MATHEMATICAL MODEL FOR THE OPTIMIZATION OF COMPRESSIVE STRENGTH OF LATERIZED SANDCRETE BLOCKS

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Abstract

In Nigeria, sandcrete blocks commonly used in the construction industry has been identified by researchers as being expensive because of the cost of its constituent components especially cement and fine aggregate (sharp river sand).This has necessitated research effort towards finding alternative materials to replace both the cement and sand either wholly or partially without adverse effect on the strength properties of the sandcrete blocks. This work applied Scheffes regression formulations to obtain mathematical model of the compressive strength of sandcrete block of alluvial deposit for various mix proportions as multivariate functions with the proportions of sandcrete ingredients serving as variables. These mathematical models are adopted for optimization of strength of sandcrete block in compression. With the model, any desired strength of sandcrete block, given any mix proportions, is easily evaluated. Basic Language is used in the development of the computer program. The maximum compressive strength predicted by the model is 2.07N/mm^2 which supports the finding of Osili [1] that says the compressive strength should be 2.07N/mm^2 for 2 or 3 storey buildings.

Keywords: sandcrete block, compressive strength, laterite, scheffe's theory

1. Introduction

In most rural communities in Nigeria especially in southern Nigeria, alluvial deposit (laterite contaminated sand) has constituted a cheap replacement of fine aggregate for moulding building blocks. This is due to affordability and the need to improve the strength of the block relative to the traditional laterite (mud building block) for there is no river in Nsukka making river sand very costly for poor masses in the area.

Adepegba [2] has reported that a concrete in which laterite fines are used instead of sand can be used as a structural material in place of normal concrete. It was found by Balogun et al [3] that when sand is mixed with laterite fines the most suitable mix for structural application is $1 : 1\frac{1}{2} : 3$ with a water/cement ratio of 0.65 provided that the laterite content is kept below 5%.

Researchers have therefore developed means and methods of improving the laterite engineering properties; and today compressed earth blocks could be obtained which can meet the minimum standards for precast concrete building blocks Berkovitch [4].

These developments coupled with its abundance, cheapness, high fire resistance, Kateregga [5], Aribisale [6] make laterite an attractive material for engineering applications, particularly building construction, especially in third world countries where the need to provide low cost housing is of economic and social imperative.

The standard practice in the building industry is to specify target strength and performance of building material products to be achieved by producers. Usually the specifications are given in empirical figures. For example, in BS8110, C30 concrete stands for minimum compressive strength of 30N/mm^2 which a concrete producer must meet if so specified by a designer. The producer is left to conduct series of trial mixes based on intelligent gueses,using different mix ratios and sometimes with component materials from different sources until the right mix that meets the specification for strength is identified.

The above mentioned problems naturally lead to the search for a model for sandcrete block that can predict either the strength of laterite contaminated sandcrete block given component mix of water, cement, laterite and sand. A better approach is to formulate mathematical models that can predict mix proportions of composites given specified strengths.

Consequently to this, it is the objectives of this research to

i) Formulate a statistically adequate model of laterite contaminated sandcrete blocks that will predict the strength of sandcrete block [the response function] at any given mix proportion of water, cement, laterite and sand.

ii) Constrain the models to generate all possible combinations of component mix proportions through computer application, to satisfy a desired target strength of laterite contaminated sandcrete blocks and vice versa.

2. Scheffes Optimization Theory

Simplex lattice is an example of such experimental design methodology which is adopted in this research study. The hypothesis used is that for the properties of a q-component mixture which depend only on component ratio not on the quantity of the components. The factor space is a regular $(q-1)$ simplex and for the mixture the relationship holds:

$$\sum x_i = 1 \tag{1}$$

Where x_i is the component fraction and q is the number of components.

To describe such surface adequately Scheffe [6] suggested ways to describe the mixture properties by reduced polynomials given thus

$$\begin{aligned} y = \; & b_0 + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_i x_k + \\ & \ldots + \sum b_{i1,i2,\ldots,i_n} x_{i1} x_{i2} x_{in} \\ & \text{where } (1 < i < q,\ 1 < i < j < q, \\ & 1 < i,\ j < k < q) \end{aligned} \tag{2}$$

mixture y is the mixture property; b is the polynomial coefficient; x is the mix component ratio in weight Scheffe [7] developed a theory for experiments with mixtures of q-components whose purpose is the empirical prediction of the response (a real-valued function) to any mixture of the components, when the response depends only on the proportion of the components and not on the total amount. His theory is one of the adaptation to this work in that formulation of a response function for compressive strength of laterite contaminated sandcrete block. This response function serves as a model whose predictions can then be tested for adequacy against control results obtained in the laboratory.

