COMPARATIVE STUDY OF VLASOV AND EULER INSTABILITIES OF AXIALLY COMPRESSED THIN-WALLED BOX COLUMNS

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ABSTRACT

Through experimental studies, Vlasov showed that Euler's critical load formula cannot be directly applied to the buckling analysis of thin-walled closed columns. In this study, Vlasov's displacement model with modification by Varbanov and Euler's elastica model were used in a comparative study to determine the flexural buckling strength of single-cell doubly symmetric thin-walled box columns with different boundary conditions. The study involved a theoretical formulation based on Vlasov's theory as modified by Varbanov and implemented the associated displacement model in analyzing flexural buckling modes. Euler's critical load formula was used to solve the same set of problems and the results thereof were compared with those obtained from Vlasov's model. The flexural behaviour showed that for all three sets of boundary conditions considered, the critical load due to flexure about the oz-axis will control design in both models. Comparison with the Euler critical load results showed that Euler's model underestimated the critical buckling load by 67.53% for hinged-hinged, 67.11% for clamped-hinged and 66.11% for clamped-clamped boundary conditions respectively. For bending about the oy-axis, the underestimation ranged from 51.14% for hinged-hinged, 50.33% for clamped-hinged to 48.52% for clamed-clamped boundary conditions. The results show that for single-cell doubly symmetric box columns, the Euler buckling strength should be increased by about 100% to 200% to obtain the Vlasov buckling strength. The actual percentage depends on the axis of symmetry and the boundary conditions under consideration.

Key words: Euler's Model, Flexural Buckling, Single-Cell Section, Thin-Walled Column, Vlasov's Model.

NOTATIONS:

$U_i(x)$: Longitudinal displacements function due to flexure about oy- and oz-axes and warping due to torsion about ox-axis.

$V_k(x)$: Transverse displacements function due to flexure about oy- and oz-axes, torsion about ox-axis, and distortion of the cross-section.

$\varphi_i(s)$: Generalized longitudinal strain fields due to flexure about oy- and oz-axes, and warping torsion about ox-axis.

$\varphi_i'(s)$: First derivative of the longitudinal strain fields with respect to the profile coordinate, S

$_k(s)$: Generalized transverse strain fields due to flexure about oy- and

oz-axes, torsion about ox-axis and distortion of the cross-section

P_{cr}:   Critical buckling load

S: Profile coordinate

E: Modulus of elasticity

G: Modulus of rigidity

(x, s): Shear stress

(x, s): Normal stress

ε_x: Longitudinal strain

$_{xs}$:   Shear strain

I_y: Moment of inertia about the oy – axis

I_z: Moment of inertia about the oz – axis

I :   Warping constant

  :   Warping function

INTRODUCTION

Thin-walled structures consist of a wide and growing field of engineering application which seek efficiency in strength and cost by minimizing material [1]. Multitude of research efforts have recently been invested in thin-walled structures because of the complexity of their behaviour, their natural optimization characteristics and the need for brief but accurate and reliable design methods [2]. Thin-walled closed structures are very economical as structural elements due to their light weight, and their high flexural and torsional rigidity [3-6]. However, owing to the thinness of their walls, these structures appear to have low resistance against buckling, consequently their instability problems need some careful and in-depth study before their reliable design as compression elements can be accomplished [5]. Closed cross-section thin-walled steel columns have at least three competing instability modes: flexural, torsional and distortional buckling [6].

Backing his theory up with experimental studies, Vlasov showed that Euler's critical load formula cannot be directly applied to the buckling analysis of thin-walled closed columns because the formula was derived on the basis of elementary beam theory which does not embrace cross section warping and distortion [7]. Subsequent research efforts have been quoting this finding by Vlasov, but none has studied the level of relationship between flexural buckling strengths obtained from the two models. Strict application of Vlasov's equation for the analysis of thin-walled structures leads to large number of kinematic unknowns in form of displacement functions. As a result of this problem, Varbanov [4] showed that by using generalized strain fields, the number of kinematic unknowns was drastically reduced. Generalized strain fields are strain fields chosen in such a way that they constitute linear combinations of the unit (elementary) strain fields used by Vlasov [8].

The main motivation for this present study is the need to establish simple closed-form relationship between flexural buckling strengths obtained using Vlasov's displacement model as modified by Varbanov and that obtained using Euler's elastic model for single-cell thin-walled box columns. Availability of such simple relationship will enable an accurate estimate of Vlasov's flexural buckling loads for single–cell box columns when a more direct Euler's value is known.

