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MODELING AND ANALYSIS OF A THREE-PHASE SOLID-STATE VAR COMPENSATOR (SSVC)

A.J. Onah[a], **M.U. Agu**[b]

[a]Department of Electrical and Electronic Engineering, Michael Okpara University of Agriculture, Umudike, Abia State. ***Email:*** *aniagbosoonah@yahoo.com*
[b]Department of Electrcial Engineering, University of Nigeria, Nsukka, Nigeria.
Email: *drmarcelagu@yahoo.com*

Abstract

The problems associated with the flow of reactive power in transmission and distribution lines are well known. Several methods of var compensation have been applied in solving these problems. This paper examines the performance of a three-phase solid-state var compensator employed in the power system for reactive power compensation. The principal component of this device is a three-phase pulse-width-modulated voltage source inverter. A mathematical model of the inverter is derived and then used for the transient analysis. Optimization of the ac input power factor can be achieved by adjusting the angle difference between the inverter fundamental output voltage and the utility supply voltage. The open-loop frequency responses obtained show a very stable, high speed performance in the frequency range of interest.

Keywords: solid state, open loop frequency, reactive power, compensation, power factor

1. Introduction

It is a well-known fact that poor power factor causes the flow of reactive power in transmission and distribution networks, resulting in (a) voltage drop at line ends, (b) a rise in temperature in the supply cables, producing losses of active power, (c) over-sizing of generation, transmission and distribution equipment, (d) over-sizing of load protection due to harmonic currents, and (e) transformer overloads.

In view of the aforementioned problems associated with poor power factor, it is necessary to improve the power factor of an installation. Before the advent of modern power electronics, shunt static capacitors/reactors and synchronous condensers were extensively used to reduce the level of reactive power flowing in transmission and distribution networks [1],[2] But these elements are costly, bulky and often relatively inefficient. As a result, extensive research developed line commutated thyristors converters in combination with some reactive components. But there is the problem of reliable controlled switching. It's effective use is only when it is force-commutated and it requires costly and complex external circuits that reduce circuit reliability. One of the major factors in the advancement of var compensators technology is the advent of fast self-commutating solid-state devices (bipolar junction transistor (BJT), insulated-gate bipolar transistor (IGBT), gate-turn-off thyristor (GTO) and power MOSFET [3]. The voltage source inverter (VSI) employing any one of these devices is an efficient equipment for reactive power compensation or reduction of harmonic injection into ac mains - in order to improve power factor. Equipment is available from 380V to 34.5KV.

