THE USE OF THE DYNAMIC MAGNIFICATION FACTOR IN THE DYNAMIC ANALYSIS OF FRAMED STRUCTURES

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ABSTRACT

This paper examined the use of the dynamic magnification factor in the analysis of framed structures. It is a method of practice in dynamic analysis of structures to magnify static response by a dynamic magnification factor in order to obtain the equivalent dynamic response. This method has been applied to the dynamic analysis of many structures including bridges and some country's codes of practice made specifications in respect of the dynamic magnification factor for the analysis and design of various types of structures subjected to dynamic excitation. The suitability of this method to the dynamic analysis of frames was investigated in this paper by carrying out static and dynamic analysis of four frames using the flexible frame model and the stiffness formulation. Dynamic responses were first obtained by direct analysis as solutions to the set of equations governing the motions of the frames and secondly by the magnification of the static responses using the dynamic magnification factors. By comparing the results obtained in both methods it was inferred that the practice of magnifying static responses to obtain their dynamic equivalents in frame analysis gives correct results only in the case of deflections and not in stresses. Finally, this practice should de discouraged or limited only to the case of deflections in the dynamic analysis of framed structures.

1.0 INTRODUCTION

Dynamic magnification factor is defined as the ratio of the dynamic deflection at any time to the static deflection which would have resulted from the static application of the external load, which is used in specifying the load-time variation [1-3]. Dynamic magnification factor has been variously referred to as Dynamic Load Factor [1,4] Impact Factor [4], Mechanical Admittance Function [5], Dynamic Magnifier [6], or Dynamic Ratio [7]. In practice the Dynamic Magnification Factor is computed as the ratio of the maximum dynamic deflection to the maximum static deflection [8] or as a function of the maximum frequency ratio [7]. The dynamic magnification factor as presented here assumes that the motion of the structure is simple harmonic.

The notion that static responses could be amplified by a dynamic magnification factor to obtain the equivalent dynamic response was investigated in this paper by carrying out static and dynamic analyses of four frames using the stiffness formulation and the flexible frame model. The dynamic magnification factors were computed. Dynamic responses (i.e. Bending, Movement, Shear Force, Axial Force, and Joint Displacement) obtained from direct forced vibration analysis using equation of motion for forced vibration and that obtained by using the dynamic magnification factor were tabulated and compared.

Previous work by Ezeokpube [6]

showed that the dynamic magnification factor of frames decreases with increase in joint stiffening. Also Smith [4] established that the dynamic magnification factor decreases with span. Lee [9] remarked that design codes often present dynamic magnification factors as formulae related to span. For example the United kingdom code [10] presented dynamic magnification factors, related to spans, by which the static bending moments must be multiplied in order to obtain the dynamic bending moment Smith [4].

2.0 THE DYNAMIC MAGNIFICATION FACTOR IN FRAMES

By definition the dynamic magnification factor is given by

$$\rho = \frac{\text{Maximum Dynamic Displacement}}{\text{Maximum Static Displacement}} \tag{1}$$

For SDOF (Single Degree of Freedom) Frames the Dynamic Magnification Factor could be expressed using Equation (1) or as a function of the frequency ratio

$$\rho = \frac{X}{\Delta} \tag{2}$$

or $\rho = 1/(1 - \theta^2/\omega^2)$ …(3)

where, X = The Amplitude of Joint Displacement due to Force Vibration

Δ = Maximum Static Joint Displacement

θ = Forcing Frequency

ω = Natural Frequency

For MDOF (Many Degrees of Freedom) Frames the dynamic magnification factor become

$$\rho_i = \frac{X_i}{\Delta_i}$$

where the subscript i indicates the floor level under consideration.