These now form the basis of a new method of sandcrete block components mix. It will do match this against the primordial arbitrariness (they call it rule of thumb) of the nominal mix design.

Transformation of components from pseudo to actual variables.

Due to ease of formulation and mathematical manipulations, the response surface preferably is modelled in a factor space whose variables xi must satisfy eqn. [1]. In practice, the actual variables z_i which are specified in terms of mix ratios such as $1:2:4$, $1:3:6$ etc. at a given water-cement ratio cannot satisfy eqn. [1]. Consequently, the components

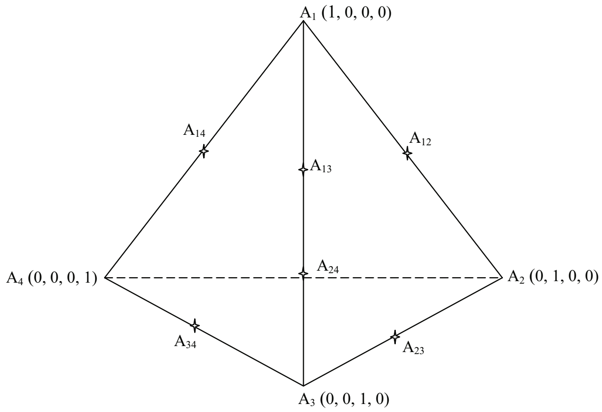

Figure 1: A (4, 2) simplex lattice in pseudo factor space.

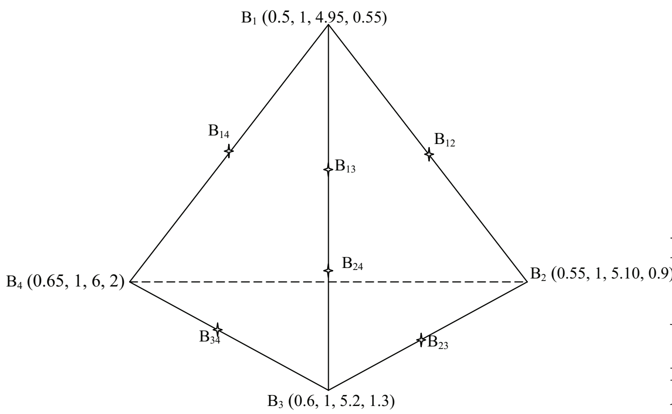

Figure 2: A (4, 2) simplex lattice in actual factor space.

given in terms of the variables x_i are termed "pseudo components". In order to establish the transformation T which for any given actual mix proportions and water-cement ratio gives the corresponding pseudo components and vice versa, we consider a factor space of the real variables Z_i which is also a tetrahedron (see fig. 2). We note that the points A_i, A_{ij} of the factor space in pseudo variables (see fig.1) correspond to the points B_i, B_{ij} respectively in the space of the actual variables.

The experimenter based on his experience and available data in literature should assign co-ordinates to the points $B_i = 1, 2, 3, 4$ in such a manner that the factor space would capture or contain the searched optimum point or the feasible region. For the problem at hand the co-ordinates of point Bi are chosen as follows:

$B_i\,(0.5, 1, 4.95, 0.55)$ $B2(0.55, 1, 5.10, 0.9)$
$B3(0.6, 1, 5.2, 1.3)$ $B4(0.65, 1, 6, 2)$

The first ordinate of each point B_i is the water-cement ratio. Arranging these coordinates in array and obtain the matrices P and Q for the actual and the pseudo components respectively.

$$P = \begin{bmatrix} 0.5 & 0.55 & 0.6 & 0.65 \\ 1 & 1 & 1 & 1 \\ 4.95 & 5.10 & 5.2 & 6 \\ 0.55 & 0.9 & 1.3 & 2 \end{bmatrix} \tag{3}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Consequently, Q and P are related as follows:

$$Q = TP \tag{5}$$

Where T is linear transformation which transforms any given point in the factor space of the actual variable to the factor space of the pseudo variables.