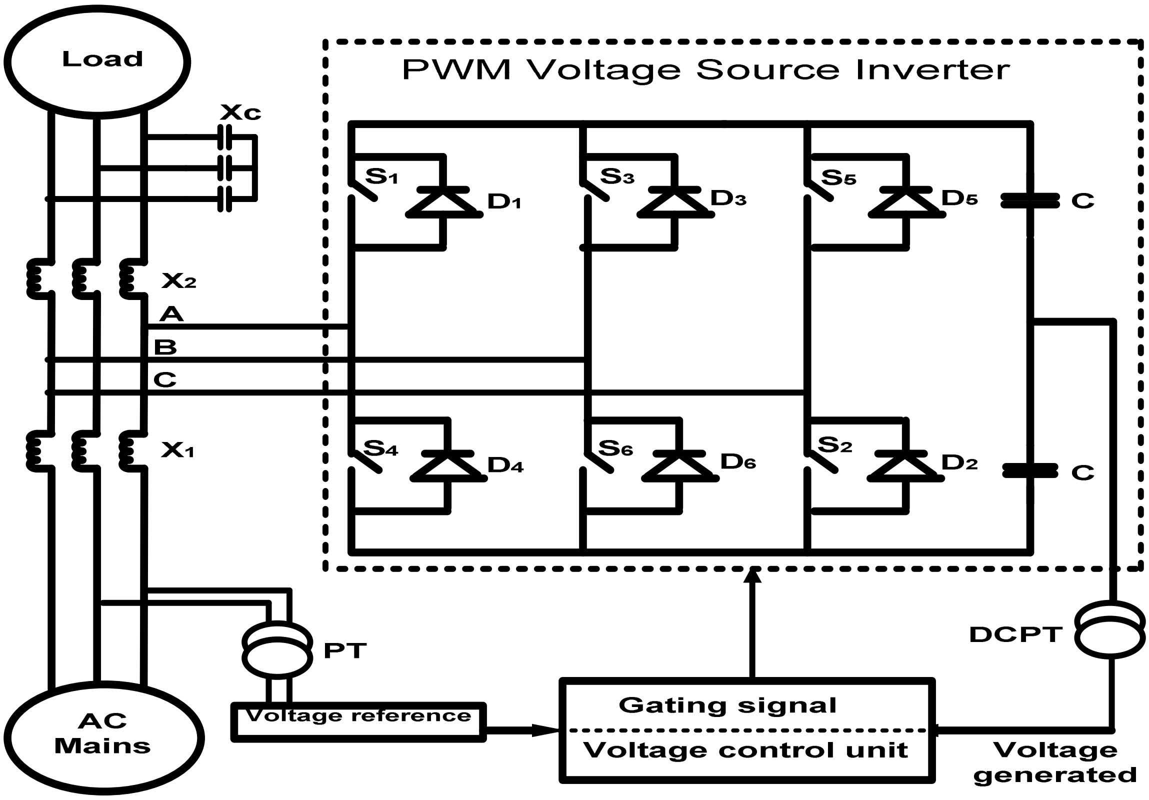


Figure 1: Solid-state var compensator configuration.

2. The three-phase solid-state var compensator (SSVC)

2.1. Description

It employs a three-phase pulse-width modulated (PWM) voltage source inverter (VSI) as shown in figure 1 [4]. The inverter is connected to the ac mains through a reactor X_1. A dc capacitor is connected to the dc side of the var compensator. The capacitor maintains a ripple-free dc voltage at the input of the inverter as well as storing reactive power. The SSVC is connected to the load through a second-order low-pass filter, X_2 and X_C, which reduces the harmonic components of currents flowing into the utility grid.

2.2. Principle of operation

With reference to figure 1, the single-phase equivalent circuit of the var compensator at fundamental frequency is as shown in figure 2.

The apparent power supplied by the ac source can be expressed as

$$S = \frac{VE_1}{X}\sin\delta + j\left(\frac{VE}{X}\cos\delta - \frac{E^2}{X}\right) \qquad (1)$$

where, δ = Phase-shift angle between the source voltage, V and the inverter ac voltage, E_1.

The real power supplied by the ac source is shared by the load and the inverter. The amplitude of the fundamental component of the inverter output ac voltage, E_1 depends on the value of the dc bus voltage, V_{dc}. So, E increases or decreases if the capacitor is charged or discharged. The voltage drop across the inductor X_1 determines the source power factor. The voltage drop across X_1 can be minimized by equalizing V and E, thus maintaining near unity power factor. The var compensator responds to fluctuations in load power factor by providing extra power required by the load or absorbing excess power from the load. If the power factor of the load increases, the load draws more real power which is transiently supplied by the inverter. The capacitor is thus discharged, leading to decrease in E. The control system takes corrective measure to make E equal to the corresponding value of V and hence maintain the source power factor at near unity. This is done by increasing the phase-shift angle, δ and more active power will flow to the inverter