3.0 PROCEDURE FOR DYNAMIC ANALYSIS USING THE FLEXIBLE FRAME MODEL WITH STIFFNESS FORMULATION.

Using the flexible frames model and the stiffness method the procedure for the dynamic analysis of MDOF Frames is as follows,

1) The dynamic degrees of freedom n is first, determined. This is equal to the number of the lumped masses.
2) The structure is idealized into a conjugate (or fundamental) system with horizontal translational restrictions (i.e. imaginary supports introduced at the points of the lumped masses).
3) Bending Moment diagrams, M_k (for k=1, 2, 3, ...n) due to displacements X_i applied at the imaginary supports of the conjugate systems are drawn. From the bending moment diagram reactions K_{ij} are determined at the imaginary supports. The complete set of these reactions K_{ij} form the stiffness matrix while that of the displacements X_j form the displacement vector.
4) Using the lumped masses (m_1, m_2, ..m_n) the forces of inertia are then introduced on the elements of the leading diagonal of the stiffness matrix to form the matrix equation for the equation of motion for free vibration.

Thus

$$\begin{bmatrix} k_{11}^* & k_{12} & k_{13} & \cdots & \cdots & \cdots & k_n \\ k_{21} & k_{22}^* & k_{23} & \cdots & \cdots & \cdots & k_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ k_{n1} & k_{n2} & k_{n3} & & & & k_{nn}^* \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdots \\ \cdots \\ \cdots \\ X_n \end{bmatrix} = 0 \tag{4}$$

where, $k^*_{ii} = k_{ii} - m_i\omega^2$

Equation (5) is solved as an eigenvalue problem in order to determine the natural frequencies ($\omega_1, \omega_2, \ldots, \omega_n$)

5) Bending Moment diagram Mp, of the conjugate system due to external load (lateral loads and gravity loads) is also drawn. From the bending moment diagrams the reaction R_{ip} (for i = 1, 2, 3, … n) are determined at the points of imaginary supports of the conjugate system. These reactions form the load vector for the equation of Motion due to Forced Vibration. Thus

$$\begin{bmatrix} k^*_{11} & k_{12} & k_{13} & \cdots & \cdots & \cdots & k_n \\ k_{21} & k^*_{22} & k_{23} & \cdots & \cdots & \cdots & k_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ k_{n1} & k_{n2} & k_{n3} & & & & k^*_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdots \\ \cdots \\ \cdots \\ X_n \end{bmatrix} + \begin{bmatrix} R_{1p1} \\ R_{2p} \\ \cdots \\ \cdots \\ \cdots \\ R_{np} \end{bmatrix} = 0 \quad \ldots(6)$$

where, $K^*_{ii} = K_{ii} - m_1\omega^2$

θ = Forcing Frequency

6) The Equation of motion for Forced Vibration, equation (6), is then solved as simultaneous linear equation to yield the amplitudes of displacement (i.e. $X_1, X_2, X_3, \ldots, X_n$)

7) Bending moment, shear force and axial force due to forced vibration are determined using the following relations:

Bending Moment, $$M = \sum_{k=1}^{n} M_k X_k + M_p$$

Shear Force, $$Q = \sum_{k=1}^{n} Q_k X_k + Q_p$$

Axial Force, $$N = \sum_{k=1}^{n} N_k X_k + N_p$$

4.0 CASE STUDY

It is required to carry out the static and dynamic analysis of the frames (Fig 1 to Fig. 4) loaded as shown and tabulate the response information for joint 1 of member 1-A in all cases. The forcing frequencies are given in Table 3. EI = 3×10^4 KNm^2. Acceleration due to gravity, g = 9.81×10^{-3}ms^{-2}

