipated over the steps for aerated region

As the flow over the aerated region is assumed to be uniform (i.e. the flow that goes into the horizontal channel to form the hydraulic jump at the toe of the spillway); then the head loss over the aerated region ( $H_a$ ) is given as  $H_a = H_1 - H_2$  where,  $H_1$  is the total energy over the steps at the point of inception,  $H_1 = Z + d_1 \cos \theta + V_1^2/2g$  where  $d_1$  and  $V_1$  are the measured water depth and the average velocity just beyond the inception point,  $Z$  and  $g$  were defined previously;  $H_2$  is the total energy at the toe of model,  $H_2 = d_1 + V_1^2/2g$  where,  $V_1 = q/d_1$ ;  $q$  is the discharge per unit width [7,9].

Based on this, the energy dissipation over the steps for the aerated region is given in Equation (14) as:

$$\frac{H_a}{H_1} = \frac{H_1 - H_2}{H_1} \quad (14)$$

### 2.2. Percentage of total energy dissipated over the steps

The percentage of total energy dissipated over the steps is given in equation (15) as:

$$H_T = \frac{H_n}{H_0} + \frac{H_a}{H_1} \quad (15)$$

where  $H_n$  [m] is the energy loss within the non-aerated region of the stepped chute and  $H_T$  is the total head loss over the entire chute length

Table 2: Flow characteristics for the different stepped spillway models.

Upstream Flow Characteristics			Downstream Flow Characteristics for Different Models											
			Model A		Model B		Model C		Model D		Model E		Model F	
$Q \times (10^{-3})$ (m <sup>3</sup> /s)	$H$ (cm)	$y_o$ (cm)	$d_1$ (cm)	$d_2$ (cm)	$d_1$ (cm)	$d_2$ (cm)	$d_1$ (cm)	$d_2$ (cm)	$d_1$ (cm)	$d_2$ (cm)	$d_1$ (cm)	$d_2$ (cm)	$d_1$ (cm)	$d_2$ (cm)
6.133	5.31	3.26	1.78	6.15	1.88	6.26	1.83	6.37	1.78	6.49	1.73	6.61	1.68	6.73
5.026	4.65	2.95	1.75	5.31	1.69	5.43	1.64	5.54	1.59	5.66	1.54	5.77	1.49	5.90
4.469	4.30	2.72	1.64	4.90	1.59	5.00	1.53	5.13	1.49	5.22	1.44	5.33	1.39	5.45
3.845	3.89	2.48	1.51	4.41	1.46	4.52	1.41	4.62	1.37	4.71	1.32	4.82	1.27	4.94
3.282	3.50	2.27	1.43	3.88	1.35	4.03	1.30	4.13	1.25	4.24	1.21	4.33	1.16	4.44
2.488	3.21	2.03	1.32	3.57	1.27	3.67	1.22	3.76	1.17	3.87	1.12	3.98	1.07	4.09

Table 3: Optimum energy characteristics over different portions of stepped spillway models.

Model	Flow	$H_0$ (cm)	$H_1$ (cm)	$H_1$ (cm)	$H_n$ (cm)	$H_a$ (cm)	$H_n/H_0$ (%)	$H_a/H_1$ (%)	$H_T$ (%)
A	Max	27.30	10.30	7.65	16.90	2.69	62.10	26.00	88.10
	Min.	26.00	15.60	4.02	10.50	11.50	40.20	74.20	114.40
B	Max	27.30	10.10	7.91	17.10	2.23	62.80	22.00	84.80
	Min.	26.00	15.50	04.19	10.60	11.30	40.60	72.90	114.00
C	Max	27.30	9.96	8.19	17.30	1.77	63.50	17.80	81.20
	Min.	26.00	15.40	4.38	10.70	11.00	40.90	71.50	112.00
D	Max	27.30	9.82	8.50	17.40	1.32	64.00	13.50	77.40
	Min.	26.00	5.30	4.61	10.70	10.69	41.00	70.00	111.00
E	Max	27.30	9.73	8.85	17.50	1.77	64.30	9.03	73.40
	Min.	26.00	15.40	4.87	10.70	11.00	41.10	68.30	109.00
F	Max	27.30	9.67	9.23	17.60	0.45	64.50	4.59	69.10
	Min	26.00	15.44	5.18	10.60	10.20	40.80	66.40	107.00

**2.3. Hydraulic jump at the toe of the stepped spillway model**

The sequent depth  $d_2$  of a hydraulic jump in a rectangular channel is related to the initial depth,  $d_1$  the discharge per unit width  $q$ , gravity acceleration  $g$ , and water density  $\rho$ . By dimensional analysis,  $f_1(d_2, d_1, q, \rho, g) = 0$  and in dimensionless term it can be re-written as in Equation (16)

$$d_2 = d_1\phi(F_1) \tag{16}$$

where  $F_1$  is the entry Froude number, and  $\phi$  is a function which is to be determined experimentally. Equation (16) is the expression for sequent depth of the jump formed at the toe of a model. The chart prepared by [10] is used in order to calculate the length of the hydraulic jump because, according to [11], the chart was desirable for the fact that it shows regularity or a fairly flat portion for the range of well-established jumps;