FORMULATION OF THE EQUILIBRIUM EQUATION:
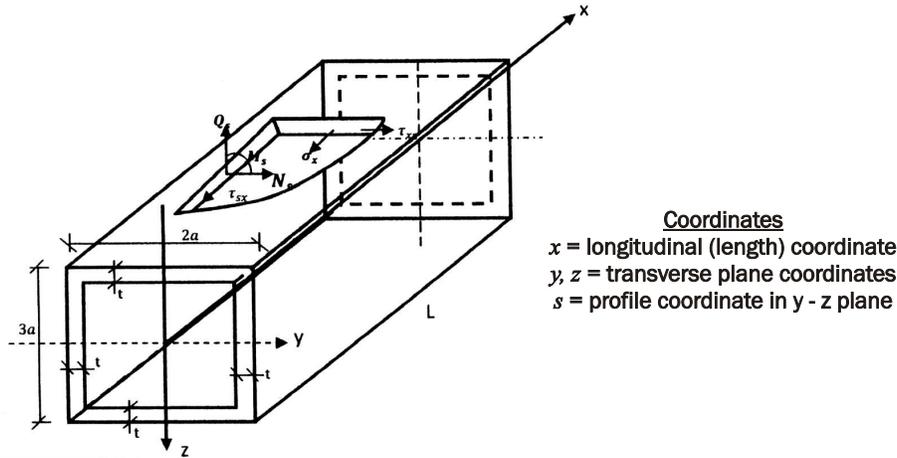
Figure 1:   Axially compressed single-cell thin-walled box column and the associated stress resultants

Figure1 shows an axially compressed thin-walled hollow column, the generated stress resultants and the box cross-section parameters. Using Lagrange's principle, Vlasov [8] expressed the displacements in the longitudinal and transverse directions, $u_{(x,s)}$ and $v_{(x,s)}$ of a thin-walled closed structure in series form as follows:

$$u(x, s) = \sum_{i=1}^{m} U_i(x)\, \varphi_i(s)$$

$$v(x, s) = \sum_{k=1}^{m} V_k(x)\, \psi_k(s)$$

Vlasov's formulation yields $(m + n)$ second order differential equations. Later work by Varbanov [4] showed that m and n can be limited to four by using generalized strain fields. Using equations (1) and (2) and basic stress-strain relations of the theory of elasticity, the expressions for normal and shear stresses become:

$$\sigma(x,s) = E\varepsilon_x = E\sum_{i=1}^{m} U_i'(x)\varphi_i(s)$$

$$\tau(x,s) = G\gamma_{xs} = G\left[\sum_{i=1}^{m} U_i(x)\varphi_i'(s) + \right.$$

$$\left. \sum_{k=1}^{n} V_k'(x)\psi_k(s)\right]$$

The bending moment induced by distortion is given by:

$$M(x, s) = \sum_{k=1}^{m} M_k(s) * V_k(x)$$

The potential energy of an axially loaded thin-walled closed structure is given by: $\qquad \prod_p = S - W$ (6)

For the structure under consideration, the strain energy and work done by the external load are given by:

$$S = \frac{1}{2}\int_L \int_S \left[\left(\sigma_{(x,s)}\varepsilon_{(x,s)} + \tau_{(x,s)}\gamma_{(x,s)}\right)t_{(s)} + \frac{M^2_{(xs)}}{EI}\right]dxds \qquad (7)$$

$$W = \frac{1}{2}\int_L \int_S Pv_{(x,s)}'^2\, dxds$$

(3)

Substituting equations (7) and (8) into equation (6) and simplifying, we obtained the

total potential energy functional as:

$$\pi_p = \frac{1}{2}\int_L \left\{ E\sum_{i=1}^{m} \sum_{j=1}^{m} a_{ij} U_i(x) U_j(x) + \right.$$