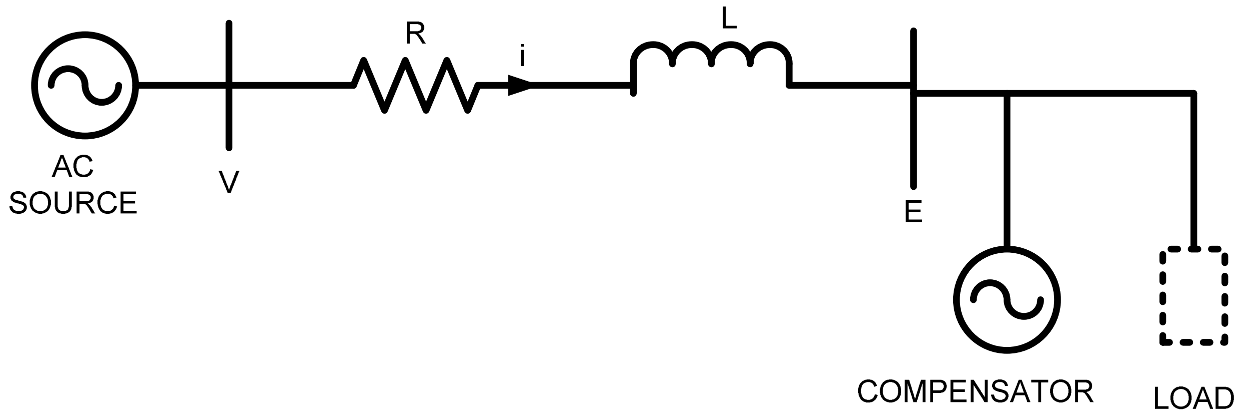


Figure 2: Equivalent circuit per phase of var compensator. V = Line-to-neutral voltage of the ac mains. E = Fundamental component of the inverter phase-to-neutral ac voltage. i = ac current. R = Losses of the system lumped. L = Line inductor.

to charge the capacitor. If the load power factor decreases, the load is taking less real power, and then the compensator absorbs the excess real power which charges the capacitor and then lead to higher output of E. To restore E to normal value, the capacitor is discharged by decreasing δ. By controlling the phase angle δ, the dc capacitor voltage levels can be changed, and thus the amplitude of the fundamental component of the inverter output voltage E can be controlled [5],[6]. Real power flows to the compensator if the load power factor is lagging (the load is drawing less active power); and the compensator supplies real power if load power factor is leading (load requires more active power).

3. Transient Analysis of the Var Compensator

3.1. Mathematical model

The speed of response of the dc capacitor (V_{dc}) to the changes in the phase-shift angle, δ determines the transient response of the var compensator. The analysis is based on the equivalent circuit shown in figure 2, where R is the equivalent resistance representing the total compensator system losses. To derive mathematical model of the solid-state var compensator, we assume that (a) the ac source is a ripple-free, balanced, three-phase voltage, (b) only fundamental components of currents and voltages are represented by the equivalent circuit, (c) since the variations in the phase-shift angle, $\Delta\delta$ are small, the system is made linear [7].

From the equivalent circuit,

$$v(t) - e(t) = R_i(t) + L\frac{di(t)}{dt} \qquad (2)$$

Let δ oscillate around a mean value δo between $\delta o - \Delta\delta$ and $\delta o + \Delta\delta$ with a frequency ω_d, then,

$$\delta(t) = \delta_{max}\cos(\omega_d t) = Re[\delta_{max}e^{j\omega_d t}]$$

Considering the inverter voltage oscillations, the voltages and current can be expressed as

$$\begin{aligned} v(t) &= Ve^{j(\omega t+\Delta\delta)} \\ e(t) &= Ee^{j(\omega t+\Delta\delta)} \\ i(t) &= Ie^{j(\omega t+\Delta\delta)} \end{aligned} \qquad (3)$$

Where $\omega = 2\pi f$ is the ac source frequency.

In $d-q$ axis,

$$\begin{aligned} v(t) &= (v_d + jv_q)[\cos(\omega t+\delta) + j\sin(\omega t+\delta)] \\ e(t) &= (e_d + je_q)[\cos(\omega t+\delta) + j\sin(\omega t+\delta)] \\ i(t) &= (i_d + ji_q)[\cos(\omega t+\delta) + j\sin(\omega t+\delta)] \end{aligned} \qquad (4)$$

From equation (3)

$$\begin{aligned} v(t) &= [v_d\cos\omega t - v_q\sin\omega t]\cos(\delta) \\ e(t) &= [e_d\cos\omega t - e_q\sin\omega t]\cos(\delta) \\ i(t) &= [i_d\cos\omega t - i_q\sin\omega t]\cos(\delta) \end{aligned} \qquad (5)$$

Equation (5) in (2) gives

$$(v_d - e_d)\cos\omega t - (v_q - e_q)\sin\omega t = \Big(Ri_d + L\tfrac{di_d}{dt} - \omega Li_q\Big)\cos\omega t - \Big(Ri_q + L\tfrac{di_q}{dt} + \omega Li_d\Big)\sin\omega t \qquad (6)$$

And then, after grouping, the steady-state equations are:

$$\begin{aligned} v_{do} - e_{do} &= Ri_{do} + L\tfrac{di_{do}}{dt} - \omega L i_{qo} \\ v_{qo} - e_{qo} &= Ri_{qo} + L\tfrac{di_{qo}}{dt} + \omega L i_{do} \end{aligned} \quad (7)$$