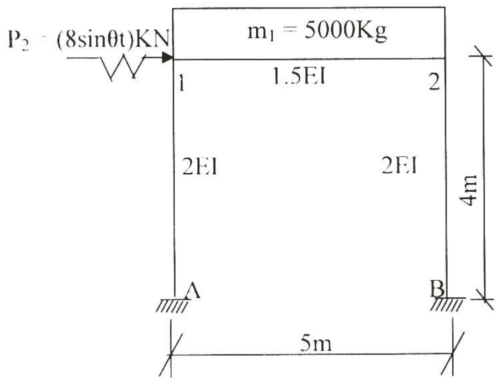

Fig.1: Frame 1

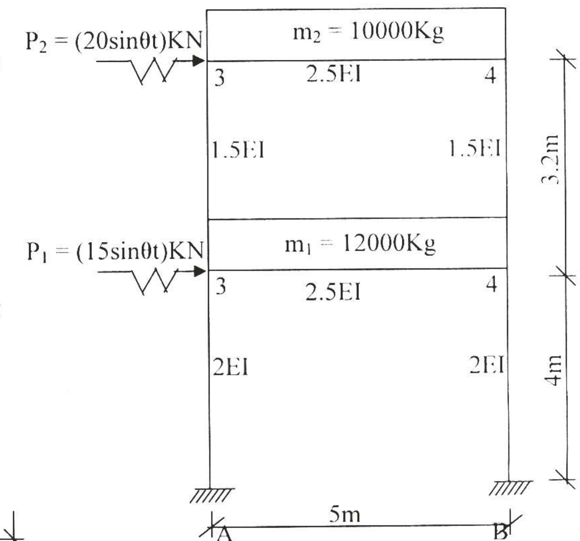

Fig.2: Frame 2

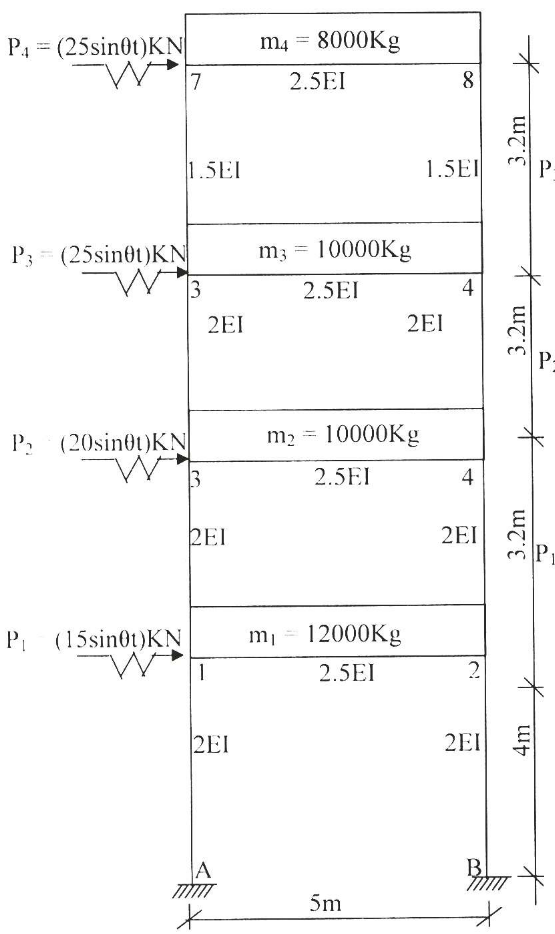

Fig.4: Frame 4

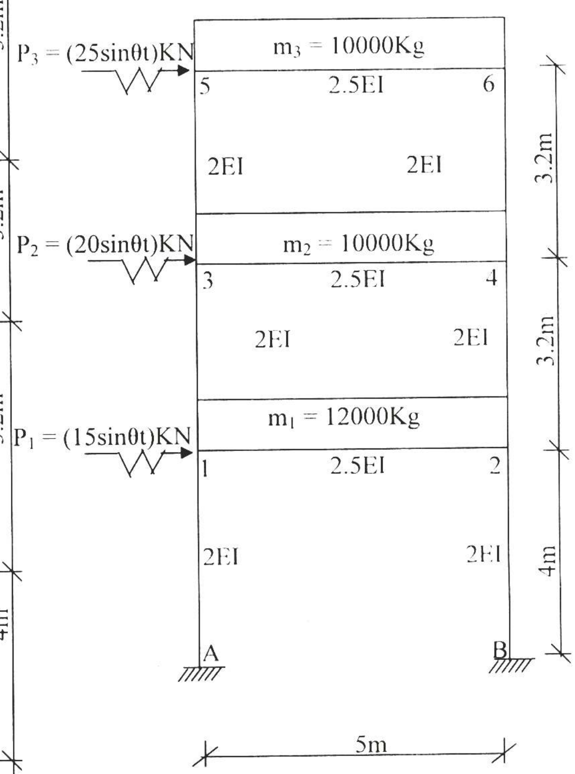

Fig.3: Frame 3

5.0 RESULTS OF ANALYSIS

Table 1. Static Response

Frame	Joint displacement	Binding Moment (Knm)	Shear Force (KN)	Axial Force (KN)
Frame 1	0.50	9.82	-2.27	-21.95
Frame 2	2.46	-12.57	11.64	-41.30
Frame 3	4.41	-30.67	23.90	-93.38
Frame 4	6.38	-49.80	36.58	-77.47