Multiplying both sides of eqn. [5] in P^{-1} and noting that Q is an identity matrix, the linear transformation T is obtained as follows:

$$QP^{-1} = TPP^{-1}$$

Consequently,

$$P^{-1} = T$$

Thus T is obtained on the inverse matrix of P given by

$$T = \begin{bmatrix} -100 & 73 & -6 & 14 \\ 170 & -134.5 & 13 & 27 \\ -60 & 63 & -8 & 12 \\ = 10 & -0.5 & 1 & 1 \end{bmatrix} \tag{6}$$

Consequently, for any given vector

$$Z = \begin{bmatrix} Z_1^{(i)} & Z_2^{(i)} & Z_3^{(i)} & Z_4^{(i)} \end{bmatrix}^T$$

of the ith experimental point, the vector of the corresponding pseudo components is given by

$$\begin{bmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{bmatrix} = \begin{bmatrix} -100 & 73 & -6 & 14 \\ 170 & -134.5 & 13 & 27 \\ -60 & 63 & -8 & 12 \\ = 10 & -0.5 & 1 & 1 \end{bmatrix} \begin{bmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \\ Z_4^{(i)} \end{bmatrix} \tag{7}$$

Conversely, the inverse transformation S that relates P with Q can be obtained as follows

$$P = SQ \tag{8}$$

By multiplying both sides in eqn. [8] by $Q-1$ we obtain that

$$PQ^{-1} = SQQ^{-1} \tag{9}$$

Since Q and $Q-1$ are identity matrices eqn. [8] gives

$$S = P$$

Thus, for any given vector $X = \begin{bmatrix} X_1^{(i)} & X_2^{(i)} & X_3^{(i)} & X_4^{(i)} \end{bmatrix}^T$ of the i^{th} coordinate point, the vector corresponding to actual components $Z = \begin{bmatrix} Z_1^{(i)} & Z_2^{(i)} & Z_3^{(i)} & Z_4^{(i)} \end{bmatrix}^T$ is given as

$$Z = \begin{bmatrix} 0.5 & 0.55 & 0.6 & 0.65 \\ 1 & 1 & 1 & 1 \\ 4.95 & 5.10 & 5.2 & 6 \\ 0.55 & 0.9 & 1.3 & 2 \end{bmatrix} \begin{bmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{bmatrix} \tag{10}$$

Eqn [10] is used to generate the actual mix ratios for the coordinate points A_{ij} and those of the control points tables [1] and [2].

Consequently, for any given vector $Z = \begin{bmatrix} Z_1^{(i)} & Z_2^{(i)} & Z_3^{(i)} & Z_4^{(i)} \end{bmatrix}^T$ of the i^{th} experimental point, the vector of the corresponding pseudo components is given by

3. Materials and methods

The specimen exhibited both vertical and peripheral cracks at failure. Failure occurred within one and a half minutes of load application. The maximum load carried by the specimen during the test was recorded and divided by the net area of the specimen. The compressive strength was obtained from the ratio.

$$Y = (\text{maximum load/cross} - \text{sectional area})\ \text{N/mm}^2$$

The results obtained are as shown in table 4

4. Result and Analysis

Crushing strength (f_c)

$$f_c = \frac{P}{A}$$

where P = The maximum load on the block (N); A = the cross sectional area of the block (mm^2). After determining coefficients,the mathematical model expressing the crushing strength of block as a multivariate function of proportions of its constituent component is given by

$$\begin{aligned} y = {} & 2.08X_1 + 1.29X_2 + 1.58X_3 + 1.09X_4 - \\ & 0.4X_1X_2 - 1.79X_1X_3 - 1.39X_1X_4 + \\ & 0.59X_2X_3 + 1.77X_2X_4 - 0.4X_3X_4 \end{aligned} \tag{11}$$

4.1. Adequacy Test for the Model

A statistical adequacy test for the mathematical model eqn. (11) against the test (control) results from the experiments is necessary. For this the statistical hypothesis is used as follows:

i. Null hypothesis, H_0: There is no significant difference between the experimental and theoretical result.

ii. Alternative hypothesis, H_1: There is a significant difference between the experimental and theoretical results.

4.1.1. Program Testing and Results

A computer programme in turbo C++ is developed. Any desired strength is specified as input. The computer then prints out all possible combinations of the mixes that yield the strength, to a tolerance of +0.001 N/mm^2. Interestingly, should there not be any matching combinations, the computer so informs the user. The program also checks

Table 1: Design matrix for a (4,2) lattice.