**3. Experimental Set up in the Hydraulics Laboratory in Ahmadu Bello University, Zaria**

The tilting flume shown in plate 1 was used as the model channel for the laboratory study. It was dimensioned 6m long, 30cm wide and 30cm deep and had

rails on top of its glass-sided walls on which a pointer-gauge trolley was moved to and fro for the measurement of its water depths. It also had a tailgate located at its outlet end to regulate the downstream water depth to the desired depth so that hydraulic jump can occur at downstream toe end of the inserted models. The flume has a continuous flow pumping mechanism, a control valve for regulating the pumped water flow from its storage tank into the rectangular channel of the flume, a stilling storage provisions at the downstream end of the flume channel and flow measuring facilities to measure the regulated flow through its channel at any time. Figures 1, 2 and 3 gives some descriptive views of some of the model fixtures, experimental setup and runs during the laboratory study.

**4. Description of the Developed Stepped Spillway Model**

Six models of a stepped chute were built at the Hydraulics Laboratory of the Department of Water Resources and Environmental Engineering, Ahmadu Bello University, Zaria. The models were fabricated from plywood. The crest of the models were broad crested [12]; the chute step height was taken after that of [7] and the models had the same step height with smooth horizontal face totaling 24cm (six steps); considering the workable depth of the laboratory flume.

Table 4: Flow characteristics for optimum energy dissipated over each model.

Model	$q * 10^2$ (m <sup>2</sup> /s)	$Fr_1$	$d_2$ (cm)	$L_j/d_2$	$L_j$ (cm)	$H_T$ (%)
A	2.044	2.43	6.15	4.82	29.64	88.10
B	2.044	2.53	6.26	4.91	30.73	84.80
C	2.044	2.64	6.37	5.02	31.98	81.20
D	2.044	2.75	6.49	5.13	33.29	77.40
E	2.044	2.87	6.61	5.24	34.64	73.40
F	2.044	3.00	6.73	5.32	35.80	69.10

Table 5: Summary of derived model parameters.

Equation No	Parameters	Coefficients	Regression Coefficient ( $R^2$ )	Error (%)
17	$K_2$	25.29	0.998	0.0025031
	$X_1$	0.054		
	$X_2$	- 0.130		
	$X_3$	0.448		
	$X_4$	0.448		
18	$X_5$	-0.084.	0.999	0.0019209
	$K_3$	1.28		
19	$X_6$	1.042	0.970	0.0072452
	$K_1$	0.5012		
	$X_0$	1.5		

Table 1 presents the geometric conditions defined for the developed stepped spillway models. In table 1,  $P$  is the model height;  $S_h$  is the step height;  $S_L = S_h / \tan \theta$  is the step length;  $L_S = S_h / \sin \theta$  is the length between the steps edges,  $\theta$  is the angle of model chute with the horizontal,  $K_S = S_h \cos \theta$  is the step roughness height,  $L = P / \sin \theta$  is the length of model chute.

**5. Results**

The discharge through the model channel of the tilting flume was measured and used to determine the coefficient of discharge (Cd) for the developed spillway models. Basically, the general form of the discharge equation for spillways is given as  $Q = C_d L^2 g H^{3/2}$ ; which can be in the form  $Q = KH^n$  where,  $Q$  is the actual discharge;  $H$  is the measured head;  $K$  and  $n$  are parameters to be determined by regression analysis. Comparing these two expressions, gives  $C_d = \frac{K}{L\sqrt{2g}}$  and the average  $C_d$  value obtained is equal to 0.38 using the average measured discharge data.

The characteristics of the flows over each of the stepped spillway models at the upstream and downstream end of the each model used in this study are presented in Tables 2. The values of the initial depths and sequent depths of the observed hydraulic jumps for each model were measured. The sequent depth was measured after the adjustment of the tail gate situated at the end of the flume, to create a hydraulic jump just downstream of the toe of model. The sequent depth

was measured at a point 70.15cm downstream of models' crest.

Using various energy levels measured within the aerated and non-aerated regions of the stepped spillways for each models, the values for  $H_0, H_1, H_2, H_a, \frac{H_n}{H_0}, \frac{H_a}{H_1}$ , and  $H_T$  as defined in equations (2), (3), (4), (5), (14) and (15) were determined. Their respective values for the maximum and minimum total energy dissipated over the steps ( $H_T$ ) are presented in Table 3. For each model, the Froude's number ( $Fr_1$ ), length of hydraulic jump ( $L_j$ ) and the sequent depth ( $d_2$ ) for the maximum total energy dissipated over the steps ( $H_T$ ) are presented in Table 4.