Applying small disturbances to variables in (7) around the operating point yields

$$\begin{aligned} &v_d = v_{do} + \Delta v_d; \quad e_d = e_{do} + \Delta e_d; \quad i_d = i_{do} + \Delta i_d \\ &v_q = v_{qo} + \Delta v_q; \quad e_q = e_{qo} + \Delta e_q; \quad i_q = i_{qo} + \Delta i_q \end{aligned} \quad (8)$$

Equation (7) becomes

$$\begin{aligned} &v_{do} + \Delta v_d - e_{do} - \Delta e_d = Ri_{do} + R\Delta i_d + L\tfrac{di_{do}}{dt} + \\ &L\tfrac{d\Delta i_d}{dt} - \omega L i_{qo} - \omega L \Delta i_q \\ &v_{qo} + \Delta v_q - e_{qo} - \Delta e_q = Ri_{qo} + R\Delta i_q + L\tfrac{di_{qo}}{dt} + \\ &L\tfrac{d\Delta i_q}{dt} + \omega L i_{do} + \omega L \Delta i_d \end{aligned} \quad (9)$$

Subtract (7) from (9),

$$\Delta v_d - \Delta e_d = R\Delta i_d + L\frac{d\Delta i_d}{dt} - \omega L \Delta i_q \quad (10)$$

$$\Delta v_q - \Delta e_q = R\Delta i_q + L\frac{d\Delta i_q}{dt} + \omega L \Delta i_d \quad (11)$$

Multiplying equation (7) by $\Delta\delta$

$$v_{do}\Delta\delta - e_{do}\Delta\delta = Ri_{do}\Delta\delta + L\frac{di_{do}}{dt}\Delta\delta - \omega L i_{qo}\Delta\delta \quad (12)$$

$$v_{qo}\Delta\delta - e_{qo}\Delta\delta = Ri_{qo}\Delta\delta + L\frac{di_{qo}}{dt}\Delta\delta + \omega L i_{do}\Delta\delta \quad (13)$$

Subtract (13) from (10), and add (12) and (11)

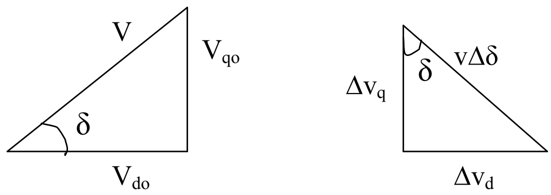
$$\begin{aligned} &\Delta v_d - \Delta e_d - v_{qo}\Delta\delta + e_{qo}\Delta\delta = R\Delta i_d + L\tfrac{d\Delta i_d}{dt} - \\ &\qquad \omega L\Delta i_q - Ri_{qo}\Delta\delta - L\tfrac{di_{qo}}{dt}\Delta\delta - \omega L i_{do}\Delta\delta \\ &\Delta v_q - \Delta e_q - v_{do}\Delta\delta + e_{do}\Delta\delta = R\Delta i_q + L\tfrac{d\Delta i_q}{dt} - \\ &\qquad \omega L\Delta i_d - Ri_{do}\Delta\delta - L\tfrac{di_{do}}{dt}\Delta\delta + \omega L i_{qo}\Delta\delta \end{aligned} \quad (14)$$

Applying the Laplace transform to equation (14),

$$\begin{aligned} &\Delta v_d - \Delta e_d - v_{qo}\Delta\delta + e_{qo}\Delta\delta = (R + sL)(\Delta i_d - \\ &\qquad i_{qo}\Delta\delta) - \omega L(\Delta i_q + i_{do}\Delta\delta) \\ &\Delta v_q - \Delta e_q + v_{do}\Delta\delta + e_{do}\Delta\delta = (R + sL)(\Delta i_q + \\ &\qquad i_{do}\Delta\delta) + \omega L(\Delta i_d - i_{qo}\Delta\delta) \end{aligned} \quad (15)$$

In matrix form, the final equations for the system are

$$\begin{bmatrix} \Delta v_d - v_{qo}\Delta\delta \\ \Delta v_q + v_{do}\Delta\delta \end{bmatrix} - \begin{bmatrix} \Delta e_d - e_{qo}\Delta\delta \\ \Delta e_q + e_{do}\Delta\delta \end{bmatrix} = \begin{bmatrix} R + sL & -\omega L \\ \omega L & R + sL \end{bmatrix} \times \begin{bmatrix} \Delta i_d - i_{qo}\Delta\delta \\ \Delta i_q + i_{do}\Delta\delta \end{bmatrix} \quad (16)$$



3.2. Transfer function of the compensator

Figure 3 is the phasor diagram of the perturbed system, where the inverter output voltage E is taken as the reference phasor with the ac voltage $d-q$ components oscillating about their quiescent values, v_{do} and v_{qo} with amplitudes Δv_d and Δv_q and frequency ω_d.