$$G\sum_{i=1}^{m} \sum_{j=1}^{m} b_{ij} U_i(x) U_j(x) +$$

$$+ G\sum_{i=1}^{m} \sum_{r=1}^{n} c_{ir} U_i(x) V_r'(x) +$$

$$G\sum_{j=1}^{m} \sum_{k=1}^{n} c_{jk} U_j(x) V_k'(x) +$$

$$+ G\sum_{k=1}^{m} \sum_{r=1}^{n} m_{kr} V_k'(x) V_r'(x) +$$

$$E\sum_{k=1}^{n} \sum_{r=1}^{n} s_{kr} V_k(x) V_r(x) -$$

$$\left. P\sum_{k=1}^{n} \sum_{r=1}^{n} h_{kr} V_k'(x) V_r'(x) \right\} dx$$

where

$$\left.\begin{array}{l} a_{ij} = \int_S \varphi_i(s)\varphi_j(s)t(s)\,ds \\[2mm] b_{ij} = b_{ji} = \int_S \varphi_i'(s)\varphi_j'(s)t(s)\,ds \\[2mm] c_{ir} = c_{ri} = \int_S \varphi_i'(s)\psi_r(s)t(s)\,ds \\[2mm] c_{jk} = c_{kj} = \int_S \varphi_j'(s)\psi_k(s)t(s)\,ds \\[2mm] m_{kr} = m_{rk} = \int_S \psi_k(s)\psi_r(s)t(s)\,ds \\[2mm] h_{kr} = h_{rk} = \int_S \psi_k(s)\psi_r(s)\,ds \\[2mm] s_{kr} = s_{rk} = \frac{1}{E}\int_S \frac{M_k(s)M_r(s)}{EI}\,ds \end{array}\right\}$$

The total potential energy functional p_p has stationary values if the following Euler-Lagrange differential equations are satisfied:

$$\frac{\partial F}{\partial U_j} - \frac{d}{dx}\left(\frac{\partial F}{\partial U_j'}\right) = 0$$

$$\frac{\partial F}{\partial V_r} - \frac{d}{dx}\left(\frac{\partial F}{\partial V_j'}\right) = 0$$

Where,

$$\pi_p = F\left(U_i, U_j, V_k, V_r, U_i', U_j', V_k', V_r'\right)$$

(13)

Using equations (11) and (12) on equation (9) and noting that for the thin-walled closed column under consideration, m=3 and n=4, we obtained the governing equations of equilibrium as:

$$\gamma\sum_{i=1}^{3} \sum_{j=1}^{3} a_{ij} U_i''(x) - \sum_{i=1}^{3} \sum_{j=1}^{3} b_{ij} U_i(x)$$

$$- \sum_{j=1}^{3} \sum_{k=1}^{4} c_{jk} V_k'(x) = 0$$

(9)

$$\sum_{i=1}^{3} \sum_{r=1}^{4} c_{ir} U_i'(x) + \sum_{k=1}^{4} \sum_{r=1}^{4} k_{kr} V_k'' -$$

$$\gamma\sum_{k=1}^{4} \sum_{r=1}^{4} s_{kr} V_k(x) = 0$$

Where, $\quad k_{kr} = \left(m_{kr} - \frac{P}{G}h_{kr}\right)$

GENERATION OF THE STRAIN MODES AND DETERMINATION OF COEFFICIENTS OF THE GOVERNING EQUATIONS OF EQUILIBRIUM:
Considering the nature of loading (axial compression), the longitudinal strain modes $\varphi_{i(s)}$ consist of bending about y- and z-axes, and warping in the longitudinal direction. The functions $\varphi_{i(s)}$ are chosen in the forms:

$\varphi_{1(s)} = y_{(s)}$; $\varphi_{2(s)} = z_{(s)}$; $\varphi_{3(s)} = $ $\quad M_{(s)}$ (16)
The transverse strain modes $\quad_{k(s)}$ consist of bending about y- and z-axes, pure rotation about the longitudinal x-axis and distortion of the cross section. The functions $\quad_{k(s)}$ are defined as follows:

(11)

$$\psi_{1(s)} = \varphi'_{1(s)} = y'_{(s)}; \ \psi_{2(s)} = \varphi'_{2(s)} = z'_{(s)}; \qquad \psi_{3(s)} = h'_{(s)}; \ \psi_{4(s)} = \varphi'_{3(s)} = \omega'_{M(s)}$$

(17)

