PROBABILISTIC APPROACH TO STRUCTURAL APPRAISAL OF A BUILDING DURING CONSTRUCTION

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Abstract

Probabilistic assessment methods are very attractive as they allow a systematic treatment of uncertainties. In this paper, probabilistic models are formulated to predict the reliability of concrete in a structure under construction, a case study of Laboratory Block for College of Continuing Education, University of Port Harcourt, Rivers State. The safety of the structure is predicted based on the safety index obtained from the probabilistic model. The design strength of concrete in the structure (grade of concrete) is obtained from schmidt hammer test. The concrete in the structure gave a safety index value of 2.83 which is less than the target reliability index value of 3.7 for concrete for safety class 1 BKR[1] and less than the target reliability index of 4.5 for slabs, 4.9 for beams in flexure, 3.6 for beams in shear and 3.9 for columns under dead and live-load combination[2] proving that the structure is not safe and is prone to risk of serious injury to persons and damage to properties.

Keywords: Probabilistic model, Uncertainities, Reliability, Safety index, Schmidt hammer

1. Introduction

The assessment of a building during construction has become a more frequent task for engineers both now and in the future due to the increasing risk of failure during and after construction. The reasons for the assessment are that different use may be proposed to the structure, new regulations with higher load requirements can be applied to the structure or there can be indications of ongoing deterioration in the structure. Deterioration is a common reason for assessment [3-6].

The use of reliability theory and probability concepts is well established as a basis for defining criteria for safety of a building construction against collapse. Structural quality is a function of human intervention at every stage of a building process. Thus, the performance of risk assessment of a building not only forms the design stage but while undergoing construction is necessary rather than sitting back until failure takes place and the building collapses[7]. According to Wilkinson[8], once the nature of the risk has been recognized the next step should be the determination and implementation of measures that will reduce the risk or reduce the effect of the loss or both at an economical cost. Eventually, the need for loss financing will be reduced in most instances and losses will be avoided or minimized[9].

It is in conventional design to seek to

achieve a low probability of getting an action value higher than the resistance of the structure. For this reason, it is usually verified that the limit states (states at which the structure no longer performs the intended purpose) are not reached when design values for actions, material properties and geometrical data are used in the design models. However, natural phenomena support the existence of inherent variability in most of the design quantities. As a result, the expected level of probability of failure becomes an issue which should be handled rationally.

According to Freudenthal[10], the best way to assess the safety of an existing deteriorating structure is by probability of failure. Because the design of a structure embodies uncertain predictions of the performance of structural materials as well as of the expected load patterns and intensities, the concept of probability forms an integral part of any rational analysis and any conceivable condition is necessarily associated with a numerical measure of the probability of its occurrence. Infact, meaningfully, it has been the directional effort of the engagement of probabilistic thinking to systematically assess the effect of uncertainty on structural performance. Although the use of probabilistic concept may not answer all issues of unknowns, it has played a very remarkable role in the integrity assessment of many engineering systems [11-12]. The strength variable is assumed randomly and stochastically. The reliability of concrete in the structural members is assessed in terms of reliability index.

The intent of this paper is to investigate the reliability of a building during construction using simple probabilistic model. The algorithm involved is simple, not involving much computational effort.

2. Schmidt Hammer Test

The rebound hammer test is described in ASTM C805[13]. It is a non-destructive test on concrete. It is based on the principle that the rebound of an elastic mass depends on the hardness of the surface against which the mass impinges. The test method started by the careful selection and preparation of the concrete surface to be tested. As the surface was chosen, it was prepared by an abrasive stone so that the test surface was ground smooth. An energy was applied by pushing the hammer against the test surface. The plunger was allowed to strike perpendicularly to the test surface. After impact, the rebound hammer readings were recorded for the individual structural members (Table 1).