From figure 3

$$\begin{aligned} v_{do} &= V\cos(\delta) \\ -\Delta v_q &= V\Delta\delta\cos(\delta) \end{aligned} \quad (17)$$

$$\begin{aligned} v_{qo} &= V\sin(\delta) \\ \Delta v_d &= V\Delta\delta\sin(\delta) \end{aligned} \quad (18)$$

Equations (17) and (18) in (16) results in

$$\begin{aligned} \Delta v_d - v_{qo}\Delta\delta &= V\Delta\delta\sin(\delta) - V\Delta\delta\sin(\delta) = 0 \\ \Delta v_q + v_{do}\Delta\delta &= -V\Delta\delta\cos(\delta) + V\Delta\delta\cos(\delta) = 0 \end{aligned} \quad (19)$$

$$\begin{aligned} &e_{do} = E; \quad \Delta e_{do} = \Delta E \\ &e_{qo} = 0; \quad \Delta e_{qo} = 0 \end{aligned} \quad (20)$$

If k is the modulation of the inverter, then, the output voltage is related to the capacitor voltage as

$$\begin{aligned} e_{do} &= kV_{dco} \\ \Delta e_d &= k\Delta V_{dc} \end{aligned} \quad (21)$$

Equations (20) and (21) in (16) yields

$$\begin{bmatrix} \Delta i_d - i_{qo}\Delta\delta \\ \Delta i_q + i_{do}\Delta\delta \end{bmatrix} = \begin{bmatrix} R + sL & -\omega L \\ \omega L & R + sL \end{bmatrix}^{-1} \times \begin{bmatrix} -k\Delta V_{dc} \\ -kV_{dco}\Delta\delta \end{bmatrix} \quad (22)$$

$$\Delta i_d = \frac{-(R + sL)k\Delta V_{dc} + \omega L k V_{dco}\Delta\delta}{R^2 + 2RsL + s^2L^2 + (\omega L)^2} + i_{qo}\Delta\delta \quad (23)$$

Equation (23) is the line current due to the oscillations of the phase-shift angle δ.