PURE BENDING ABOUT O-Y AXIS   PURE BENDING ABOUT O-Z AXIS   WARPING STRAIN MODE
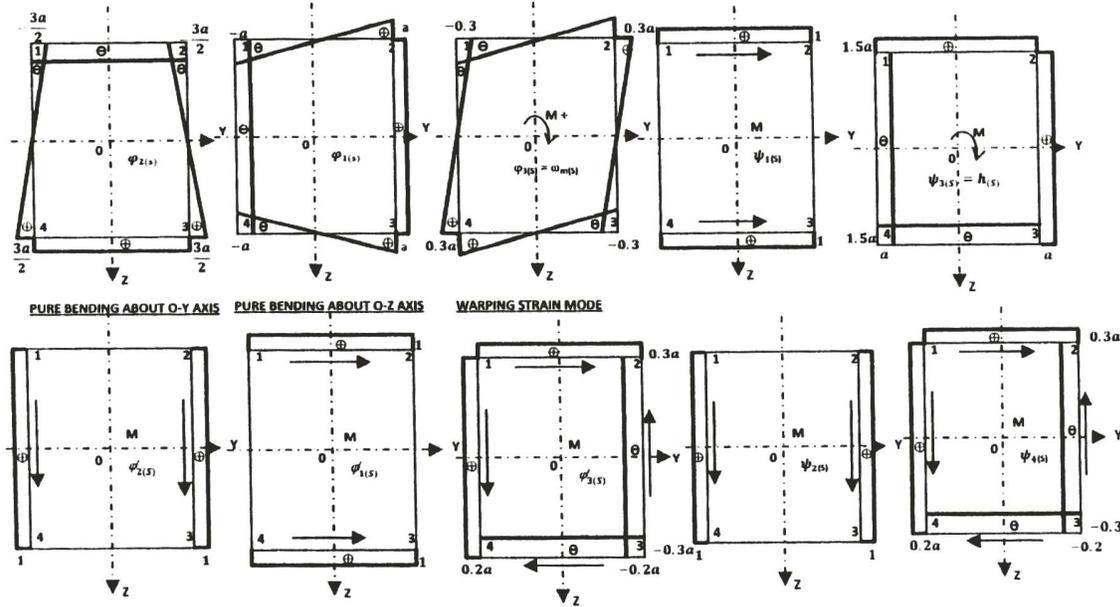
$\varphi_{1(s)}, \ \varphi'_{1(S)}$ and $\psi_{1(S)}$ Due to flexure about oz-axis

$\varphi_{2(s)}, \varphi'_{2(S)}$ and $\psi_{2(s)}$ Due to flexure about oy-axis

$\varphi_{3(s)}, \ \varphi'_{3(S)}$ and $\psi_{4(s)}$ Due to torsion about ox-axis

$\psi_{3(s)} = h(s)$ Due to pure rotation about ox-axis

$M_{4(S)}$, Moment generated by distortion
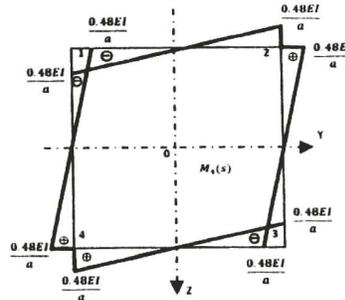
The elements of the coefficient matrix of the governing differential equations of equilibrium were determined for the cross sections by first generating and plotting the strain fields as shown in figure 2. Diagram multiplication technique was then used in determining the elements from the strain mode diagrams as follows:

Fig. 2. Generalized strain fields for single-cell doubly symmetric section

$a_{ij} = a_{ji} = \int_s \varphi_i(s)\varphi_j(s)\, t(s)\, ds$

$a_{11} = \int_s \varphi_1(s)\varphi_1(s)\, t(s)\, ds = 7.333a^3t$

$a_{22} = \int_s \varphi_2(s)\varphi_2(s)\, t(s)\, ds = 13.5a^3t$

$a_{12} = a_{21} = \int_s \varphi_1(s)\varphi_2(s)\, t(s)\, ds = 0$

$a_{13} = a_{31} = \int_s \varphi_1(s)\varphi_3(s)\, t(s)\, ds = 0$

$a_{23} = a_{32} = \int_s \varphi_2(s)\varphi_3(s)\, t(s)\, ds = 0$

$a_{33} = \int_s \varphi_3(s)\varphi_3(s)\, t(s)\, ds = 0.3a^5t$

$b_{ij} = b_{ji} = \int_s \varphi_i'(s)\varphi_j'(s)\, t(s)\, ds$

$b_{11} = \int_s \varphi_i'(s)\varphi_1'(s)\, t(s)\, ds = 4at$

$b_{12} = b_{21} = \int_s \varphi_1'(s)\varphi_2'(s)\, t(s)\, ds = 0$

$b_{13} = b_{31} = \int_s \varphi_1'(s)\varphi_3'(s)\, t(s)\, ds = 0$

$b_{22} = \int_s \varphi_2'(s)\varphi_2'(s)\, t(s)\, ds = 6at$

$b_{23} = b_{32} = \int_s \varphi_2'(s)\varphi_3'(s)\, t(s)\, ds = 0$

$b_{33} = \int_s \varphi_3'(s)\varphi_3'(s)\, t(s)\, ds = 0.6a^3t$

$c_{ir} = c_{ri} = \int_s \varphi_i'(s)\,_r(s)\, t(s)\, ds$

$c_{11} = \int_s \varphi_1'(s)\,_1(s)\, t(s)\, ds = 4at$

$c_{12} = c_{21} = \int_s \varphi_1'(s)\,_2(s)\, t(s)\, ds = 0$

$c_{13} = c_{31} = \int_s \varphi_1'(s)\,_3(s)\, t(s)\, ds = 0$

$c_{14} = \int_s \varphi_1'(s)\,_4(s)\, t(s)\, ds = 0$

$c_{22} = \int_s \varphi_2'(s)\,_2(s)\, t(s)\, ds = 6at$

$c_{23} = \int_s \varphi_2'(s)\,_3(s)\, t(s)\, ds = 0$

$c_{24} = \int_s \varphi_2'(s)\,_4(s)\, t(s)\, ds = 0$

$c_{33} = \int_s \varphi_3'(s)\,_3(s)\, t(s)\, ds = 0.6a^3t$

$c_{34} = \int_s \varphi_3'(s)\,_4(s)\, t(s)\, ds = 0.6a^3t$

$m_{kr} = m_{rk} = \int_s \,_k(s)\,_r(s)\, t(s)\, ds$

$m_{11} = \int_s \,_1(s)\,_1(s)\, t(s)\, ds = 4at$

$m_{12} = m_{21} = \int_s \,_1(s)\,_2(s)\, t(s)\, ds = 0$

$m_{13} = m_{31} = \int_s \,_1(s)\,_3(s)\, t(s)\, ds = 0$

$m_{14} = m_{41} = \int_s \,_1(s)\,_4(s)\, t(s)\, ds = 0$

$m_{22} = \int_s \,_2(s)\,_2(s)\, t(s)\, ds = 6at$

$m_{23} = m_{32} = \int_s \,_2(s)\,_3(s)\, t(s)\, ds = 0$

$m_{24} = m_{42} = \int_s \,_2(s)\,_4(s)\, t(s)\, ds = 0$

$m_{33} = \int_s \,_3(s)\,_3(s)\, t(s)\, ds = 15a^3t$

$m_{34} = \int_s \,_3(s)\,_4(s)\, t(s)\, ds = 0.6a^3t$

$m_{44} = \int_s \,_4(s)\,_4(s)\, t(s)\, ds = 0.6a^3t$

$h_{kr} = h_{rk} = \int_s \,_k(s)\,_r(s)\, ds$

$h_{11} = \int_s \phi_1(s)\phi_1(s)ds = m_{11}/t = 4a$

$h_{12} = h_{21} = \int_s \phi_1(s)\phi_2(s)ds = 0$

$h_{13} = h_{31} = \int_s \phi_1(s)\phi_3(s)ds = 0$

$h_{14} = h_{41} = \int_s \phi_1(s)\phi_4(s)ds = 0$

$h_{22} = \int_s \phi_2(s)\phi_2(s)ds = 6a$

$h_{23} = h_{32} = \int_s \phi_2(s)\phi_3(s)ds = 0$

$h_{24} = h_{24} = \int_s \phi_2(s)\phi_4(s)ds = 0$

$h_{33} = \int_s \phi_3(s)\phi_3(s)ds = 15a^3$

$h_{34} = \int_s \phi_3(s)\phi_4(s)ds = 0.6a^3$

$h_{44} = \int_s \phi_4(s)\phi_4(s)ds = 0.6a^3$

$s_{kr} = s_{rk} = \dfrac{1}{E}\int_S \dfrac{M_k(s)M_r(s)}{EI}ds$

$s_{44} = \dfrac{1}{E}\int_S \dfrac{M_4(s)M_4(s)}{EI}ds = \dfrac{0.768It}{a}$

But, $I = t^3/12$ for all the plates

$\Rightarrow s_{44} = \dfrac{0.768t}{a} * \dfrac{t^3}{12} = \dfrac{0.064t^4}{a}$

DERIVATION OF INDEPENDENT EQUATIONS IN LATERAL DISPLACEMENT QUANTITIES $V_K(x)$:
Substituting the zero coefficients as obtained into the matrix form of the governing equations of equilibrium (14 & 15), and assuming the cross-section to be rigid (non-deformable), we obtained:

$$\gamma \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} U_1'' \\ U_2'' \\ U_3'' \end{bmatrix}$$

$$- \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$- \begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{22} & 0 \\ 0 & 0 & c_{33} \end{bmatrix} \begin{bmatrix} V_1' \\ V_2' \\ V_3' \end{bmatrix} = 0$$

$$\begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{22} & 0 \\ 0 & 0 & c_{33} \end{bmatrix} \begin{bmatrix} U_1' \\ U_2' \\ U_3' \end{bmatrix} + \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{bmatrix} \begin{bmatrix} V_1'' \\ V_2'' \\ V_3'' \end{bmatrix} = 0$$

(19)

Where, $k_{11} = \left(m_{11} - \dfrac{P}{G}h_{11} \right)$;

$k_{22} = \left(m_{22} - \dfrac{P}{G}h_{22} \right)$;   etc.

Expanding equation (18), we obtained:

$$\left.\begin{array}{l} \gamma a_{11} U_1'' - b_{11} U_1 - c_{11} V_1' = 0 \\[2mm] \gamma a_{22} U_2'' - b_{22} U_2 - c_{22} V_2' = 0 \\[2mm] \gamma a_{33} U_3'' - b_{33} U_3 - c_{33} V_3' = 0 \end{array}\right\}$$

Expanding equation (19), we obtained:

$$\left.\begin{array}{l} c_{11} U_1' + k_{11} V_1'' = 0 \\[2mm] c_{22} U_2' + k_{22} V_2'' = 0 \\[2mm] c_{33} U_3' + k_{33} V_3'' = 0 \end{array}\right\}$$

Eliminating $U_i(x)$ and their derivatives from equations (20) and (21), we obtained:

$$V_1^{iv} + a_{11}^2 V_1'' = 0$$

$$V_2^{iv} + a_{22}^2 V_2'' = 0$$

$$V_3^{iv} + a_{33}^2 V_3'' = 0$$

Where, $a_{11}^2 = \left[\dfrac{c_{11}^2 - b_{11} k_{11}}{\gamma a_{11} k_{11}}\right]$;

$$a_{22}^2 = \left[\dfrac{c_{22}^2 - b_{22} k_{22}}{\gamma a_{22} k_{22}}\right];$$

$$a_{33}^2 = \left[\dfrac{c_{33}^2 - b_{33} k_{33}}{\gamma a_{33} k_{33}}\right]$$

The independence of the above three equations shows that there are three possible independent buckling modes, namely: flexural buckling about oz- and oy-axes (22) and (23) and torsional buckling mode about

ox-axis (24).The general solution of the flexural modes (22) and (23) are given by:

$$V_{1(x)} = C_1 \cos_{11} x + C_2 \sin_{11} x + \qquad + C_3 x + C_4 \qquad (20) \qquad (25)$$

$$V_{2(x)} = C_1 \cos_{22} x + C_2 \sin_{22} x \qquad + C_3 x + C_4 \qquad (26)$$

The constants C_1, C_2, C_3 and C_4 were evaluated from the boundary conditions as follows:

(i)   Hinged-Hinged condition:

$$V_1 = V_2 = 0(x = 0, l);$$

$$\frac{d^2 V_1}{dx^2} = \frac{d^2 V_2}{dx^2} = 0(x = 0, l)$$

(ii)   Clamped–Hinged condition:

$$\left.\begin{array}{l} V_1 = V_2 = 0(x = 0, l); \\[3mm] \dfrac{dV_1}{dx} = \dfrac{dV_2}{dx} = 0(x = 0) \\[3mm] \dfrac{d^2 V_1}{dx^2} = \dfrac{d^2 V_2}{dx^2} = 0(x = l) \end{array}\right\}$$

(iii)   Clamped–Clamped condition:

$$V_1 = V_2 = 0;$$

$$\frac{dV_1}{dx} = \frac{dV_2}{dx} = 0(x = 0, l)$$

Applying the boundary conditions (27), (28) and (29) to equations (25) and (26) and noting that for nontrivial solutions or nonzero values of the constants, the determinant of the coefficients of C_1 ... C_4 must varnish, we obtained the following:

(i)   Hinged-Hinged conditions:-

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ \cos\alpha_{11}l & \sin\alpha_{11}l & l & 1 \\ \cos\alpha_{11}l & \sin\alpha_{11}l & 0 & 0 \end{bmatrix} = 0$$

$$\text{and} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ \cos\alpha_{22}l & \sin\alpha_{22}l & l & 1 \\ \cos\alpha_{22}l & \sin\alpha_{22}l & 0 & 0 \end{bmatrix}$$

(ii)   Clamped–Hinged conditions:

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \alpha_{11} & 1 & 0 \\ \cos\alpha_{11}l & \sin\alpha_{11}l & l & 1 \\ \cos\alpha_{11}l & \sin\alpha_{11}l & 0 & 0 \end{bmatrix} = 0$$

$$\text{and} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \alpha_{22} & 1 & 0 \\ \cos\alpha_{22}l & \sin\alpha_{22}l & l & 1 \\ \cos\alpha_{22}l & \sin\alpha_{22}l & 0 & 0 \end{bmatrix} = 0$$

(iii)   Clamped-Clamped conditions:

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \alpha_{11} & 1 & 0 \\ \cos\alpha_{11}l & \sin\alpha_{11}l & l & 1 \\ -\alpha_{11}\sin\alpha_{11}l & a_{11}\cos\alpha_{11}l & 1 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \alpha_{22} & 1 & 0 \\ \cos\alpha_{22}l & \sin\alpha_{22}l & l & 1 \\ -\alpha_{22}\sin\alpha_{22}l & a_{22}\cos\alpha_{22}l & 1 & 0 \end{bmatrix} = 0$$

...(32)

Equations (30), (31) and (32) represent the stability matrices for equations (22) and (23) for the two axes of symmetry and the different boundary conditions. Expanding equations (30), (31), and (32), we obtained the critical buckling loads for the respective boundary conditions as follows:

(i)

$$P_{cr} = \left| m_{11} - \frac{c_{11}^2}{\dfrac{n^2\pi^2}{l^2}*\gamma a_{11}+b_{11}} \right| \frac{G}{h_{11}}$$

(ii)

$$P_{cr} = \left| m_{11} - \frac{c_{11}^2}{\dfrac{20.19}{l^2}*\gamma a_{11}+b_{11}} \right| \frac{G}{h_{11}}$$

(iii)

$$P_{cr} = \left| m_{11} - \frac{c_{11}^2}{\dfrac{4n^2\pi^2}{l^2}*\gamma a_{11}+b_{11}} \right| \frac{G}{h_{11}}$$

(i)

$$P_{cr} = \left| m_{22} - \frac{c_{22}^2}{\dfrac{n^2\pi^2}{l^2}*\gamma a_{22}+b_{22}} \right| \frac{G}{h_{22}}$$

(ii)

$$P_{cr} = \left[m_{22} - \frac{c_{22}^2}{\frac{20.19}{l^2} * \gamma a_{22} + b_{22}} \right] \frac{G}{h_{22}}$$

(iii)

$$P_{cr} = \left[m_{22} - \frac{c_{22}^2}{\frac{4n^2\pi^2}{l^2} * \gamma a_{22} + b_{22}} \right] \frac{G}{h_{22}}$$

NUMERICAL STUDY

A numerical study was performed for single-cell thin-walled steel box column with the following parameters:

$E = 210 \times 10^3 MN/m^2$, $G = 81 \times 10^3 MN/m^2$, $L = 4.5m$, $a = 0.08m$, and $t = 0.0005m$ to $0.02m$.

The critical loads associated with the two flexural buckling modes were evaluated for the different boundary conditions and the results presented on tables 1 and 2.

THE EULER BUCKLING LOAD EQUIVALENT:

The Euler's buckling load is given by:

$$P_{cr} = \frac{\pi^2 EI}{l_{eff}^2}, \qquad l_{eff} = \text{effective length}$$

Bending about oz-axis: $I_z = a_{11} = 7.333a^3t$, $a = 0.08m$, $L = 4.5m$

Hinged-hinged condition; $l_{eff} = L = 4.5m$;

$$\therefore P_{cr} = \frac{\pi^2 EI}{l_{eff}^2} = 384.278t$$

Clamped-hinged condition $l_{eff} = 0.7L$;
$$\therefore P_{cr} = 784.241t$$
Clamped-clamped condition; $l_{eff} = 0.5L$;
$$\therefore P_{cr} = 1537.112t$$

Bending about oy-axis: $I_y = 13.5a^3t$
Hinged-hinged condition:
$$P_{cr} = 707.453t$$
Clamped–hinged condition:
$$\therefore P_{cr} = 1443.782t$$
Clamped–clamped condition:
$$\therefore P_{cr} = 2829.813t$$

Again, the critical loads associated with the two flexural buckling modes were evaluated for the different boundary conditions and the results presented on tables 1 and 2.