3. Model Development

The reliability prediction is achieved using normal probabilistic (μ, σ) model. The probability density function of a normal variate is given by

$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right];\ -\infty \le x \le \infty \tag{1}$$

Where μ and σ are the mean and standard deviation of the distribution respectively. For standard and normal distribution N(0,1), the PDF of the standard normal variate u is given by:

$$f_u(u) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}u^2\right];\ -\infty \le x \le \infty \tag{2}$$

Let $\phi_{(u)}$ represent CDF. Hence $\phi_{(u)}$ is the cumulative probability of a standard normal variate given by

$$\phi_{(u)} = f_u(u) = P(U \le u) \tag{3}$$

Where PDF, CDF represent probability density function and cumulative density function of the standard normal variate respectively.

The PDF and CDF of u are as shown in figure 1 below.

Conversely, u_1 at a cumulative probability of P1 is given by:

$$u_1 = \phi^{-1}(P_1) \tag{4}$$

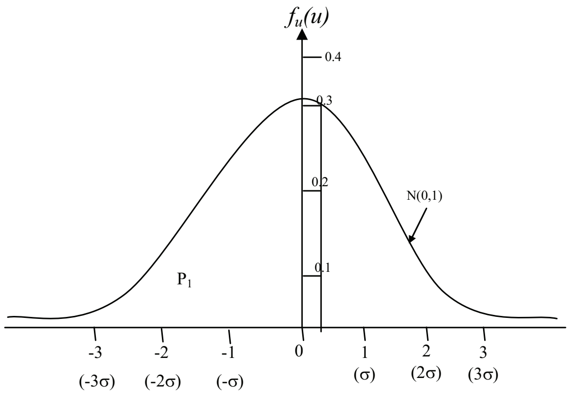

Figure 1: Formulation of safety analysis in normalized coordinates[2].

$$\phi(-u_2) = 1 - \phi(u_2) \tag{5}$$

If

$$\phi(-u_2) = P_2 \tag{6}$$

Then

$$u_2 = -\phi^{-1}(P_2) \tag{7}$$

The distribution is symmetrical. The CDF of x with distribution N(μ, σ) is given by

$$F_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2\right] dx \tag{8}$$

The normally distributed variable (X) can be reduced to a standard normal variable by using the transformation given in equation (9) Let

$$u = \frac{X-\mu}{\sigma} \tag{9}$$

Therefore,

$$dx = \sigma du \tag{10}$$

Substituting for dx in equation (8) above gives:

$$F_x(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X-\mu/\sigma} \exp\left(-\frac{u^2}{2}\right) du \tag{11}$$

The integral in equation (11) is the area under the standard normal density curve between $-\infty$ and $\frac{X-\mu}{\sigma}$. Hence,

$$f_x(x) = \varphi\left(\frac{X-\mu}{\sigma}\right) \tag{12}$$

Therefore,

$$F_x(x) = \varphi(u) \tag{13}$$

Equation (14) represents the cumulative distribution function of x.

Using normal probability tables, the probability of failure of concrete in a structure in compression can be obtained.

4. Reliability Analysis

Let x be a basic variable, μ_x be the true mean of x, and δ_x be the true coefficient of variation of x.
$\overline{X}$ = sample mean
$\overline{\delta_x}$ = sample coefficient of variation
x and $\overline{\delta_x}$ are determined from data collected under carefully controlled conditions. $\overline{\delta_x}$ determines the inherent variability. Let the bias and coefficient of variation of uncertainties be given by $\overline{M}$ and $\overline{\delta_x}$. Then μ_x and δ_x are estimated as:

$$\mu_x = \overline{M}\,\overline{X} \tag{14}$$

$$\delta_x = \left(\overline{\delta_x}^2 + \delta_M^2\right)^{1/2} \tag{15}$$

Considering n factors,

$$\delta_M = \left(\delta_1^2 + \delta_2^2 + \ldots + \delta_n^2\right)^{1/2} \tag{16}$$

Let y be the cube strength of concrete and x be the strength of concrete in structure. The mean value of the concrete strength in structure = 0.67 times the mean value of the cube strength of concrete.