Table 2. Free-Vibration Analysis

Frame	Natural Frequencies ($\sqrt{EI} \times 10^{-3}$rad/sec)			
	ω_1	ω_2	ω_3	ω_4
Frame 1	9.909	-	-	-
Frame 2	4.042	14.466	-	-
Frame 3	2.922	9.173	17.571	-
Frame 4	2.360	7.203	13.053	19.064

Table 3. Dynamic Response Using Forced Vibration Analysis

Frame	Forcing Frequency, θ (↓ EI×10^{-3} rad/sec)	Joint displacement	Binding Moment (Knm)	Shear Force (KN)	Axial Force (KN)
Frame 1	5.00	0.67	7.94	-1.08	-21.19
Frame 2	3.03	5.61	-48.45	33.95	-52.82
Frame 3	2.50	16.34	-156.70	104.72	81.91
Frame 4	1.76	14.23	-129.76	88.66	71.22

Table 4. Dynamic Magnification Factor

Floor Level	Frame 1	Frame 2	Frame 3	Frame 4
1st Floor	1.34	2.28	3.71	2.23
2nd Floor	-	2.28	3.73	2.24
3rd Floor	-	-	3.74	2.25
4th Floor	-	-	-	2.25
Mean Value	1.34	2.28	3.73	2.24

Table 5: Dynamic response using application of the Dynamic Magnification Factor

Frame	Joint Displacement (mm)	Bending Moment (KNm)	Shear Force (KN)	Axial Force (KN)
Frame 1	0.67	13.16	-3.04	-29.41
Frame 2	5.61	-28.66	26.54	-94.16
Frame 3	16.45	-114.40	89.15	-348.31
Frame 4	14.29	-111.55	81.94	-173.53

Table 6. Percentage Different in Absolute Values of Dynamic Response

Frame	Joint Displacement	Bending Moment	Shear force	Axial Force
Frame 1	0.0	65.7	181.5	38.8
Frame 2	0.0	40.8	21.8	78.3
Frame 3	0.7	27.0	14.9	325.2
Frame 4	0.4	14.0	7.6	143.7

6.0 DISCUSSION OF RESULTS

Dynamic and static analyses were carried out in four frames. Response information were recorded for the first floor, precisely for joint 1 of member 1-A and tabulated for all four frames. Dynamic Magnification factors were computed and shown in Table 4. dynamic responses obtained by using forced vibration analysis (see Table 3.) and that obtained by using the application of the Dynamic Magnification factor (See Table 5.) were compared by percentage Difference as shown in Table 6.

The results of the comparison showed that joint displacement (i.e. translations) obtained by direct forced vibration analysis are the same with those obtained by using the application of the Dynamic Magnification Factor but internal stresses obtained in both cases did not show any agreement whatsoever. Of particular importance is the staggering difference in a dynamic response of frame 3 where the dynamic axial force for member 1-A portrays tension in Table 3 and compression in Table 5 with magnitudes of 81.91KN and 348.31KN respectively.

7.0 CONCLUSIVE REMARKS AND RECOMMENDATION

In conclusion, therefore, it can be inferred that the relationship between the dynamic and static translations, in terms of the dynamic modification factor, is a direct linear variation and so the dynamic modification factor can be used to multiply static translations in frames in order to obtain the dynamic equivalent for a given forcing frequency. However, the results of the analyses showed that this relationship is not true in the case of other responses. Consequently the results of the dynamic bending moment, shear force and axial force, obtained by the magnification of their static equivalent using the dynamic magnification factor do not agree with that of the exact method. Therefore, it is recommended that actual dynamic responses, other than translations, in frames can only be obtained by using the exact method and not by the magnification of their static equivalent using the dynamic magnification factor.

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