RESULTS AND DISCUSSION

The results as presented on tables (1 and 2) show that the flexural buckling load about oz-axis is less than that about oy-axis. Hence, the flexural behaviour shows that for all the three sets of boundary conditions considered, the critical load due to flexure about the oz-axis will control the design. Comparison of Vlasov critical load values in table 1 with the Euler critical load results showed that the Euler model underestimated the critical buckling loads by 67.53% for hinged-hinged, 67.11% for clamped-hinged and 66.11% for clamped-clamped boundary conditions under flexure about the oz-axis. However, under flexure about oy-axis, the percentages of underestimation by Euler model were reduced to 51.14% for hinged-hinged, 50.33% for clamped-hinged and 48.52% for clamped-clamped boundary conditions respectively. In both axes, the results also indicate that improved fixity reduces the degree of underestimation by Euler model.

The results generally show that for single-cell doubly symmetric box columns, the Euler buckling strength could be used to estimate the Vlasov flexural buckling strength by increasing the Euler value by about 100% to 200% depending on the axis of symmetry and the boundary conditions.

CONCLUSION

This study has resulted in a better understanding of the level of difference between the more accurate Vlasov critical loads for single-cell thin-walled box columns and the underestimated Euler critical loads. The results confirms that Euler's critical load model is indeed inadequate for instability design of thin-walled box columns. A critical comparison of the results show that the simpler Euler flexural buckling loads can now be used for making a reasonable estimate of the more accurate and more rigorous Vlasov flexural buckling value.

Table 1: Vlasov and Euler critical buckling loads for single-cell doubly symmetric thin-walled box column under the different boundary conditions and bending about oz-axis

t(m)	Hinged-Hinged (MN)		Clamped-Hinged (MN)		Clamped-Clamped (MN)	
	Pcr(Vlasov)	Pcr (Euler)	Pcr(Vlasov)	Pcr (Euler)	Pcr(Vlasov)	Pcr (Euler)
0.02	23.673	7.686	47.686	15.685	90.714	30.742
0.0175	20.713	6.725	41.725	13.724	79.374	26.899
0.015	17.754	5.764	35.764	11.764	68.035	23.057
0.0125	14.795	4.803	29.803	9.803	56.696	19.214
0.01	11.836	3.843	23.843	7.842	45.357	15.371
0.0075	8.877	2.882	17.882	5.882	34.018	11.283
0.005	5.918	1.921	11.921	3.921	22.678	7.686
0.0025	2.959	0.961	5.961	1.961	11.339	3.843
0.001	1.184	0.384	2.384	0.784	4.536	1.537
0.00075	0.888	0.288	1.788	0.588	3.402	1.153
0.0005	0.592	0.192	1.192	0.392	2.263	0.769

Table 2: Vlasov and Euler critical buckling loads for single-cell doubly symmetric thin-walled box column under the different boundary conditions and bending about oY-axis

	Hinged-Hinged (MN)		Clamped-Hinged (MN)		Clamped-Clamped (MN)	
t(m)	Pcr(Vlasov)	Pcr (Euler)	Pcr(Vlasov)	Pcr (Euler)	Pcr(Vlasov)	Pcr (Euler)
0.02	28.958	14.149	58.137	28.876	109.963	56.596
0.0175	25.338	12.380	50.87	25.266	96.194	49.522
0.015	21.718	10.612	43.603	21.657	82.452	42.447
0.0125	18.099	8.843	36.335	18.047	68.710	35.373
0.01	14.479	7.075	29.068	14.438	54.968	28.298
0.0075	10.859	5.306	21.801	10.828	41.226	21.224
0.005	7.239	3.537	14.534	7.219	27.484	14.149
0.0025	3.620	1.769	7.267	3.609	13.742	7.075
0.001	1.448	0.707	2.907	1.444	5.497	2.830
0.00075	1.086	0.531	2.180	1.083	4.123	2.122
0.0005	0.724	0.354	1.453	0.722	2.748	1.415

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