$$\Longrightarrow \mu_x = 0.67\mu_y[14] \tag{17}$$

Taking into account the uncertainties involved in testing procedure (δ_{test}) and in-situ variation of concrete strength ($\delta_{in\,situ}$), the coefficient of variation of concrete strength using equation (16) can be given as

$$\delta_x^2 = \delta_y^2 + \delta_{test}^2 + \delta_{in\,situ}^2 \tag{18}$$

$\delta_{test} = 0.05$ and $\delta_{in\,situ} = 0.10$ [2]. Then,

$$\delta_x^2 = \delta_y^2 + 0.125 \tag{19}$$

Table 1: Results of non-destructive test on concrete. *

S/No	Location	Rebound Hammer readings	Average Rebound	Concrete strength from Rebound Test
1	Middle Panel	23.23	23	18
2	Edge Panel	23.23	23	18
3	Beam 2	20,20	20	14
4	Slab 2	24,24	24	20
5	Slab 1	18,19	19	8
6	Middle column	35,27	31	29
7	Corner column	27,27	27	2.5
8	Beam 1	21,12	12	5
9	Stair case	23.3,19	21.2	15
10	Column footing	12.5,6	9	4

* $\mu_y = \sum_{i=1}^{10} \frac{y_i}{10} = 15\text{N/mm}^2$

The value of δ_y is a function of the design mix. Equation (19) represents the net variation in concrete strength.

When the stress developed in the ith structural member exceeds the allowable stress, its safety is jeopardized. Hence, the probability of failure of an i_{th} structural member can be given as:

$$P_{f_i} = P(X_i < f_a) \tag{20}$$

Where X_i is the random variable representing the strength or resistance of the i_{th} structural member and f_a = allowable stress of concrete in compression $= 0.34f_{cu}$ [14]. X is assumed to be normally distributed. Therefore,

$$P_{f_i} = \varphi\left(\frac{f_a - \mu_x}{\delta_x}\right) \tag{21}$$

Hasofer and Lind [15] gave the relation between safety index and probability of failure as described by equation (22).

$$P_f = 1 \times 10^{-\beta} \tag{22}$$

(22) where β = safety or reliability index.

5. Results and Discussion

From Table 2, $\mu_y = 17.56\text{N/mm}^2$, $\sigma_y = 2.69\text{N/mm}^2$ and $\delta_y = 15.33\% = 0.1533$. Using equation (20),

$$\delta_x = \sqrt{0.1533^2 + 0.125} = 0.19 \tag{23}$$

From equation (18),

$$\mu_x = 0.67\mu_y = 0.67 \times 17.56 = 11.76\text{N/mm}^2$$

$$\sigma_x = 0.67\sigma_y = 0.67x0.19x17.56 = 2.24\text{N/mm}^2$$

$$\delta_y = \frac{\sigma_y}{\mu_y}$$

Using equation (22) the probability of failure of concrete in structure is:

$$P_f = \varphi\left(\frac{5.10 - 11.76}{2.24}\right) = \varphi(-2.97) = 1.49x10^{-3}$$

From equation (23), $0.00149 = 10^{-\beta}$, $\beta = 2.83$.

6. Discussion of Results and Conclusion

Probabilistic approach to structural appraisal of a building during construction has been discussed. From Table 1, it can be observed that Rebound test gave an average concrete strength of about 15N/mm^2 (grade 15). The as-constructed concrete strength gave a probability of failure value of 1.49×10^{-3} and the corresponding reliability index value of 2.83 which is less than the target reliability index of 3.7 BKR[1] safety class 1 and less than reliability index of 4.5 for slabs, 4.9 for beams in flexure, 3.6 for beams in shear and 3.9 for columns under dead and live load combination[2]. In conclusion, the structure is not safe and stands a risk of serious injury to persons and damage to properties. The floor slab and beams are therefore recommended for careful demolition to give way to a new structural frame which should be properly designed to reflect the "as-constructed" structural arrangement of the columns and a more competent contractor should be considered for the re-construction while supervision should be more stringent.

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Table 2: Statistics of basic variable [2]. *

Variable	**Mix**	**Specified strength**	**Mean** (μ_y) N/mm^2	**Standard deviation** (σ_y) N/mm^2	**COV.** (δ_y) %	**Probability Distribution**	**Quality Control**
Cube strength	Grade 15	15	17.56	2.69	15.33	Normal	Design mix

* $f_a = 0.33 \times 15 = 5.10$N/mm^2

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