Power balance equation of the inverter is

$$\frac{3}{2}(e_d i_d + e_q i_q) = V_{dc}C\frac{dV_{dc}}{dt} \quad (24)$$

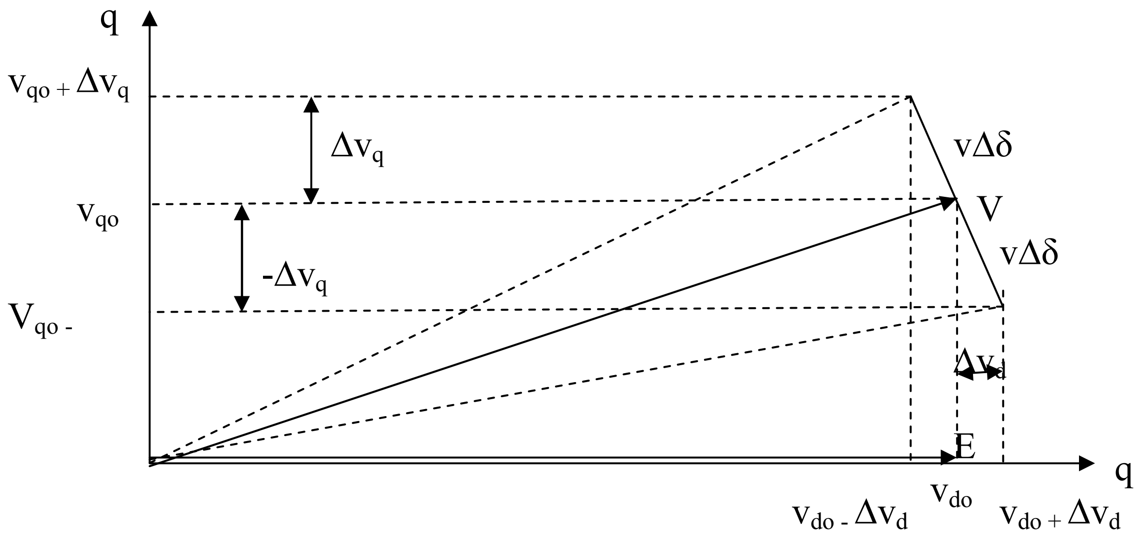


Figure 3: Phasor diagram of the perturbed system.

where C is the dc capacitor. Applying small perturbations around the steady-state operating point,

$$e_q = 0$$

$$\frac{3}{2}(e_{do}+\Delta e_d)(i_{do}+\Delta i_d) = (V_{dco}+\Delta V_{dc})C\frac{d}{dt}(V_{dco}+\Delta V_{dc}) \tag{25}$$

Subtracting the steady-state equation from equation (25) gives

$$\frac{3}{2}(e_{do}i_{do} + e_{do}\Delta i_d + \Delta e_d \Delta i_d - e_{do}i_{do} = V_{dco}C\frac{d}{dt}V_{dco} + V_{dco}C\frac{d}{dt}\Delta V_{dc} + \Delta V_{dc}C\frac{d}{dt}V_{dco} + \Delta V_{dc}C\frac{d}{dt}\Delta V_{dc} - V_{dco}C\frac{d}{dt}V_{dco}$$

But,

$$\frac{d}{dt}V_{dco} = 0$$

$$\frac{3}{2}(e_{do}\Delta i_d + \Delta e_d i_{do} + \Delta e_d \Delta i_d) = V_{dco}C\frac{d}{dt}\Delta V_{dc} + \Delta V_{dc}C\frac{d}{dt}\Delta V_{dc}$$

Neglecting second order terms,

$$\frac{3}{2}(e_{do}\Delta i_d + \Delta e_d i_{do}) = V_{dco}C\frac{d}{dt}\Delta V_{dc} \tag{26}$$

i_{do} corresponds to the steady-state current component that provides the losses of the var compensator. Since the losses of the system are small, the product $\Delta e_d i_{do}$ can be neglected Therefore,

$$\frac{3}{2}e_{do}\Delta i_d = V_{dco}C\frac{d}{dt}\Delta V_{dc} \tag{27}$$

Recall,

$$e_{do} = kV_{dco}$$

Hence,

$$\begin{aligned} \frac{3}{2}kV_{dco}\Delta i_d &= V_{dco}C\frac{d}{dt}\Delta V_{dc} \\ \frac{3}{2}k\Delta i_d &= C\frac{d}{dt}\Delta V_{dc} \end{aligned} \tag{28}$$

Applying Laplace transform,

$$\begin{aligned} \frac{3}{2}k\Delta i_d &= Cs\Delta V_{dc} \\ \Delta i_d &= \frac{2Cs\Delta V_{dc}}{3k} \end{aligned} \tag{29}$$

From equations (23) and (29),

$$\frac{2Cs\Delta V_{dc}}{3k} = \frac{-k\Delta V_{dc}(R+sL) + \omega LkV_{dco}\Delta\delta}{L^2s^2 + 2RLs + (\omega L)^2 + R^2} + i_{qo}\Delta\delta$$

$$\Delta V_{dc}\left[\frac{2s^3CL^2+4RCLs^2+2sC\omega_o^2L^2+2sCR^2+3k^2R+3k^2sL}{3k(L^2s^2+2RLs+\omega_o^2L^2+R^2)}\right] = \Delta\delta\left[\frac{i_{qo}\left(L^2s^2+2RLs+\omega_o^2L^2+R^2\right)+\omega_oLkV_{dco}}{L^2s^2+2RLs+\omega^2L^2+R^2}\right]$$

$$\frac{\Delta V_{dc}}{\Delta\delta} = \frac{3ki_{qo}\left(L^2s^2+2RLs+\omega_o^2L^2+R^2\right)+3k^2\omega_oLV_{dco}}{2CL^2s^3+4RCLs^2+\left(2\omega_o^2L^2C+2R^2C+3k^2L\right)s+3k^2R} \tag{30}$$

This is the transfer function of the var compensator system.