**5.1. Developed mathematical relationships for predicting flow conditions over the stepped spillway**

Equations (17), and (18) respectively derived from equations (2) and (16) are used for predicting the sequent depth ratio and the total energy dissipated (TED) over any stepped spillway.

Equation (17) is an expression which can be used to calculate the percentage of energy dissipation over the non-aerated region of the steps of any of the stepped spillway models. Equation (18) is an expression for determining the sequent depth of the jump at the toe of any of the models.

$$\frac{H_n}{H_0} = K_2 (Re)^{X_1} (Fr)^{X_2} \left(\frac{K_s}{h_w}\right)^{X_3} \left(\frac{L_i}{L_s}\right)^{X_4} tg(\theta)^{X_5} \tag{17}$$

$$d_2 = K_3 d_1 (Fr_1)^{X_6} \tag{18}$$

Equation (19) is the discharge-head relationship used to calculate the flow rate over the spillway models in this experiment.

$$Q = K_1 H^{X_0} \tag{19}$$

Using the values for various flow parameters measured experimentally in the laboratory, the regression analysis approach was used to develop and obtained parameters for the three mathematical relationships expressed in equations 17, 18 and 19. These developed mathematical models were evaluated to ascertain the level of their reliability (performance criteria) for hydraulic modeling of stepped spillway under similar flow conditions.

The parameters values for each of the developed relationships are shown in Table 5. In addition, the values of the dependent parameters predicted in equations (17) and (18) were compared with their respective experimental values calculated directly from their respective values obtained from experiments. The results of the comparative analysis are respectively shown in Tables 6 and 7.

$K_1, K_2$  and  $K_3$  are the constants;  $X_1, X_2, X_3, X_4, X_5$  are the second, third, fourth, fifth and the sixth terms in equation (17) right hand side;  $R^2$  is an indicator of reliability or performance level of each of these relationships.

Table 6: Comparison of the range of values obtained for Energy Dissipation in the Non-aerated flow region with those predicted from eq. (17).

Model	Actual $H_n/H_0$	Theoretical $H_n/H_0$	Percentage Difference (%)
A	40.20 - 62.10	40.13 - 62.04	0.17 - 0.10
B	40.60 - 62.80	40.52 - 62.87	0.20 - 0.11
C	40.90 - 63.50	40.78 - 63.40	0.29 - 0.16
D	41.00 - 64.00	40.87 - 63.82	0.32 - 0.28
E	41.15 - 64.30	40.90 - 63.89	0.61 - 0.64
F	41.34 - 64.50	41.00 - 64.38	0.82 - 0.19

Table 7: Comparison of measured Sequent Depth downstream of hydraulic jump with those predicted from eq. (18).

Model	Actual $d_2$	Theoretical $d_2$	Percentage Difference (%)
A	6.15	6.23	1.28
B	6.26	6.33	1.11
C	6.37	6.44	1.09
D	6.49	6.54	0.76
E	6.61	6.64	0.45
F	6.73	6.76	0.44

## 6. Discussion of Results

The varying flow conditions observed in Table 2 for each of the stepped models studied is typical and agrees with the findings of [7] which clearly indicate that the initial and sequent depths varies with flow rate and the geometry of the chute. Similarly, as shown in Tables 3, the optimum energy levels observed within the aerated regions of the stepped spillway models decreases as the chute angles changes in each of the model studied. The vice versa of this scenario is observed with optimum energy levels within the non-aerated regions of these models. As further shown in Table 4, the varying slope of the spillway chute is a major factor that determines the level of the optimum total energy dissipated over each of these models. Hence obtaining models as in equations (17), (18) and (19) for the prediction of this and other very relevant hydraulic parameters that are required to understand the influence of flow over these stepped spillways becomes very relevant and useful in practice.

Furthermore, a quick evaluation of the predictive capability of each of these modeled equations shows that their marginal error level is lower than 5% for each of the equations modeled. The comparison shown in Table 6 indicating the error levels between 0.10% and 0.64% also signifies that there is a good agreement between the measured and the predicted values of the rate of energy dissipation over the non-aerated region.

Similarly, the error level between the observed actual and predicted (theoretical) sequent depth downstream of the hydraulic jump also varies between 0.44 and 1.28% indicative that the model equation (18) can effectively predict the location of the sequent depth which is very useful for locating and sizing of the consequent stilling structure to be put in place.

The application of these developed mathematical mod-

els becomes obviously relevant in highly constraining conditions in practice where the chute geometry may be required at an unusual specific slope angle to achieve a required optimum dissipated energy to control and protect lives and properties at their downstream section.

## 7. Conclusion

This study has developed some mathematical models for predicting some hydraulic parameters for flow over stepped spillway with varying chute geometry. The application of these models becomes relevant to practicing engineers and water resources managers for predicting sequent depth ratio and energy dissipated over non-aerated portion of the stepped spillway prototypes under a Froude scale ratio and obtaining relevant information for decision making in the overall management of these water resources infrastructures.

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