S/No	Pseudo Coordinates					Response	Actual Coordinates				
	Coordinate points	X_1	X_2	X_3	X_4	Y_{exp}	**Coordinate points**	Z_1	Z_2	Z_3	Z_4
1	A_1	1	0	0	0	Y_1	B_1	0.5	1	4.95	0.55
2	A_2	0	1	0	0	Y_2	B_2	0.55	1	5.10	0.9
3	A_3	0	0	1	0	Y_3	B_3	0.60	1	5.2	1.3
4	A_4	0	0	0	1	Y_4	B_4	0.65	1	6	2
5	A_{12}	1/2	1/2	0	0	Y_{12}	B_{12}	0.525	1	5.025	0.725
6	A_{13}	1/2	0	1/2	0	Y_{13}	B_{13}	0.55	1	5.075	0.925
7	A_{14}	1/2	0	0	1/2	Y_{14}	B_{14}	0.575	1	5.475	1.275
8	A_{23}	0	1/2	1/2	0	Y_{23}	B_{23}	0.575	1	5.15	1.10
9	A_{24}	0	1/2	0	1/2	Y_{24}	B_{24}	0.6	1	5.55	1.45
10	A_{34}	0	0	1/2	1/2	Y_{34}	B_{34}	0.625	1	5.6	1.65

Table 2: Design matrix for control points of a (4,2) lattice.

S/No	Pseudo Coordinates					Response	Actual Coordinates			
	Coordinate points	X_1	X_2	X_3	X_4	Y_{exp}	Z_1	Z_2	Z_3	Z_4
1	C_1	1/3	1/3	1/3	0	Y_{C1}	0.55	1	5.08	0.91
2	C_2	1/3	1/3	0	1/3	Y_{C2}	0.57	1	5.35	1.15
3	C_3	1/3	0	1/3	1/3	Y_{C3}	0.59	1	5.38	1.28
4	C_4	0	1/3	1/3	1/3	Y_{C4}	0.60	1	5.43	1.40
5	C_5	1/4	1/4	1/4	1/4	Y_{C5}	0.58	1	5.32	1.20
6	C_6	1/2	1/4	0	1/4	Y_{C6}	0.55	1	5.26	1.01

and prints the maximum strength obtainable with the model. Whenever more than one combination of the basic ingredients yields specified input strength, the preference of any combination over the others after results from consideration of cost of constituents materials, availability etc. See Appendix for the program.

An executed program for compressive strength using Scheffe's Model.

```
Enter desired strength 2
For compressive strength using Scheffes model

Count  X1    X2    X3    X4     Y    Z1    Z2    Z3    Z4

1     0.940 0.050 0.000 0.010 2.000 0.504 1.000 4.968 0.342
2     0.940 0.050 0.010 0.000 2.000 0.504 1.000 4.960 0.335
3     0.950 0.030 0.000 0.020 2.000 0.505 1.000 4.976 0.344
4     0.950 0.030 0.010 0.010 2.000 0.504 1.000 4.968 0.337
5     0.960 0.010 0.000 0.030 1.999 0.505 1.000 4.983 0.347
6     0.960 0.010 0.010 0.020 2.000 0.505 1.000 4.975 0.339
7     0.960 0.010 0.020 0.010 2.001 0.504 1.000 4.967 0.332

The Maximum Value Of Strength Predictable By This

Model Is 2.079998 N/sq.mm.
```

5. Conclusion

It has been shown from this work that Scheffes simplex lattice theory for mixture design have been successfully applied in generating a mathematical models for the compres-

Table 3: Results of crushing strength test.

Exp. No (r)	Repetition	Point	wt of Block (g)	Response Yr (N/mm^2)
1	A	y_1	27.1	2.37
	B		26.0	1.78
2	A	y_2	25.6	0.99
	B		25.9	1.58
3	A	y_3	25.2	1.58
	B		26.2	1.58
4	A	y_4	25.0	1.09
	B		25.6	1.09
5	A	y_{12}	25.6	1.09
	B		26.5	1.38
6	A	y_{13}	25.6	1.78
	B		25.0	1.38
7	A	y_{14}	25.8	1.28
	B		25.0	1.19
8	A	y_{23}	25.0	1.78
	B		25.6	1.38
9	A	y_{24}	26.1	1.48
	B		25.6	1.78
10	A	y_{34}	26.0	1.48
	B		24.1	0.99
Control Point				
11	A	C_1	26.0	1.98
	B		27.0	1.68
12	A	C_2	26.4	1.38
	B		26.6	1.19
13	A	C_3	26.5	1.57
	B		25.2	1.09
14	A	C_4	25.2	1.28
	B		26.5	1.48
15	A	C_5	27.0	1.78
	B		25.8	2.07
16	A	C_6	25.8	2.48
	B		26.5	2.07

sive strengths of sandcrete block as a multivariate function of the proportions of its constituents ingredients: water, cement, sand and laterite fines. This goes further to prove that the strength of sandcrete blocks varies with the proportion of the ingredients and not the total amount.