The following values of the system parameters may be used to illustrate the transient response of the system: $C = 800\mu$F; $L = 20$mH; $R = 0.4\Omega$; $V_{dco} = 100$V; $i_{qo} = 5$A; $k = 1$; $f = 60$Hz.

Applying Bode plots, the open-loop frequency response of the system represented by equation

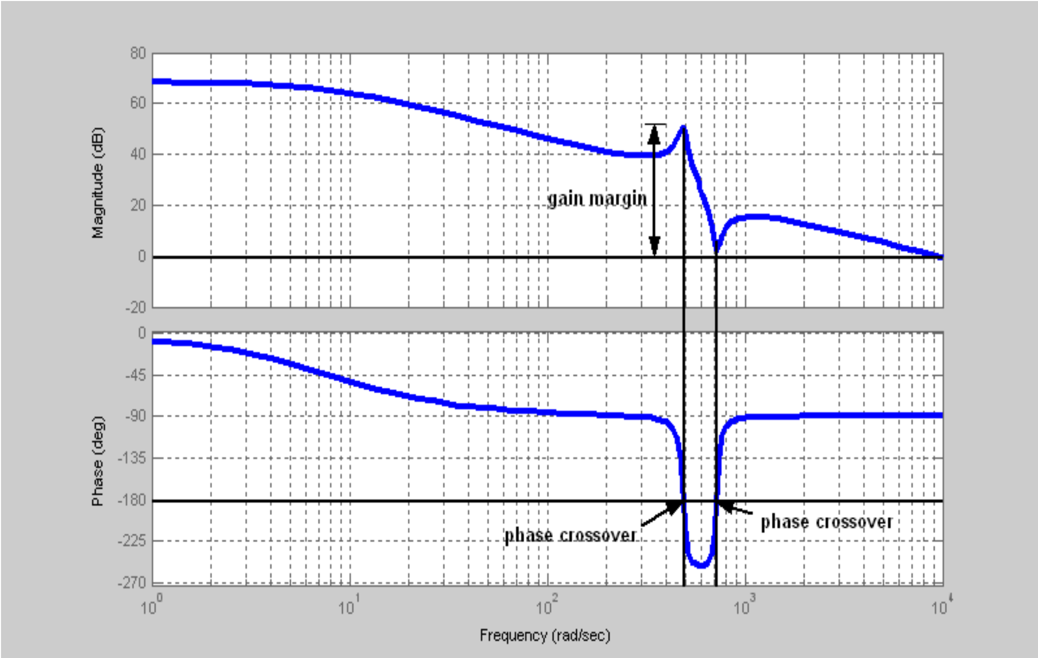


Figure 4: Open loop frequency response of var compensator at rated operating condition ($i_{qo} = 1$ p.u).

(30) is shown in figure 4. The points where the phase curve intersects the 180° axis are two in number, as shown in the phase plot. So, the phase crossover frequencies, ω_p are 500 rads/sec and 700 rads/sec. The corresponding gain margins (GM), measured from the 0 - dB axis on the magnitude plot are -50 dB and -4 dB. They are negative because they lie above the 0 - dB axis, and for this reason the system is unstable in this frequency range (500 – 700 rads/sec.) [8], [9].

At no-load ($i_{qo} = 0$), the var compensator transfer function simplifies to:

$$\frac{\Delta V_{dc}}{\Delta \delta} = \frac{3k^2\omega_o L V_{dco}}{2CL^2s^3 + 4RCLs^2 + (2\omega_o^2 L^2 C + 2R^2C + 3k^2 L)\,s + 3k^2 R} \tag{31}$$

The bode plot of the transfer function (31) is shown in figure 5. From the plot, Gain crossover frequency, $\omega_g = 1700$ rads/s. Phase margin (PM) $= -90°$. It is negative because it is measured below the 180° axis. Since the phase margin is negative, the system is unstable. Phase crossover frequency $\omega_p = 500$ rads/sec. Gain margin (GM) = -50 dB. The gain margin is negative because it is measured above the 0 - dB axis. Since the gain margin is negative, the system is unstable. Thus, the system is unstable at frequencies above 500rad/sec.

Substituting these values of the parameters in equation (31) results in

$$\frac{\Delta V_{dc}}{\Delta \delta} = \frac{2262}{6.4\times10^{-7}s^3 + 2.56\times10^{-5}s^2 + 0.1512s + 1.2}$$

$$\frac{\Delta V_{dc}}{\Delta \delta} = \frac{14976.4\omega_n^2}{(s+7.95)\left(s^2 + 2\varepsilon\omega_n s + \omega_n^2\right)} \tag{32}$$

$$\frac{\Delta V_{dc}}{\Delta \delta} = \frac{1884}{(j\omega T + 1)\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\varepsilon\left(\frac{j\omega}{\omega_n}\right) + 1\right]} \tag{33}$$

Time constant, $T = 0.1258$s Natural frequency of oscillation, $\omega_n = 485.5$rads/sec Damping ratio, $\varepsilon = 0.033$

At low frequencies, equation (33) approximates to a first order system

$$\frac{\Delta V_{dc}}{\Delta \delta} = \frac{1884}{j\omega T + 1} = \frac{A_o}{Ts + 1} = \frac{A_o}{s/\omega_b + 1} \tag{34}$$

ω_b = break frequency.

4. Conclusions

The features and operational principles of a var compensator employing PWM voltage source inverter with self-controlled dc bus, otherwise known as solid-state var compensator (SSVC) have been discussed in this paper. In the analysis of the SSVC, a model was derived, using d-q reference axis, for a var compensator operating with the δ phase-shift angle control method.

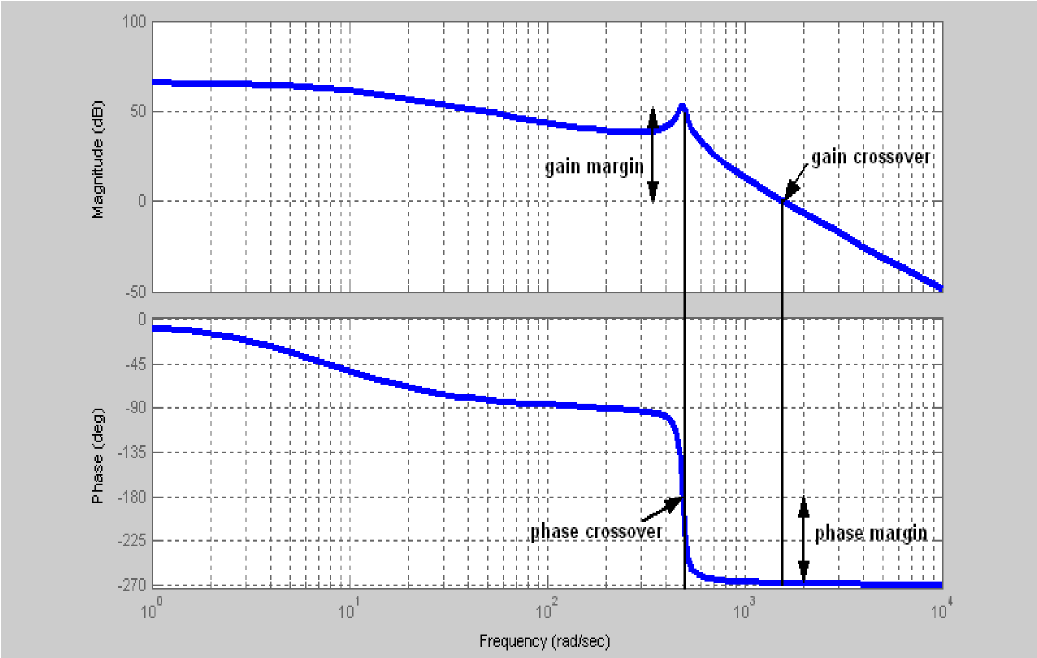


Figure 5: Open loop frequency response of var compensator at no-load operating condition ($i_{qo} = 0$).

The model has been used to analyze the performance of the compensator, applying the bode plots. Results of analysis show that the compensator under discussion is stable for low frequencies (under 60Hz). Selective harmonic elimination method of PWM was used. Hence reduction in low-frequency harmonic components of the inverter ac output voltage, allowing considerable decrease in size, cost and weight of reactive elements. Besides, fast, accurate and continuous var control is realized without computation of the required reactive power.

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