It was also established that the maximum mean strength obtained in Scheffes model is 2.07N/mm^2 which is in agreement with earlier results.

Adequate tests show second degree polynomial can model the response surface with very high degrees of accuracy. The models and their associated computer programs provide efficient tool for fast realization of various mix proportions of the constituent ingredients of sandcrete block for any stipulated strength in a direct manner without trial and error.

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Appendix

```
10 REM A QBasic program that optimises the proportion
of sandcrete mixes
15 REM Scheffes Model for compressive strength
20 REM Variable used:
30 REM Z1, Z2, Z3, Z4, X1, X2, X3, X4, Ymax, Yout, Yin
40 REM begin main program
41 OPEN "MAMABO.BOM" FOR APPEND AS #1
50 LET Count = 0
60 CLS
70 GOSUB 100
71 CLOSE #1
80 END
90 REM End of main program
100 REM Procedure Begin
110 LET Ymax = 0
120 PRINT #1,
130 PRINT #1,
140 PRINT #1, ''MATHEMATICAL MODELS FOR OPTIMIZATION
OF THE MECHANICAL PROPERTIES OF THE SANDCRETE BLOCK
MADE FROM RIVER SAND AND LATERITE''
160 PRINT #1,
170 PRINT #1,
180 INPUT ''ENTER DESIRED STRENGTH''; Yin
185 PRINT #1, ''ENTER DESIRED STRENGTH''; Yin
186 PRINT #1,
187 PRINT #1,
190 GOSUB 400
200 FOR X1 = 0 TO 1 STEP .01
210 FOR X2 = 0 TO 1 - X1 STEP .01
220 FOR X3 = 0 TO 1 - X1 - X2 STEP .01
230 LET X4 = 1 - X1 - X2 - X3
240 LET Yout = 2.08 * X1 + 1.29 * X2 + 1.58 * X3 +
1.09 * X4 - .4 * X1 * X2 - 1.79 * X1 * X3 - 1.39 * X1
* X4 + .59 * X2 * X3 + 1.77 * X2 * X4 - .4 * X3 * X4
250 GOSUB 500
260 IF (ABS(Yin - Yout) <= .001) THEN 270 ELSE 290
270 LET Count = Count + 1
280 GOSUB 600
290 NEXT X3
291 NEXT X2
292 NEXT X1
295 PRINT #1,
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```
300 IF (Count > 0) THEN GOTO 310 ELSE GOTO 340
310 PRINT #1, ``The Maximum Value Of Strength
Predictable By This Model Is ''; Ymax; ``N / sq.mm.'';
``  ''
320 SLEEP (2)
330 GOTO 360
340 PRINT #1, ``Sorry!  Desired Strength Out Of Range
Of Model.''
350 SLEEP 2
360 RETURN
400 REM Procedure Print Heading
410 PRINT #1,
420 PRINT #1, TAB(1); ``Count''; TAB(7); ``X1'';
TAB(15); ``X2''; TAB(23); ``X3''; TAB(31); ``X4'';
TAB(39); ``Y''; TAB(47); ``Z1''; TAB(55); ``Z2'';
TAB(63); ``Z3''; TAB(71); ``Z4''
430 PRINT #1,
440 RETURN
500 REM Procedure Check Max
510 IF Ymax < Yout THEN Ymax = Yout ELSE Ymax = Ymax
520 RETURN
600 REM Procedure Out Results
610 LET Z1 = .5 * X1 + .55 * X2 + .6 * X3 + .65 * X4
620 LET Z2 = X1 + X2 + X3 + X4
630 LET Z3 = 4.95 * X1 + 5.1 * X2 + 5.2 * X3 + 6 * X4
640 LET Z4 = .55 * X1 + .9 * X2 + 1.3 * X3 + 2 * X4
650 PRINT #1, TAB(1); Count; USING ``####.###'';
X1; X2; X3; X4; Yout; Z1; Z2; Z3; Z4
660 RETURN
```